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The Mass Growth Factor – Snowball Effects in Aircraft Design

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Project

The Mass Growth Factor – Snowball Effects in Aircraft Design

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Abstract

Purpose – This project work shows a literature survey, clearly defines the mass growth factor, shows a mass growth iteration, and derives an equation for a direct calculation of the factor (without iteration). Definite values of the factor seem to be missing in literature. To change this, mass growth factors are being calculated for as many of the prominent passenger aircraft as to cover 90% of the passenger aircraft flying today. The dependence of the mass gain factor on requirements and technology is examined and the relation to Direct Operating Costs (DOC) is pointed out.
Methodology – Calculations start from first principles. Publically available data is used to calculate a list of mass growth factors for many passenger aircraft. Using equations and the resulting relationships, new knowledge and dependencies are gained.
Findings – The mass growth factor is larger for aircraft with larger operating empty mass ratio, smaller payload ratio, larger specific fuel consumption (SFC), and smaller glide ratio. The mass growth factor increases much with increasing range. The factor depends on an increase in the fixed mass, so this is the same for the payload and empty mass. The mass growth factor for subsonic passenger aircraft is on average 4.2, for narrow body aircraft 3.9 and for wide body aircraft (that tend to fly longer distance) 4.9. In contrast supersonic passenger aircraft show a factor of about 14.
Practical implications – The mass growth factor has been revisited in order to fully embrace the concept of mass growth and may lead to a better general understanding of aircraft design.
Social implications – A detailed discussion of flight and aircraft costs as well as aircraft development requires detailed knowledge of the aircraft. By understanding the mass growth factor, consumers can have this discussion with industry at eye level.
Originality/value – The derivation of the equation for the direct calculation of the mass growth factor and the determination of the factor using the iteration method for current aircraft was not shown in the examined literature.

The Mass Growth Factor - Snowball Effects in Aircraft Design

Task for the project

Background

The mass growth factor is fundamental for the preliminary design of the aircraft. It is usually defined as the ratio of an increase in total mass (take-off mass) due to an arbitrary increase in local mass (empty mass), which is determined after a complete iteration in the aircraft design to achieve the original performance requirements (payload and range). The iteration of the aircraft construction provides a further increase of the take-off mass after each loop, so that an initial (local) mass increase worsens the situation like a snowball turning into an avalanche. Hence, the pseudonym snowball effect. The concept of the mass growth factor is probably as old as aviation. It was intensively discussed from the 1950s to the 1970s and is still mentioned repeatedly today. Nevertheless, it seems not to be understood well enough today. Perhaps its importance has decreased due to modern computing power, which provides quite accurate mass estimates in every design phase, but keeps the engineer from having a feel for the numbers.

Task

The task of this project work is to determine the mass growth factor. This shall be done by using the iteration method. Furthermore an equation shall be derived, with which the mass growth factor can be calculated directly. The calculation methods shall be used to calculate mass growth factors for many of the common passenger aircraft. The following sub items shall be considered:

- Literature research: Collecting previous findings
- Derive methods for calculating the mass growth factor
- Determining the mass growth factor for passenger aircraft flying today
- Sensitivity test of the mass growth factor

The results should be documented in a report. When preparing the report, the relevant DIN standards must be observed.

Content

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List of Symbols

$m_{take-off}$	Take-off mass
m_{drive}	Mass of the drive system
$m_{structure}$	Mass of the structure
m_{fuel}	Fuel mass
W_p	Propulsion system weight
W_g	Gross weight
W_f	Fuel weight
W_{str}	Structure weight
W_F	Fuel weight
W_{S+VE}	Weight of Structure and Variable Equipment
W	Gross weight
F	Fixed weight
S	Size
n	Strength or structural criteria
Q	Quality
P	Performance
W_g	Gross weight
W_o	Fixed weight
m_F	Fuel Mass
m_{MPL}	Maximum Payload
m_{PL}	Payload
m_{MTO}	Maximum take-off mass
m_{OE}	Operating empty mass
Δm_L	Local mass growth
Δm_G	Global mass growth
k_{MGF}	Mass growth factor
m_{ZP}	Zero Passenger Weight
c	Specific fuel consumption
R	Range
B	Breguet factor
E	Glide ratio in cruise flight
v_{cr}	Cruising speed
m_1	Take-off mass
m_2	Landing mass
C_{DEP}	Depreciation
C_{INT}	Interest
C_{INS}	Insurance
C_F	Fuel
C_M	Maintenance
C_C	Crew

