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FROM PRELIMINARY AIRCRAFT CABIN DESIGN TO CABIN OPTIMIZATION - PART II -

Mihaela NIȚĂ[1], Dieter SCHOLZ[2]

This paper conducts an investigation towards main aircraft cabin parameters. The aim is two-fold: First, a handbook method is used to preliminary design the aircraft cabin. Second, an objective function representing the "drag in the responsibility of the cabin" is created and optimized using both an analytical approach and a stochastic approach. Several methods for estimating wetted area and mass are investigated. The results provide optimum values for the fuselage slenderness parameter (fuselage length divided by fuselage diameter) for civil transport aircraft. For passenger aircraft, cabin surface area is of importance. The related optimum slenderness parameter should be about 10. Optimum slenderness parameters for freighters are lower: about 8 if transport volume is of importance and about 4 if frontal area for large items to be carried is of importance.

These results are published in two parts. Part I includes the handbook method for preliminary designing the aircraft cabin. Part II includes the results of the optimization and the investigations of the wetted areas, masses and "drag in the responsibility of the cabin".

Keywords: preliminary cabin design, optimization, evolutionary algorithms

1. Analytical Cabin Optimization

1.1 Introduction

This section aims to determine and minimize the objective function that relates the "aircraft drag being in the responsibility of the cabin" to the fuselage slenderness parameter, l_F / d_F, (fuselage length divided by fuselage diameter) which in turn is a function of cabin layout parameters like n_{SA}, (number of seats abreast):

$$D_F = D_{0,F} + D_{i,F} = f\left(l_F(n_r), d_F(n_{SA}), \lambda_F\right) \quad . \tag{1}$$

Based on these results, a broader examination, extending on a larger number of parameters is foreseen for future work.

The objective function relates cabin parameters to fuselage parameters, with the purpose to minimize the fuselage drag and mass. This reduces fuel consumption and allows for an increase in payload.

[1] PhD, e-mail: Man.MihaelaFlorentina@yahoo.com
[2] Prof., Hamburg University of Applied Sciences, Aero - Aircraft Design and Systems Group, e-mail: info@ProfScholz.de

For sure, the fuselage shape follows cabin parameters. But at a second glance it can be seen that the empennage size also depends on cabin geometry, because the cabin length determines the lever arm of the empennage and hence the area of the horizontal and vertical tail.

The drag expressed in (1) represents the drag being in the responsibility of the cabin, consisting of zero-lift drag (surface of fuselage and tail) and induced drag (mass of fuselage and tail). Hence, it is necessary to:

- estimate fuselage drag and mass,
- perform a preliminary sizing of the empennage,
- estimate empennage drag and mass,
- calculate total drag from zero lift drag and induced drag as a function of the fuselage slenderness parameter, $\lambda_F = l_F / d_F$, which represents the objective function.

The fuselage *drag being in the responsibility of the cabin* is:

$$D_F = q\,S \cdot \left(C_{D,0,F} + k\,C_{L,F}{}^2\right)$$
$$q = \frac{1}{2}\rho V^2; \quad k = \frac{1}{\pi \cdot A \cdot e} \quad , \qquad (2)$$

where $C_{L,F} = \dfrac{2 \cdot m_F g}{\rho \cdot V^2 \cdot S_w}$.

Typical values for the aspect ratio, A, range between 3 and 8. The Oswald efficiency factor, e, ranges from 0.7 to 0.85 [1].

1.2 Fuselage Drag and Mass

For the aircraft, as well as for aircraft components, such as the fuselage, the drag calculated as the sum of zero-lift drag and induced drag is expressed through the drag coefficients

$$C_D = C_{D,0} + k \cdot C_L{}^2 \quad . \qquad (3)$$

The zero-lift drag (also called parasite drag) consists primarily of skin friction drag and is directly proportional to the total surface area of the aircraft or aircraft components exposed ('wetted') to the air [1].

There are two ways of calculating the zero-lift drag [1]: *First*, by considering an equivalent skin friction coefficient, C_{fe} which accounts for skin friction and separation drag:

$$C_{D,0,F} = C_{fe} \cdot \frac{S_{wet,F}}{S_W} \quad . \qquad (4)$$

Second, by considering a calculated flat-plate skin friction coefficient, C_f, and a form factor, F, that estimates the pressure drag due to viscous separation. This estimation is done for each aircraft component, therefore an interference factor, Q, is also considered. The fuselage drag coefficient is then:

$$C_{D,0,F} = C_{f,F} \cdot F_F \cdot Q_F \cdot \frac{S_{wet,F}}{S_W} \quad . \qquad (5)$$

The first approach considers in general the aircraft as a whole. The second approach allows a component-based examination and is potentially more accurate. Further on, each factor of the zero-lift drag will be calculated.

For the fuselage wetted area there are several calculation possibilities. Chosen was (6), from [2], which has a slenderness ratio dependency:

$$S_{wet,F} = \pi \cdot d_F \cdot l_F \cdot \left(1 - \frac{2}{\lambda_F}\right)^{2/3} \left(1 + \frac{1}{\lambda_F^2}\right) \quad . \qquad (6)$$

The form factor is given in [1] as:

$$F_F = 1 + \frac{60}{\lambda_F^3} + \frac{\lambda_F}{400} \quad . \qquad (7)$$

The fuselage has an interference factor of $Q_F = 1$, because (by definition) all other components are assumed to be related in their interference to the fuselage [1].

The friction coefficient depends on the Reynolds number, Mach number and skin roughness. The contribution to the skin friction drag is mainly depending on the extent to which the aircraft has a laminar flow on its surface. A typical fuselage has practically no laminar flow. Laminar flow normally can be found only over 10 % to 20 % of wing and tail [1]. For turbulent flow, the friction coefficient can be calculated with (8):

$$C_{f,turbulent} = \frac{0.455}{(\log_{10} Re)^{2.58}(1 + 0.144M^2)^{0.65}} \quad , \qquad (8)$$
$$Re = V\, l_F / \upsilon$$

where υ represents the kinematic viscosity of the air, which depends on the air temperature and thus flight altitude.