C_{FEE}	Fees and charges
$C_{a/c,a}$	Aircraft annual costs
$n_{t,a}$	Number of Flights per year
$C_{s,m}$	Seat-mile costs
$C_{a\ /c,t}$	Aircraft trip cost
n_{PAX}	Number of passengers
$U_{a,f}$	Annual aircraft use
t_f	Defined aircraft time

Indices

MTO	Maximum Take-Off
OE	Operating empty
PL	Payload
cr	Cruise
a/c	Aircraft
a	Annual Costs
t	Flight, cycle or trip

List of Abbreviations

SAWE	Society of Allied Weight Engineers
LTH	Luftfahrttechnisches Handbuch
MGW	Mass growth
TLAR	Top Level Aircraft Requirements
DOC	Direct Operating Costs
GF	Growth Factor
MD	McDonnell Douglas
A	Airbus
ATR	Avions de Transport Régional
DHC	De Havilland Canada
neo	New Engine Option
PAX	Passenger
SFC	Specific Fuel Consumption
TU	Tupolev

Definitions

Preliminary sizing

The project phase of aircraft development consists of the activities preliminary sizing and design. The most important design parameters are determined in the preliminary sizing of the aircraft.

The preliminary sizing according to ... can be carried out without the need for an exact aircraft geometry. However, there should already be some ideas about the configuration to be chosen and the propulsion system. Only then reasonable assumptions about the parameters to be selected can be made. (Scholz 2015, Section 2, p.1)

Maximum take-off mass

The maximum take-off mass is the maximum mass of an aircraft that may be used for a take-off. It is the operating empty mass plus the maximum permissible combination of payload and operating materials (fuel). (DIN 9020, Scholz 2015, Section 10)

Operating empty weight

The operating empty weight is the take-off weight of an aircraft minus the payload and the operating materials. In other words, the operating empty weight is the manufacturer's empty weight (structure, propulsion system, standard equipment) plus mass deviations, fixed and mobile operating equipment (DIN 9020, Scholz 2015, Section 10)

Fuel

Aviation fuels are specialized types of petroleum-based fuel used to power aircraft. They are refined to a higher specification under greater quality control than fuels used in less critical applications, such as heating or road transport and often contain additives to reduce the risk of liquid fuel icing at low temperatures or the explosion of fuel vapor at high temperatures. (SKYbrary 2020)

Payload

The payload of an aircraft or spacecraft is the amount or weight of things or people that it is carrying. (Collins 2020)

Sensitivity

Synonyms for sensitivity: sensitiveness, delicacy… (Duden 2019)

1 Introduction

1.1 Motivation and Objectives

Hardly any other market segment is developing as dynamically as industrial lightweight construction. Even in today's aircraft development, it is this construction method that is most important. Less weight means lower fuel consumption. Lightweight aircraft are therefore more economical and environmentally friendly. The most important requirement in aircraft construction, in addition to the relevant construction regulations for the certification of aircraft, is weight reduction.

In the project phase of aircraft development, an aircraft is dimensioned taking into account the requirements and boundary conditions. This means that during this phase one tries to fulfill a certain performance catalog. Design changes can lead to the growth of mass. If an increase of the operating empty mass is necessary, this will result in a revision of the design draft. This in turn influences the take-off mass.

This is where this project work starts and deals with the topic of mass increase and the resulting mass snowball effect. An increase of the operating empty mass causes a further increase in this area. This leads to an iterative process. This also applies to the reduction of the operating empty mass, which leads to mass reductions. The growth factor can be used to determine the extent to which an increase in the operating empty mass affects the take-off mass of the aircraft.

The development of modern design synthesis programs has made it possible to quickly determine the influence of weight changes on the overall configuration. Although this has somewhat reduced the need to use mass growth factors to determine weight change effects, they are still widely used in rapid conceptual trade studies and as plausibility checks of analysis results.

The aim of this project is to find out how the mass growth factor can be determined. This shall be done by applying the "first law of aircraft design", which is used to estimate the maximum take-off mass. The goal is to show a mass growth iteration and to derive an equation for a direct calculation of the factor.