The drag-due-to-lift (also called induced drag) which falls in the responsibility of the cabin can be estimated first based on the fuselage-tail group weight. The lift produced by the wing in order to keep the fuselage respectively cabin in the air (noted with m_F) equals the weight of the fuselage-tail group (represented by the sum $m_f + m_h + m_v$)

$$L_F = m_F \cdot g \Rightarrow qS \cdot C_{L,F} = (m_f + m_h + m_v) \cdot g \quad . \qquad (9)$$

Thus the induced drag

$$D_{i,F} = k \cdot \frac{(m_f + m_h + m_v)^2 \cdot g^2}{qS} \quad , \qquad \text{where } f - \text{fuselage},\ h - \text{horizontal tail},\ v - \text{vertical tail} \quad . \qquad (10)$$

The mass of the fuselage can be calculated from [2]:

$$
\begin{aligned}
m_F &= 0.23 \cdot \sqrt{V_D \frac{l_H}{2d_F}} \cdot S_{wet,F}{}^{1.2} \\
V_D &= M_D \cdot a \\
M_D &= M_{CR} + \Delta M \\
\Delta M &\approx 0.05...0.09 \quad .
\end{aligned} \tag{11}
$$

l_H is the lever arm of the horizontal tail. The value is in many cases close to 50 % of the fuselage length (see Table 3). In this case, (11) can be written as a function of the slenderness parameter:

$$m_F = 0.115 \cdot \sqrt{V_D \cdot \lambda_F} \cdot S_{wet,F}{}^{1.2} \quad . \tag{12}$$

1.3 Empennage Preliminary Sizing

The empennage provides trim, stability and control for the aircraft. The empennage generates a tail moment around the aircraft center of gravity which balances other moments produced by the aircraft wing – in the case of the horizontal tail, or by an engine failure – in the case of the vertical tail. Figure 1 shows possible tail arrangements. More information with respect to the characteristics of each configuration can be found in the literature, such as [1].

Once the configuration is chosen, other parameters can and must be preliminarily estimated:

- Aspect ratio
- Taper ratio
- Sweep
- Span
- Thickness to chord ratio at tip and root

Typical values for aspect and taper ratios for the vertical and horizontal tail are indicated in Table 1. The leading-edge sweep of the horizontal tail is usually 5° larger than the wing sweep. The vertical tail sweep ranges from 35° to 55°. The thickness ratio is usually similar to the thickness ratio of the wing [1].

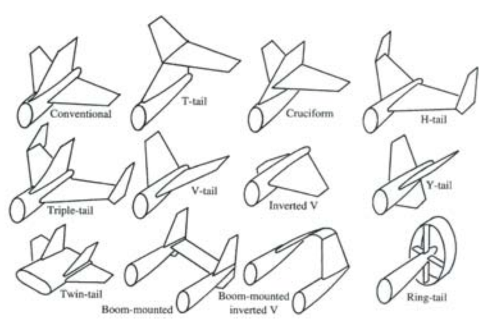

Fig. 1 Empennage configurations [1]

Table 1

Typical values for aspect and taper ratios of the empennage [1]

	Horizontal tail		Vertical tail	
	A	λ	A	λ
Fighter	3…4	0.2…0.4	0.6…1.4	0.2…0.4
Sailplane	6…0	0.3…0.5	1.5…2.0	0.4…0.6
T-Tail	–	–	0.7…1.2	0.6…1.0
Others	3…5	0.3…0.6	1.3…2.0	0.3…0.6

Further on, preliminary values of the parameters defining the empennage are required. For the initial estimation of the tail area the ‘tail volume method’ can be used. The tail volume coefficients C_H and C_V are defined as:

$$C_H = \frac{S_H \cdot l_H}{S_W \cdot c_{MAC}}; \quad C_V = \frac{S_V \cdot l_V}{S_W \cdot b} \tag{13}$$

One of the most important considerations, especially for this paper, is that the moment arm of the empennage should be as large as possible in order to have smaller empennage surfaces, and thus reduced mass and drag. The moment arm is reflected in the slenderness parameter: a longer moment arm gives a larger value for the fuselage slenderness.

Tail volume coefficients can be extracted from historical data, as showed in Table 2. The moment arms can be estimated using statistics (see Table 3).

Based on the data from Table 3, (13) can be rewritten as a function of the fuselage length. The tail surface areas in question are

$$S_H = \frac{C_H \cdot S_W \cdot c_{MAC}}{l_H}; \quad S_V = \frac{C_H \cdot S_W \cdot b}{l_V} \tag{14}$$

Table 2

Typical values for the tail volume coefficient [1]

	Horizontal C_H	Vertical C_V
Sailplane	0.50	0.02
Homebuilt	0.50	0.04
General aviation – single engine	0.70	0.04
General aviation – twin engine	0.80	0.07
Agricultural	0.50	0.04
Twin turboprop	0.90	0.08
Flying boat	0.70	0.06
Jet trainer	0.70	0.06
Jet fighter	0.40	0.07
Military cargo / bomber	1.00	0.08
Jet transport	1.00	0.09

Table 3

Statistical values for the empennage moment arms [1]

Aircraft configuration	Moment arms, l_H and l_V
Front-mounted propeller engine	60 % of the fuselage length
Engines on the wing	50-55 % of the fuselage length
Aft-mounted engines	45-50 % of the fuselage length
Sailplane	65 % of the fuselage length
Canard aircraft	30-50 of the fuselage length

1.4 Empennage Drag and Mass

The drag of the empennage can be calculated with the same procedure as for the fuselage. Equation (5) remains valid. The wetted area depends on the geometrical characteristics of the empennage ‘wing’ (horizontal and vertical) (Table 1):

$$S_{wet,H} = 2 \cdot S_{\exp,H}\left(1 + 0.25 \cdot (t/c)_r \cdot \frac{1+\tau_H \cdot \lambda_H}{1+\lambda_H}\right) \quad , \tag{15}$$

$$\tau_H = (t/c)_t / (t/c)_r$$

$$S_{wet,V} = 2 \cdot S_{\exp,V}\left(1 + 0.25 \cdot (t/c)_r \cdot \frac{1+\tau_V \cdot \lambda_V}{1+\lambda_V}\right) \quad . \tag{16}$$

$$\tau_V = (t/c)_t / (t/c)_r$$

The tail thickness ratio is usually similar to the wing thickness ratio; for high speed aircraft the thickness is up to 10 % smaller [1]. According to [1] the root of the wing is about 20 % to 60 % thicker than the tip chord (which means τ is about 0.7).

The form factor of the empennage is the same as the form factor for the wing:

$$F = \left[1 + \frac{0.6}{x_t}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right] \cdot \left[1.34 \cdot M^{0.18} \cdot \cos(\varphi_m)^{0.28}\right] \quad , \tag{17}$$

where φ_m represents the sweep of the maximum-thickness line and x_t is the chord-wise location of the airfoil maximum thickness point.

The position of maximum thickness, x_t, is given by the second digit of the NACA four digit airfoils, which are frequently chosen for the empennage.

The interference factor for the conventional empennage configurations has the value Q = 1.04. An H-Tail has Q = 1.08 and a T-Tail has Q = 1.03 [1].

The empennage may have laminar flow over 10 % to 20 % of its surface. The friction coefficient for the laminar flow is:

$$C_{f,laminar} = 1.328/\sqrt{Re} = 1.328\sqrt{(V \cdot c_{MAC,H,V})/\upsilon} \quad . \tag{18}$$

The mean aerodynamic chord of the horizontal, respectively vertical tail becomes the characteristic length for the Reynolds number.

The final value of the friction coefficient accounts for the portions of turbulent and laminar flow:

$$C_f = k_{laminar} \cdot C_{f,laminar} + (1 - k_{turbulent}) \cdot C_{f,turbulent} \tag{19}$$

The empennage mass (horizontal and vertical tail) is given by [2]:

$$m_H = k_H \cdot S_H \cdot \left(62 \cdot \frac{S_H{}^{0.2} \cdot V_D}{1000 \cdot \sqrt{\cos\varphi_{H,50}}} - 2.5\right) \quad ; \tag{20}$$

$$m_V = k_V \cdot S_V \cdot \left(62 \cdot \frac{{S_V}^{0.2} \cdot V_D}{1000 \cdot \sqrt{\cos \varphi_{V,50}}} - 2.5 \right) \quad , \tag{21}$$

with:

$k_H = 1$ *for fixed stabilizers*

1.1 *for variable incidence tails*

$k_V = 1$ *for fuselage mounted horizontal tails*

$k_V = 1 + 0.15 \cdot \frac{S_H \cdot z_H}{S_V \cdot b_V}$ *for fin mounted stabilizers*

z_H – *height of the horizontal tail above the fin root*

φ_{50} – *sweep angle at* 50 % *of the chord* .

When putting together the equations listed above, the function of the "total drag being in the responsibility of the cabin" is obtained.

1.5 Objective Function

The basic objective function of the fuselage-tail group has the form:

$$D_F = D_F(l_F, d_F) \quad . \tag{22}$$

Optimizing this function means finding that combination of fuselage length and diameter which produces the lowest drag. Hypothesizes taken into account are:

1) The aircraft has a conventional configuration.
2) The results from preliminary aircraft sizing are known.

Preliminary sizing of the aircraft has the primary purpose to obtain optimum values for wing loading and thrust to weight ratio. It delivers the main input parameters required for the cabin optimization process: the wing area, the aircraft cruise speed and the cruise altitude. For obtaining the results in this paper the wing area and Mach number of the ATR 72 were used.

1.6 Optimization Results

1.6.1 Total Drag of the Fuselage-Tail Group

This sub-section calculates the zero-lift drag of the fuselage-tail group and the induced drag of the fuselage-tail group, which takes into account lift to carry the fuselage-tail mass.

The variation of the total drag of the fuselage-tail group with the fuselage length and diameter is shown in Figure 2. In order to have a better visualization of the results, it makes sense to illustrate the relative drag. For passenger transport aircraft meaningful conclusions can be drawn based on the representation of the drag relative to the *cabin surface* – drag divided by the product ($l_F \cdot d_F$) – as a function of the fuselage length and diameter (see Figure 3). The cabin surface depends directly on the number of passengers; therefore drag relative to cabin surface plays an important role for this paper.

For freighter aircraft, a better visualization of the dependency can be obtained when the “drag in the responsibility of the cabin” is represented relative to the frontal area, respectively volume (see Figures 4 and 5).

It is to be noticed that the friction coefficient and form factor used to calculate the zero-lift drag of the empennage highly depend on the geometry, which in turn depend on the geometry of the aircraft wing (as shown in Section 1.4). The influence of the empennage on the total drag is first of all contained in the wetted area estimation, while the type of surface and profile where considered for a selected aircraft (i.e. the ATR 72).

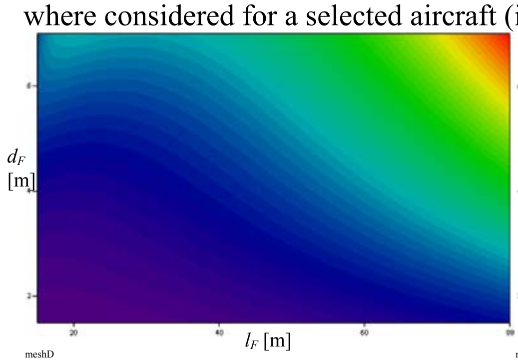

Fig. 2 Total drag of the fuselage-tail group as a function of fuselage length and diameter

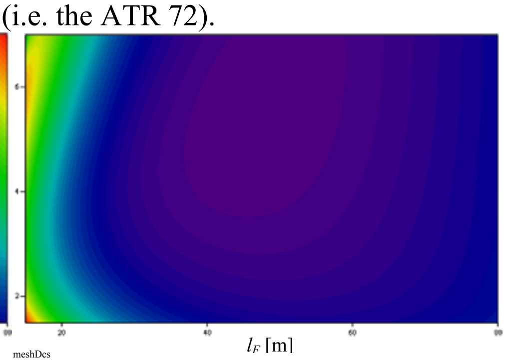

Fig. 3 Total drag of the fuselage-tail group relative to ($l_F \cdot d_F$) as a function of l_F and d_F

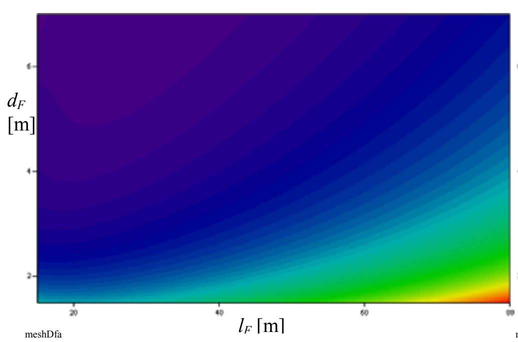

Fig. 4 Total drag of the fuselage-tail group relative to ($d_F^2 \cdot \pi/4$) as a function of l_F and d_F