In this context, the calculation methods serve to calculate the mass growth factors for many of the common passenger aircraft. In addition, previous findings on this topic are to be cited and new ones gained. Among other things, the dependence of the mass growth factor on requirements and technology will be investigated.

1.2 Title Terminology

The title of the project work is "The Mass Growth Factor - Snowball Effects in Aircraft Design". Each of the terms contained in the title is defined below.

Mass growth factor

> *The so-called mass growth factor quantifies, how much the take-off mass is finally increased, provoked by a unit mass increase (e.g., 1kg) of the empty mass due to the original design change (without iteration). Its value strongly varies with aircraft type and requirements…* (Risse 2016, S.54f.)

Snowball effect

> *In aerospace engineering, it [the snowball effect] is used to describe the multiplication effect in an original weight saving. A reduction in the weight of the fuselage will require less lift, meaning the wings can be smaller. Hence less thrust is required and therefore smaller engines, resulting in a greater weight saving than the original reduction. This iteration can be repeated several times, although the decrease in weight gives diminishing returns.* (Wiki 2018)

Aircraft Design

The aircraft design process is generally divided into three phases: Project phase, definition phase and development phase.

According to Scholz 2015 (Section 1, p.1) aircraft design is described as follows:

> *The task of aircraft design in the practical sense is to 'provide the geometric description of a new aircraft'. The new aircraft is described by a three-view drawing, a fuselage crosssection, a cabin layout, and a list of aircraft parameters.*

1.3 Literature Review

Risse (2016) provides as an introduction to the topic. In this project, a short presentation of the topic is given, including the aeronautical manual from Fürst **(**1999), which in turn refers to a SAWE manual. SAWE (Society of Allied Weight Engineers) is a society of engineers specialized in the field of mass properties.

In Section 2 of this project, a literature search is conducted first. For this purpose, the book by SAWE (2019) as well as papers (Ballhaus 1954 and Saelman 1973**)** on the subject of the mass growth factor are consulted. Furthermore, books from the field of aircraft design are used to provide further insights on this topic. This literature includes Jenkinson 1999, Howe 2000, Müller 2003, Roskam 1989 and Torenbeek 1988.

The reference to the lecture notes according to Scholz 2015 occurs repeatedly at various points throughout the work. Most of the information from this script is taken from Section 5, which deals with preliminary sizing.

The research of the necessary data about the masses to determine the mass growth factor for the different aircraft comes from the books Taylor 1989, Lambert 1991 and Jackson 2011. Detailed and reliable data can be obtained from these books. For data not available in these books, further different sources are used, which are listed in the corresponding chapter. These include Jenkinson 2019 and Airbus and Boeing manufacturer websites.

1.4 Structure of the Work

This work is divided into the following sections:

Section 2 describes the basics of the mass snowball effect and mass growth factor. Previous investigations or findings on this topic are cited by means of literature research.

Section 3 deals with the determination of the mass growth factor using the "first law of aircraft design" and the iteration method. With this calculation method, the factor is calculated for so many known passenger planes that they cover 90% of the passenger planes flying today.

Section 4 first presents the results derived from the previous section. For typical mass fractions, values for the mass growth factor are given. An equation for a direct calculation of the factor is derived. Furthermore, the dependence of the mass growth factor on requirements and technology is examined and the relationship to direct operating costs (DOC) is highlighted.

Section 5 discusses the main results of this work

Section 6 summarizes the most important results of this work and gives an outlook.

Appendix A includes the evaluation of current passenger aircraft.

Appendix B contains the results of the mass growth factor for aircraft categories.

Appendix C contains a part of the derivation of the equation for a direct calculation of the factor.

2 Fundamentals and Literature Review

The determination and investigation of the mass growth factor is carried out using known formulas from the preliminary sizing of aircraft from the field of aircraft design. The basic knowledge for this shall not be repeated in this project and is a prerequisite. If necessary, the lecture notes Scholz 2015 are recommended.