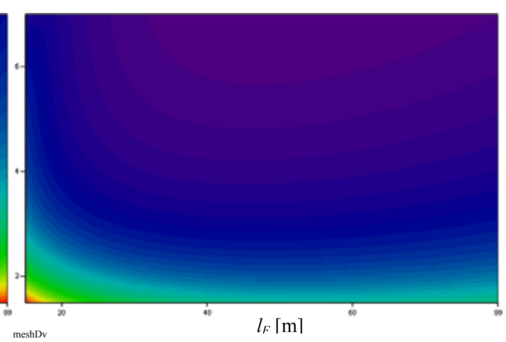

Fig. 5 Total drag of the fuselage-tail group relative to ($d_F^2 \cdot l_F \cdot \pi/4$) as a function of l_F and d_F

The following conclusions can be drawn:

- Figure 2 presents an expected drag variation: the smaller the fuselage the smaller the drag; the variation shows also that it’s better to keep the fuselage longer rather than stubbier.
- Figure 3 indicates a zone of minimum relative drag for fuselages with lengths between 30 (e.g. A 318) and 70 meters (e.g. A 340, A380). Extremities (very small length, very high diameters) produce significant relative drag.
- Figs. 4 and 5 reflect the cargo transportation requirements: for transporting large items it is important to have a large fuselage diameter; if volume

transport is of importance, a large diameter at a length of about 50 m would be best.

It is interesting to note that (maybe with exception of Beluga) no civil freighter exists that was designed specifically for this purpose. All civil freighters have been derived from passenger aircraft, while military freighters play only a minor role for civil freight transport. If the aircraft manufacturer would like to keep the advantages of this practice, the passenger transport aircraft – or the future freighter – should be designed according to the range of dimensions *common to both graphs* shown in Figures 3 and 5. This range is approximately $l_F \in [40,60]; d_F \in [3.8,7]$, which means that B747 or A380 type of aircraft are better suitable for freighter conversions than single aisle aircraft.

1.6.2 Total Drag of the Fuselage

This sub-section calculates the zero-lift drag of the fuselage and the induced drag of the fuselage, which takes into account lift to carry the fuselage mass.

In order to be able to relate the drag to the slenderness and draw other meaningful conclusions, the empennage contribution will be neglected in this section (see (23) and Figures 6 to 9):

$$D_F = f(\lambda_f, l_F \cdot d_F) \quad (23)$$

According to Figure 6 it seems that for larger fuselage length-diameter products the slenderness should lie between 5 and 10 for an optimal drag. For smaller aircraft it seems the designer has the flexibility to choose a convenient slenderness, according also to other criteria than drag. When looking at the fuselage drag relative to cabin surface a zone of optimal slenderness can be delimited. Previously, Figure 4 allowed us to favor longer fuselages instead of stubbier ones. In the same way, Figure 6 delimits a range between 5 and 10 for the value of slenderness for the larger aircraft. Figure 7 shows now, that for passenger transportation, smaller aircraft can have an increased slenderness up to 16.

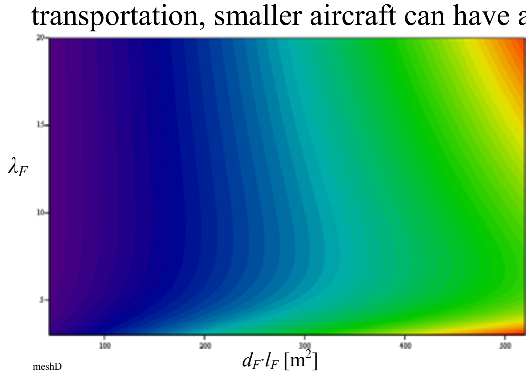

Fig. 6 Total fuselage drag as a function of the slenderness and ($l_F \cdot d_F$)

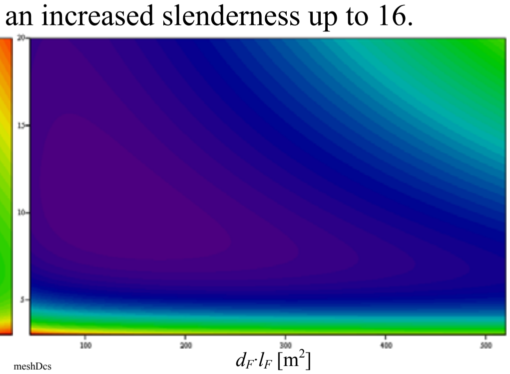

Fig. 7 Total fuselage drag relative to ($l_F \cdot d_F$) as a function of the slenderness and ($l_F \cdot d_F$)

Drag relative to frontal area and volume reflect the characteristics of cargo transport aircraft. Figures 8 and 9 show that the design of a freighter, in comparison to a passenger transport aircraft, could look quite different: a much smaller slenderness would be required (up to 7).

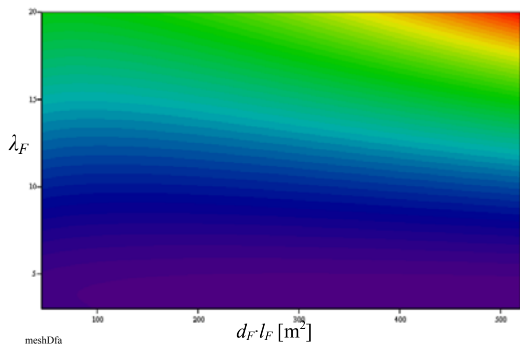

Fig. 8 Total fuselage drag relative to frontal area ($d_F^2 \cdot \pi/4$) as a function of λ_F and ($l_F \cdot d_F$)

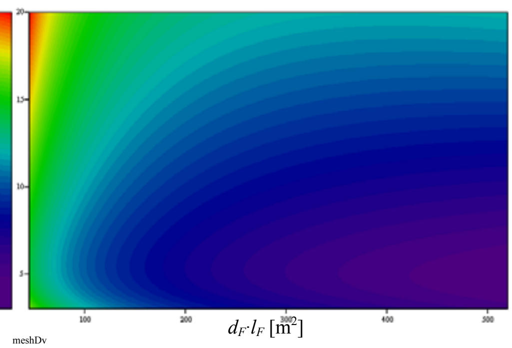

Fig. 9 Total fuselage drag relative to volume ($l_F \cdot d_F^2 \cdot \pi/4$) as a function of λ_F and ($l_F \cdot d_F$)

1.6.3 Considerations with Respect to the Fuselage Wetted Area Calculation

In order to understand how the fuselage shape affects the drag, the zero lift drag was expressed as a function of the slenderness parameter and the influence of the empennage was removed (see Section 1.6.4). The estimation method used for the wetted area - as a component of the drag, depending on the slenderness – has a great impact on the results. Three different ways of calculating $S_{wet,F}$ were chosen:

- Torenbeek approach [2], as given in (6);
- Three-parts-fuselage approach, as indicated in Figure 10 and (24);
- Simple approach (aircraft as a cylinder) as indicated in (25).