2.1 Risse 2016

The design of an aircraft is an iterative process. A first assumption, which describes the maximum take-off mass, is necessary to get a first preliminary sizing. Then the operating empty mass can be estimated. Based on this operating empty mass and the design payload it is possible to estimate the required fuel. A new assumption, which describes the maximum take-off mass, can be generated by summing the operating empty mass, payload and fuel mass. This design loop has to be executed until convergence is reached. Requirements and boundary conditions have to be met.

For aircraft that are still in the design process or already in operation, the mass can increase due to specific changes in design or structure and the integration of new technologies or systems. If the assumptions in the design process are too optimistic, the planned values may be exceeded and finally the weight of the aircraft may be too high. The operating empty mass of an aircraft in service increases during its life cycle. This occurs when basic equipment (new technologies) or operating equipment (for example, a new entertainment system) is retrofitted. The operating empty mass also increases due to repairs to the aircraft.

As the operating empty mass of the aircraft increases, the fuel consumption in a given design area increases. The increase in fuel mass may even require larger fuel tanks, which primarily increases the maximum take-off mass. For structural components (e.g. wings), this can in turn result in further adjustments by increasing flight or ground loads. The thus increased component masses again result in an increase of the fuel mass and consequently an increase of the take-off mass, which results in an iterative process (according to Risse 2016, Chapter 3, p.54f.).

The iterative interaction between the masses of the components of the structure, system or drive and the take-off mass is generally referred to as the mass snowball effect (according to Risse 2016, Chapter 3, p.54f.).

The so-called mass growth factor quantifies how much the take-off mass is finally increased, caused by an increase in the operating empty mass due to the original design change. Its value varies greatly depending on the aircraft type and requirements (according to Risse 2016, Chapter 3, p.54f.).

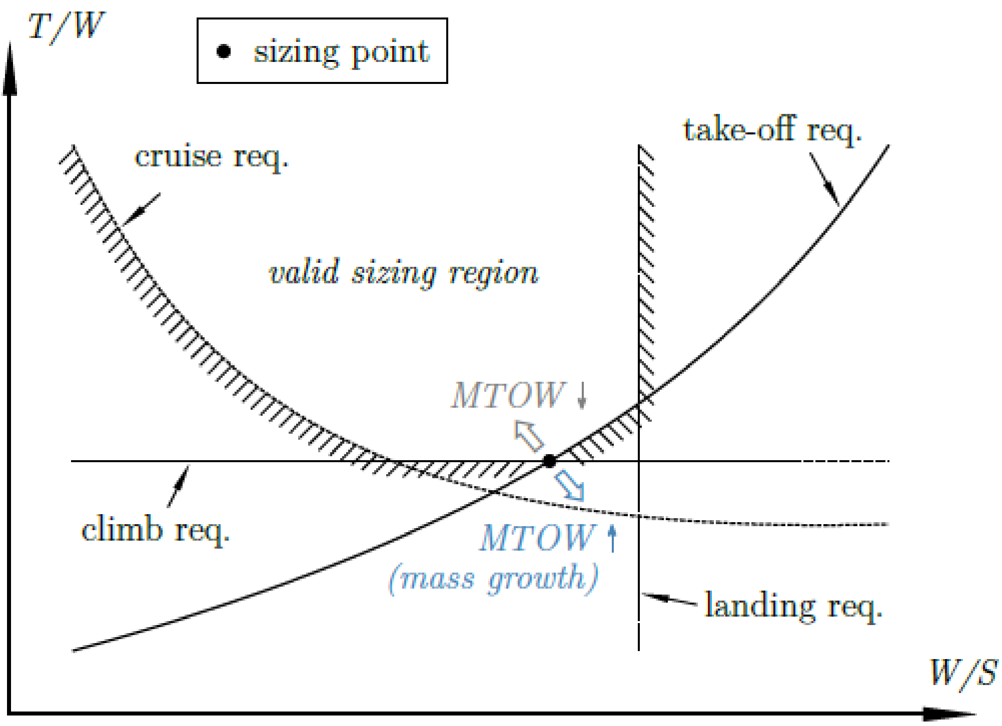

Figure 2.1 Influence of the variation of the MTOW on the compliance with the TLARs (according to Risse 2016, Chapter 3, p.55)

Figure 2.1 shows the consequence of a significant variation of the take-off mass in the form of a loss of performance or even a difference to the requirements. If the geometry of the airframe and the engines remains unchanged, an increase of the maximum take-off mass (MTOW) results in an increased wing loading (W/S) and a reduced thrust-to-weight ratio (T/W) (according to Risse 2016, Chapter 3, p.55f.).

The curves of the limitation, which show the requirements for take-off, climb, cruise flight and landing, contain a valid size range. Within this range an optimal combination of T/W and W/S must be determined. Deviation from this (ideal) design point usually results in reduced overall performance and may cause requirements (TLARs) not to be met (according to Risse 2016, Chapter 3, p.55f.).

In Figure 2.1, the selected sizing point is located near the boundary lines. There is a susceptibility to deviations for the start as well as for the climb requirements with increasing mass (increasing MTOW). This can result in necessary design adjustments (according to Risse 2016, Chapter 3, p.55f.).

The reverse case of a later mass reduction becomes particularly relevant for a retrofit design, e.g. for the integration of a fuel-saving technology. If the airframe and engine components are kept constant again, the retrofitted design with reduced MTOW has a reduced W/S and an increased T/W. This is rather uncritical with regard to compliance with the TLARs. However, a lower wing loading can also result in a loss of performance, and an increased T/W can include a drive system that is too heavy (according to Risse 2016, Chapter 3, p.55f.).

2.2 Sinke 2019

As described in the previous section, the increase in operating empty mass brings with it further increases. This leads to a so-called mass snowball effect, which in turn leads to a build-up of snow.

In Figure 2.2. the mass snow ball effect is displayed. In this view it is assumed that the fuel mass and payload do not change.

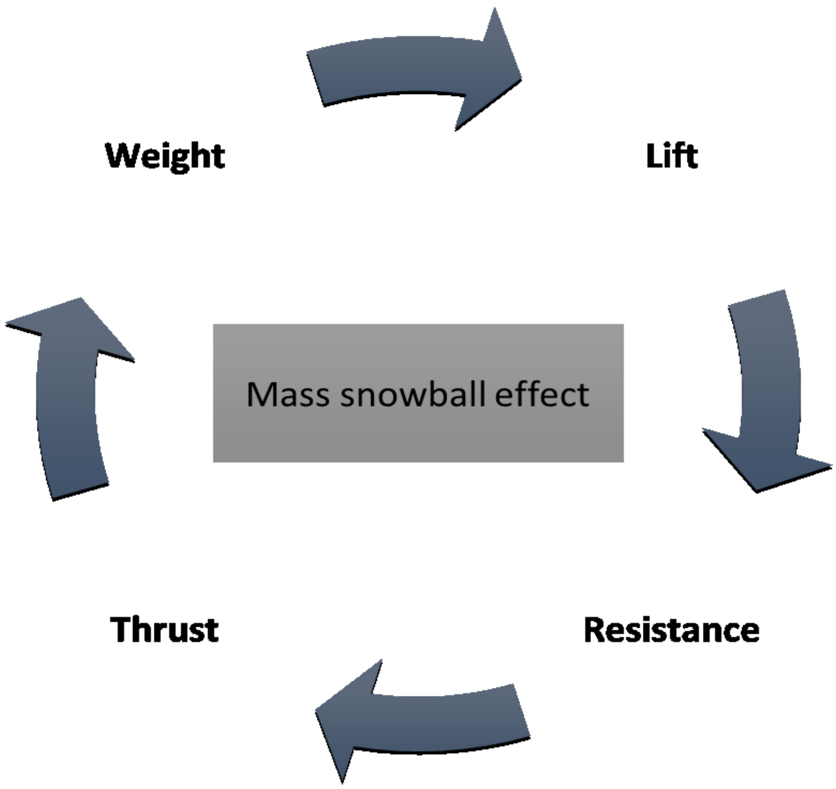

Figure 2.2 Representation of the mass snowball effect (according to Sinke 2019, p.54)

If the weight of e.g. structures or systems is increased, this leads to an increase of the total aircraft weight. This leads to

- → greater required lift
- → larger wings
- → this increases the resistance, including aerodynamic resistance
- → therefore the thrust must be increased
- → this leads to larger engines
- → this increases the weight again (according to Sinke 2019, p.54)

This process is iterative and continues until the weight converges. If the weight is reduced at the beginning, this leads to further reductions of the sizes shown in Figure 2.2 above.