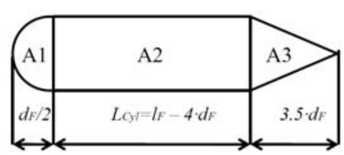

Fig. 10 Three parts approximation for the calculation of the wetted area

$$\begin{cases} A_1 = \pi \cdot d_F^{\,2}; \;\; A_2 = L_{Cyl} \cdot \pi \cdot d_F; \;\; A_3 = (d_F/2) \cdot s\pi \\ s = \sqrt{(3.5 \cdot d_F)^2 + (d_F/2)^2} \\ S_{wet,F} = A_1 + A_2 + A_3 \end{cases} \tag{24}$$

$$S_{wet,F} = \pi \cdot d_F \cdot l_F \tag{25}$$

The visualization of the three wetted areas in a single graph, relative to $d_F^{\,2}$, shows the different validity domains for each case.

The following observations can be extracted from Figure 11:

- The simple approach (red) is valid also for small slenderness λ_F.
- The Torenbeek approach and the 3-parts aircraft approach are valid for the rest of the aircraft, but not for aircraft with a small diameter.
- The Torenbeek approach (green) is valid for aircraft having a slenderness $\lambda_F > 2$.
- The 3-parts approach (blue) is valid for aircraft having a slenderness[3] $\lambda_F > 4$.

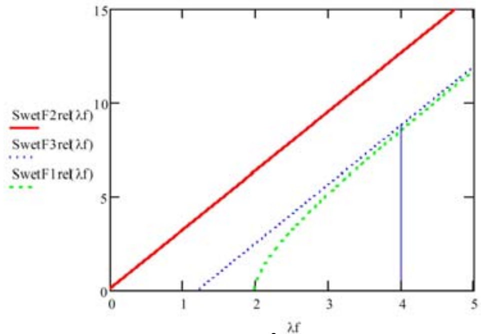

Fig. 11 Wetted area of the fuselage relative to $d_F^{\ 2}$ as a function of the slenderness parameter: Torenbeek (green); Fuselage as a cylinder (red); Fuselage as a sum of cockpit, tail and cabin (blue)

1.6.4 Fuselage Zero-Lift Drag

For each type of wetted area, the zero lift drag of the fuselage was then represented relative to[4] : Cabin surface: $d_F \cdot l_F$; Cabin frontal area: $\pi \cdot d_F^{\ 2} / 4$

A summary of the studied cases is presented in Table 4.

Table 4

Cases studied for the illustration of the zero lift drag as a function of the slenderness

ID	Case	Variable Name	Figure
1	Torenbeek approach		
1.a	Relative to cabin surface	$D_{0,Tcs}$	Fig. 12
1.b	Relative to frontal area	$D_{0,Tfa}$	
2	Three-parts aircraft approach		
2.a	Relative to cabin surface	$D_{0,Pcs}$	Fig. 13
2.b	Relative to frontal area	$D_{0,Pfa}$	
3	Simple approach		
3.a	Relative to cabin surface	$D_{0,Scs}$	Fig. 14
3.b	Relative to frontal area	$D_{0,Sfa}$	

The optimal values of the slenderness parameter, calculated with the wetted area from Torenbeek, are as follows (see Fig. 12):

- $\lambda_F = 9.8$ when the cabin surface is constant (red)
- $\lambda_F = 3.5$ when the frontal area is constant (blue)

3 The cylindrical part of the fuselage becomes too small and the equation is no longer valid (see Figure 10).

4 The zero lift drag relative to volume cannot be expressed as a function of the slenderness.

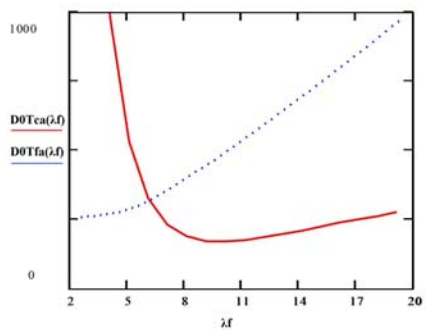

Fig. 12 Fuselage Zero Lift Drag as a function of slenderness: Cases 1.a, 1.b

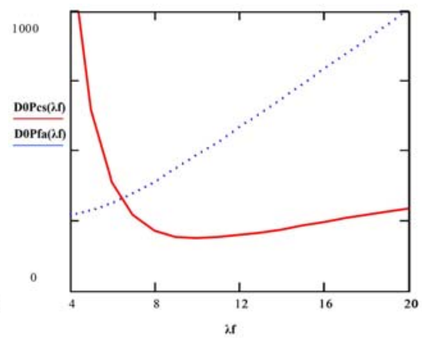

Fig. 13 Fuselage Zero Lift Drag as a function of slenderness: Cases 2.a, 2.b

The optimal values of the slenderness parameter, calculated with the wetted area from (24), are as follows (see Fig. 13):

- $\lambda_F = 10.7$ when the cabin surface is constant (red)
- $\lambda_F = 4$ when the frontal area is constant (blue)

The optimal values of the slenderness parameter, calculated with the wetted area from (25), are as follows (see Fig. 14):

- $\lambda_F = 16.4$ when the cabin surface is constant (red)
- $\lambda_F = 5$ when the frontal area is constant (blue)

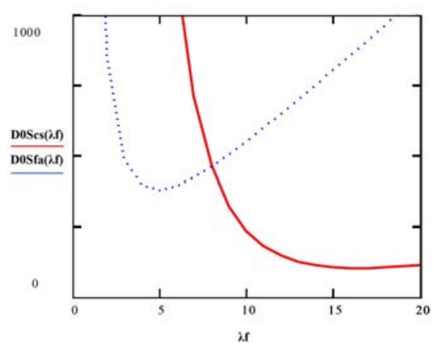

Fig. 14 Fuselage Zero Lift Drag as a function of fuselage slenderness: Cases 3.a, 3.b

However, this simple consideration (25) leads to larger, unrealistic values for the slenderness parameter in comparison to the values in the first two cases.

1.6.5 Considerations with Respect to the Fuselage Mass Calculation

The same type of evaluation that was made for the wetted areas can be conducted for the mass estimations, by looking at different authors. So far the Torenbeek approach was used to calculate the fuselage mass (see (12)). Another approach is indicated in [3] as shown in (26), called "Markwardt's approximation".

$$m_F = 13.9 \cdot S_{wet,F} \cdot \log(0.0676\, S_{wet,F}) \qquad (26)$$

Equation (26) represents the analytical interpretation of the statistical data gathered in Fig. 15.