2.3 Fürst 1999

The manual first describes the mass growth factor. According to this, the mass growth factor states what effect "an increase in empty mass of e.g. 1.0 kg has on the total mass of the aircraft" (Fürst 1999, p.1). The smallest possible value for the mass growth factor is 1.0. "However, this value is only achieved if a reduction in performance caused by the increase in mass is accepted." (Fürst 1999, p.1). As a rule, the mass growth factor is greater than 1.0, since the mass increase will be higher at the take-off mass than at the empty mass. If an increase of the empty mass is necessary in the conception and preliminary design phase of an aircraft, this has an influence on the take-off mass. In addition, the design draft must be revised.

Furthermore, the description of the mass snowball effect is given. Under the assumption that the range of the aircraft remains the same, the correlations with the mass growth factor are to be presented clearly.

"If the range is to remain constant, each increase in mass requires an additional amount of fuel. Increased fuel volume and associated enlarged fuel tanks are also necessary. Structural components that are dimensioned according to the flight loads become heavier the more the take-off mass increases. Changes in volume and structure in turn result in a larger quantity of fuel, which in turn increases the take-off mass. This process is iterative and continues until the interdependent functions - increase of the take-off mass and the resulting increase in mass of structure and fuel - converge." (Fürst 1999, p.1).

The empty mass of an aircraft can be divided into variable and fixed mass components. The variable components, such as the structure, are components "whose mass is influenced by the respective take-off mass" (Fürst 1999, p.1). Fixed mass components, such as avionics, include components "whose mass is not affected by the take-off mass" (Fürst 1999, p.1). Both mass increases in the variable and in the fixed mass component of the empty mass influence the level of the mass growth factor.

The mass growth factor MGF can be determined with the following equation:

$$MGF = \frac{1}{1-\left(\frac{m_{drive}}{m_{take-off}}+\frac{m_{structure}}{m_{take-off}}+\frac{m_{fuel}}{m_{take-off}}\right)} \tag{2.1}$$

Finally, values for the mass growth factor are given for certain types of aircraft. A subsonic transport aircraft with a moderate set of requirements has such a factor of about 1.5, whereas for a fighter aircraft with a demanding set of requirements this factor is much higher.

"For most aircraft, the factors are between 3.0 and 5.0. The average value for subsonic aircraft is 3.4 and for supersonic aircraft 4.6. For already built vertical take-off aircraft or some STOVL projects, mass growth factors of 2.5-5.0 have been determined." (Fürst 1999, p.1).

2.4 SAWE 2019

According to the definition in this book, the mass growth factor is the increase in total weight required per pound of empty weight. Here, the mass snowball effect is also described as an iterative process. As the load increases, the addition to the total weight results in an increase in structural components. In addition, additional volume and an adequate fuel absorption structure are necessary. The integration of volume and structure increases the fuel requirement, which leads to an even higher total weight. This process is continued until there is a convergence between the increase in weight of the components and increased fuel and total weight.

This is followed by a description of the classic mass growth factor definition. According to this definition, the mass growth factor determines how the total weight increases for each additional pound of the fixed weight portion of the empty weight. Two categories of empty weights can be identified. One changes when the total weight is changed (variable weight components), the other is not affected by changes in total weight (fixed weight components). The fixed weight category includes items such as avionics. The variable weight category includes structural elements.

The mass growth factor GF (growth factor) is calculated as follows:

$$GF = \frac{1}{1 - \left(\frac{W_p}{W_g} + \frac{W_{str}}{W_g} + \frac{W_f}{W_g} \right)} \tag{2.2}$$

In the classical mass growth factor definition, the total weight growth, caused by changed components with fixed weight, is taken into account. A similar effect is also produced by modified components with variable weight. For example, an increase in wing area will result in an increase in the total weight of the configuration, which is necessary to maintain performance.

2.5 Ballhaus 1954

In this paper, the mass growth factor is rejected as the ratio of the total weight to the given fixed weight.

$$\mathrm{G} = \frac{\text{Change in Gross Weight}}{\text{Fixed Weight Added or Change in Fixed Weight}} \tag{2.3}$$

First, the relationship between a change in weight and the requirements of the aircraft is explained. An aircraft is designed taking into account certain design requirements and specifications. The aircraft will therefore have a specific engine, total weight and specific geometric, structural and aerodynamic layout. If, for any reason, the fixed weight changes, the total weight of the aircraft must be adjusted so that the aircraft still meets the specified requirements exactly.