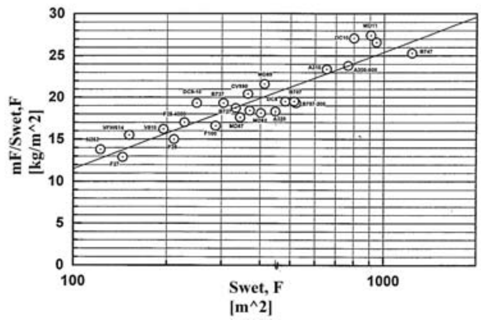

Fig. 15 $m_F/S_{wet,F}$ as a function of the wetted area [3]

When representing the two possibilities of expressing the mass relative either to d_F^2 or to $d_F \cdot l_F$ (see Fig. 16), the following observations can be extracted[5]:

- The mass is zero for $\lambda_F = 2$; this results from the wetted area equation.
- Markwardt's approximation climbs faster than Torenbeek's approximation, which means the mass penalty with Markwardt's approximation is greater for slenderness values of conventional fuselages.
- Torenbeek's approximation becomes unrealistic for large slenderness values.

Reference [4] presents the results of an investigation towards different mass estimations for aircraft components. For aircraft investigated, Markwardt's approach (26) returned a deviation of approximately $\pm 4\,\%$ from the original aircraft mass data, while Torenbeek's approach deviated from $-9\,\%$ up to $-20.6\,\%$. The wetted area calculated from Torenbeek (6) showed a deviation from $-4.7\,\%$ up to $-8.6\,\%$.

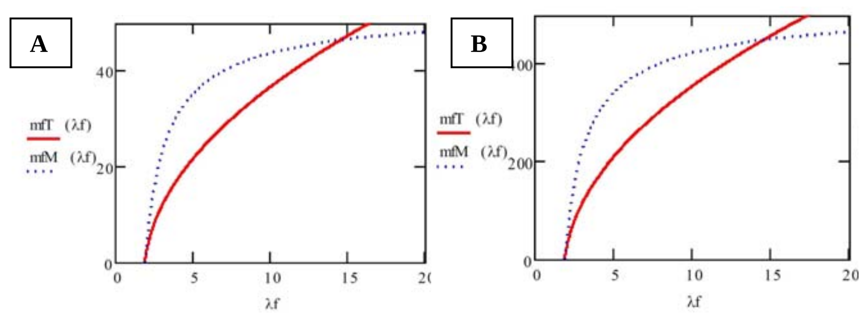

Fig. 16 Relative fuselage mass after Torenbeek (red) and Markwardt (blue) as a function of the slenderness parameter: A – relative to $l_F \cdot d_F$; B – relative to d_F^2

[5] In both cases the wetted area was calculated with Equation (6) [2].

1.6.6 Considerations with Respect to the Cabin Parameters

The basic requirement when designing the fuselage is the number of passengers (or the payload) that need to be transported. For a given (i.e. constant) number of passengers, it makes sense to optimize the number of seats abreast in connection to the fuselage drag and fuselage slenderness.

An easy way of calculating the number of seats abreast n_{SA} for a given number of passengers is given by [1] (see (1), Section 2.2, Part I). A practical question arises: if the so calculated n_{SA} has the value of 5.76 (as it is the case for the A 320 aircraft, which has 164 passengers in a two class configuration – see Table 5) which is the optimal value between the value of 5 and the value of 6? Table 5 gathers some examples in order to compare the calculated value with the real value of the n_{SA} parameter.

In order to find the optimum and to answer the above question, the following procedure was followed:

- A reference value of the parameter n_{SA} was calculated from (1).
- The resulting value was varied under and above the reference value.
- For the obtained values the corresponding fuselage length and diameter were calculated with (4) and (8), from Part I.
- For each length-diameter pair the drag and the drag relative to the cabin surface was calculated with (22) and graphically represented.

The results are indicated in Figs. 17 to 20.

Table 5

n_{SA} parameter for selected commercial transport aircraft

Aircraft type	Number of passengers	n_{SA} calculated from Reference [1]	n_{SA} real
ATR 72	74	3.87	4
A 318	117	4.87	6
A 319	134	5.21	6
A 320	**164**	**5.76**	**6**
A321	199	6.35	6
A330-300	335	8.24	8
A 340-600	419	9.21	8

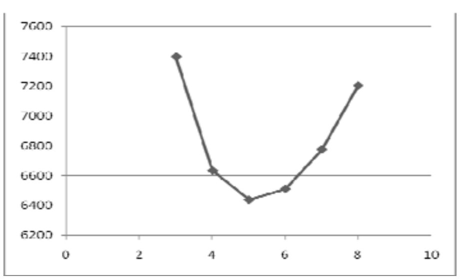

Fig. 17 Total drag of the fuselage-tail group as a function of n_{SA} (n_{PAX} = const)

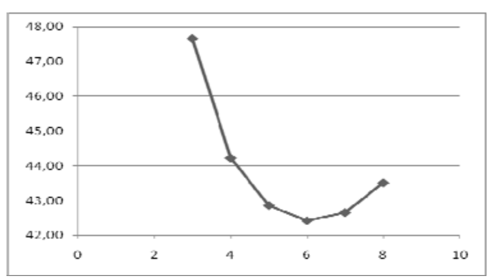

Fig. 18 Total drag of the fuselage-tail group rel. to cabin surface ($l_F \cdot d_F$) as a function of n_{SA} (n_{PAX} = const)

For the reference aircraft (A320), with 164 passengers in a standard configuration, Fig. 17 indicates that the value of 5 provides a slightly smaller drag than the real value of 6 seats abreast. On the other hand, when looking at the relative drag (Fig. 18), the value of 6 is favored.

With this approach, the effect of the empennage can now also be expressed in connection with the slenderness. The total drag and the drag relative to cabin surface of the fuselage-empennage group are shown in Figures 19 and 20. The first chart indicates an optimal value of 12.5 while the second chart indicates an optimal value of 10.2.

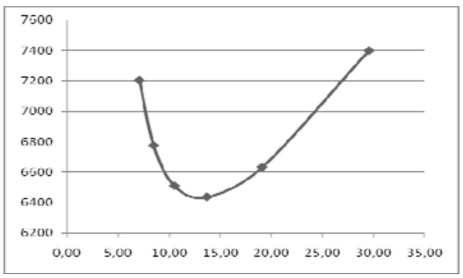

Fig. 19 Total drag of the fuselage-tail group function of λ_F (n_{PAX} = const) for the selected values of the parameter n_{SA}

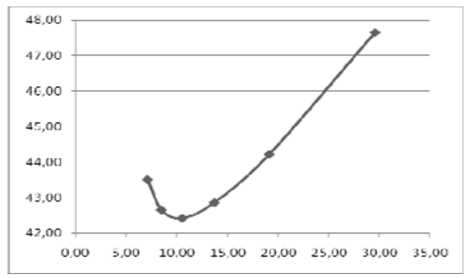

Fig. 20 Total drag of the fuselage-tail group as a rel. to ($l_F \cdot d_F$) as a function of λ_F (n_{PAX} = const) for the selected values of the parameter n_{SA}

2. Stochastic Cabin Optimization

2.1 Introduction

This section presents the results after coding and running a genetic algorithm with the purpose to find the optimal fuselage shape that minimizes the "drag in the responsibility of the cabin". Although this approach is especially valid for complex objective functions, depending on a large number of variables, the purpose here is to apply the algorithms in a simple case and to compare the results with the ones obtained in Section 1. Therefore the same two variables are intended to be optimized here: the fuselage length and diameter.

All the variations made to find an optimum start from a baseline aircraft model described by corresponding input values for the variables. The baseline model used for this research is the ATR 72 – a propeller driven commercial regional transport aircraft.

2.2 Chromosome-Based Algorithms

The values of each parameter are coded such as the genes are coded in the chromosomal structure. Each variable is associated with a bit-string with the length of 6. This length is argued by [5]. Two variables, each represented by 6 bits give $(2 \cdot 6)^4$ = 20736 possible fuselage - empennage shape variations that minimize drag.

Starting from the aircraft baseline, an initial population of chromosomes, representing random values within an interval for each parameter, is defined. In all the chromosome-based routines, the initial population is created by using a digital random number generator to create each bit in the chromosome string. Then, this string is used to change the input variables of the baseline design, creating a unique "individual" for each chromosome string defined. Where the optimizers differ is how they proceed after this initial population is created. Selection of the "best" individual or individuals is based primarily on the calculated value of the objective function [5].

The next essential step is the concept of crossover, equivalent to mating in the real world of biology. Crossover is the method of taking the chromosome/gene strings of two parents and creating a child from them. Many options exist, allowing a nearly limitless range of variations on GA methods [5]:

Single-Point Crossover – The first part of one parent's chromosome is united with the second part of the other's. The point where the chromosome bit-strings are broken can be either the midpoint or a randomly selected point.

Uniform Crossover – Combines genetic information from two parents by considering every bit separately. For each bit, the values of the two parents are inspected. If they match (both are zero or both are one), then that value is recorded for the child. If the parents' values differ, then a random value is selected.

Parameter-Wise Crossover – Combines parent information using entire genes (each 6 bits) defining the design parameters. For each gene, one parent is randomly selected to provide the entire gene for the child.

For the selection of the parents there are as well several possibilities [5]:

Roulette Selection – The sizes of the "slots" into which the random "ball" can fall are determined by the calculated values of the objective function based on actual data.

Tournament Selection – Selects four random individuals who "fight" one-vs.-one; the superior of each pairing is allowed to reproduce with the other "winner".

Breeder Pool Selection – A user-specified percentage (default 25%) of the total population is then placed into a "breeder pool"; then, two individuals are randomly drawn from the breeder pool and a crossover operation is used to create a member of the next generation.

Best Self-Clones with Mutation. A type of evolutionary algorithm, different than genetic algorithms through the lack of crossover, uses the concept of 'queen' of the population. The queen is the variant which gives best values for the objective functions. She is the only one allowed to further reproduce. The next generation is created by making copies (clones) of the queen's chromosome bit-string and applying a high mutation rate to generate a diverse next generation.

Monte Carlo Random Search. Using the same chromosome/gene string definition different versions are randomly created and analyzed, without considering any evolutionary component. Due to the binary definition of the design variables, a number of $2^{(2\cdot 6)}$ variants can be analyzed. However, in practice, the analysis is reduced to a smaller number (Reference [5] generated 20 population packages of 500 individuals each, which yields a number of 20000 aircraft variants to be analyzed out of the total design space).

2.3 Results

The aim of the genetic algorithm is to find the fuselage length and diameter which minimize the two variable objective function plotted in Figure 3. It is to be remembered that Figure 3 shows the variation of the total drag of the fuselage-tail group relative to cabin surface. The values read from the plot are than easily compared with the results generated by the genetic algorithm. The advantages of the Genetic Algorithms are, however, decisive when the objective functions have more than two variables and plotting is no longer possible.

The procedure used to program the genetic algorithm that finds the best values for the two variables is shown in Figure 21. The detailed steps followed for programming the algorithm are described in Table 6, while the results are listed in Table 7.

The following important observations can be extracted:

- The Roulette selection is made for 90% of the members, while for the rest 10%, the best parents are directly chosen
- If the percent of the very good members going directly to the next generation is to high, then the diversity of the members drops considerably and the risk of a convergence towards local (instead of global) minimums grows.
- There is no convergence criteria – after a relatively small number of generations (imposed from the beginning), no significant improvement in the values of the objective function is registered.
- The optimal number of generations, the optimal number of members for each generation and the percent of the very good members going directly to the next generation must be found based on experience.

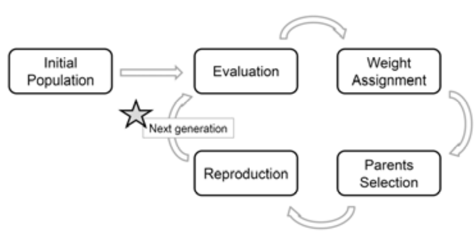

Fig. 21 The procedure used for programming the genetic algorithm (Based on [6])

The shape of the objective function plays a decisive role in choosing the right optimization algorithm. If the shape is rather linear, then there is no risk in finding just a local minimum. However, if the function is very complicated then the stochastic algorithms, such as GA, are better, as the risk of finding just a local minimum decreases.

Table 6

The steps of the genetic algorithm

Input data	- Number of bits for each chromosome[1] - Number of generations - Number of members for each generation - Definition domain for each variable of the objective function
Algorithm steps	- Creation of the initial population of members (first generation): o The crossover is made by concatenating randomly selected numbers[2,3] between 0 and 2^n-1. - Evaluation of the objective function[4]: o The numbers are scaled to the definition domain. - Creation of the next generations[5] (chosen was the Roulette method for the parents selection): o Calculation of a total weight[6] representing the total 'surface' of the roulette, where each member has a partial surface proportional to its weight[7]. o Random generation of a number between 0 and the total weight[8] for choosing the first parent. o The same procedure for the second parent. - Crossover of the chromosomes of the two parents[9] - Evaluation of the objective function[10]
Out-put data	- After the creation of the last generation, display of the best values of the objective functions, and the values of the corresponding variables

[1] A chromosome is associated with each variable of the objective function.
[2] n represents the number of bits contained by each chromosome (6 bits were chosen in this case)
[3] For the concatenation to be possible, the numbers are first transformed in base 2 numbers.
[4] For the evaluation the numbers are transformed back in base 10.
[5] Every generation represents the result of the crossover of the members from the previous generations.
[6] If it is intended to find the maximal value of the objective function, then the total weight represents the sum of the values of the objective function for each member of the population; if the purpose is to find the minimal value (our case) then the total weight represents the sum of the inverse of these values.
[7] In other words, proportional to how 'good' the value of the objective function is for the respective member.
[8] The better a member is, the greater the surface is, and so the chances to be selected become greater as well.
[9] One chromosome from one parent and one from the other (for a two variable function).
[10] In the same way as for the first generation.

In Figure 3 a zone of minimum relative drag can be identified. In practice it is difficult to 'read' the optimum values for fuselage length and diameter. The use of genetic algorithms brings its contribution in detecting the most likely minimum value of the drag and the corresponding fuselage dimensions with enough (predefined) accuracy. The results listed in Table 7, corresponding to a slenderness of $\lambda_F = 8.85$ match with the minimum zone indicated in Fig. 3.

Table 7

Results of the genetic algorithm

Input Parameters	- Number of bits for each chromosome: 6 - Number of generations: 6 - Number of members for each generation: 100 - Definition domain for each variable of the objective function: [,] for l_F, [,] for d_F		
Variables	l_F	d_F	Drag/(l_F*d_F)
Generation 1	41.4286	**5.7286**	41.8312
Generation 2	50.5211	4.7190	41.7160
Generation 3	51.4286	5.3079	41.5289

Generation 4	47.8571	5.4762	41.2689
Generation 5	**50.7143**	**5.7286**	**41.2655**
Generation 6	**50.7143**	**5.7286**	**41.2655**
Observations	The last two generations provide identical results, up to the 4th digit behind the decimal point, showing the desired convergence. Only six generations are required to obtain an optimum, due to the small number of variables of the objective function. For such a simple function, with no local minimums, the GA approach does not represent the optimal choice. However, this exercise sets the basis for future work.		

3. Summary and Conclusion

This paper dealt with two major aspects related to the aircraft cabin:

1) The cabin preliminary design, with the aim to describe the basic methodology, as part of aircraft design.
2) The cabin optimization, with the aim to find the optimum of relevant parameters.
3) With respect to cabin optimization, this paper sought the answer to the following questions:
4) What length minimizes the drag given a certain maximum diameter?
5) What slenderness minimizes the drag and the relative drag given a certain number of passengers?
6) What number of seats abreast is optimal given a certain number of passengers?
7) Which is the influence of the wetted area calculation method upon the results?
8) Which is the influence of the mass calculation method upon the results?

In order to find the answers, the fuselage "drag being in the responsibility of the cabin" was calculated.

Two approaches were selected to conduct the cabin optimization:

An in depth analytical approach, based on the available handbook methods, was used as *basic method*.

An exemplarily stochastic approach, based on chromosomal algorithms, was used as *reference method*, for the case of further extension of the research.

A two variable objective function was used in both cases. The use of two variables – either the fuselage length and fuselage diameter, or the fuselage slenderness and fuselage length multiplied by diameter – allowed plotting and therefore the visualization of each variation, and, as a consequence, no difficulty was encountered in reading the minimum from the plot.

In order to find an exact number of the minimum, an optimization method had to be applied. A Genetic Algorithm was chosen which confirmed the minimum of the plot and yielded an accurate number for the minimum depending on the number of iterations.

The results are summarized in Table 8. The main observation is that the slenderness parameter for freighter aircraft should be considerable smaller than for civil transport aircraft, especially if large items are to be transported and hence frontal area is of importance.

For a passenger aircraft, the results show that a slenderness of about 10 minimizes the "drag in the responsibility of the cabin" relative to cabin surface. For a freighter aircraft, a slenderness of about 4 minimizes the "drag in the responsibility of the cabin" relative to frontal area. A slenderness of about 8 minimizes the "drag in the responsibility of the cabin" relative to cabin volume.

Table 8

Summary of results

Aircraft	Drag relative to…	Model	Drag calculated with... zero-lift drag	Drag calculated with... induced drag	Plot	d_F	l_F	λ_F	Remark
Pax	cabin surface area	Fuselage-Tail	x	x	Fig. 3	5.7286	47.8571	8.85	Genetic
Freighter	frontal area	Fuselage-Tail	x	x	Fig. 4	7	25	3.6	Algorithm
Freighter	volume	Fuselage-Tail	x	x	Fig. 5	7	50	7.1	
Pax	cabin surface area	Fuselage	x	x	Fig. 7			10.0	
Freighter	frontal area	Fuselage	x	x	Fig. 8			3.0	
Freighter	volume	Fuselage	x	x	Fig. 9			5.0	
Pax	cabin surface area	Fuselage	x		Fig. 12			9.8	Torenbeek
Freighter	frontal area	Fuselage	x		Fig. 12			3.5	Torenbeek
Pax	cabin surface area	Fuselage	x		Fig. 13			10.7	Three-parts
Freighter	frontal area	Fuselage	x		Fig. 13			4.0	Three-parts
Pax	cabin surface area	Fuselage	x		Fig. 14			16,4	Simple
Freighter	frontal area	Fuselage	x		Fig. 14			5.0	Simple
Pax	cabin surface area	Fuselage-Tail	x	x	Fig. 20			10.2	n_{SA} variation

In order to obtain more accurate results, a multidisciplinary approach would be required. Cabin and fuselage design should be considered as part of the whole aircraft design sequence. In this way all "snow ball" effects could be accounted for. The use of stochastic optimization algorithms seems to be a good solution for multidisciplinary design optimization. This approach should be broadened and is foreseen for the future work.

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