For a more precise determination of the mass growth factor, the mathematical derivation is used. In this method, the total weight is divided into fixed weight, which does not change with total weight, size, thickness and quality, and variable weight, which changes with the mentioned quantities.

The following therefore applies to the gross weight:

$$\text{Gross weight} = \text{Fixed weight} + \text{Variable weight} \tag{2.4}$$

$$W = F + f(W, S, n, Q, P) \tag{2.5}$$

Here is $f(W, S, n, Q, P)$ the function that combines the variable weight with W, S, n, Q and P in relation to each other.

Reordering of Equation (2.5) leads to:

$$F = W - f(W, S, n, Q, P) \tag{2.6}$$

And by differentiation follows:

$$\frac{dF}{dW} = 1 - \frac{df}{dW} \tag{2.7}$$

With the connection (2.3) the mass growth factor is obtained:

$$G = \frac{dW}{dF} \tag{2.8}$$

Respectively

$$G = \frac{1}{\frac{dF}{dW}} \tag{2.9}$$

By inserting Equation (2.7) into (2.9) finally follows:

$$G = \frac{1}{1 - \frac{df}{dW}} \tag{2.10}$$

This equation is a general expression for the mass growth factor. The factor then depends only on the variable weight in the aircraft and especially on the total derivative of the variable weight in relation to the total weight.

However, since the mass growth factor only exists with constant strength criteria (constant quality and performance), the variable weight only depends on the total weight and size. The new expression is $g(W,S)$ and is the function that combines the variable weight W and S in relation to each other.

The variable weight is made up of the weight of the propulsion system, the fuel weight and the weight of the structure and variable equipment. The new expression for the variable weight is then:

$$g(W,S) = W_P + W_F + W_{S+VE} \tag{2.11}$$

The power, the thrust-to-weight ratio, the wing loading and the thrust-specific weight of the drive system are assumed to be constant. W_P and W_F depend only on the total weight, whereas W_{S+VE} depends on both, the total weight and the size. The new total derivative of the variable weight in relation to the total weight is

$$\frac{dg}{dW} = \frac{\partial W_P}{\partial W} + \frac{\partial W_F}{\partial W} + \frac{dS}{dW}\frac{\partial W_{S+VE}}{\partial S} + \frac{\partial W_{S+VE}}{\partial W} \tag{2.12}$$

The mass growth factor for constant strength criteria (constant quality and performance) is then obtained:

$$G = \frac{dW}{dF} = \frac{1}{1 - \left[\frac{\partial W_P}{\partial W} + \frac{\partial W_F}{\partial W} + \frac{S}{W}\frac{\partial W_{S+VE}}{\partial S} + \frac{\partial W_{S+VE}}{\partial W}\right]} \tag{2.13}$$

In order to determine the mass growth factor, mass fractions are used in the total derivation of variable weight. This is illustrated by the following example, in which the mass growth factor is calculated for an aircraft with a range of 5000 NM. The aircraft has the following masses:

W = 100000 lbs
W_F = 50000 lbs
W_P = 15000 lbs

This results in the following mass fractions of $\frac{W_F}{W}$ = 0.50 and $\frac{W_P}{W}$ = 0.15.

The remaining sizes are given as $\frac{S}{W}\frac{\partial W_{S+VE}}{\partial S}$ = 0.14 and $\frac{\partial W_{S+VE}}{\partial W}$ = 0.09.

For the aircraft, Equation (2.13) gives a mass growth factor of:

$$G = \frac{1}{1 - [0.15 + 0.50 + 0.14 + 0.09]} = 8.33$$

Next, the effect of increasing the range is mentioned. If the range of the aircraft is to be increased from 5,000 NM to 6,000 NM, more fuel must be carried in addition. For this purpose, 6,500 lbs are subtracted from the fixed weight and 6,500 lbs more fuel is added. The fuel mass is then W_F = 56,500 lbs and the fuel mass ratio accordingly $\frac{W_F}{W}$ = 0.565. The new mass growth factor is then obtained:

$$G = \frac{1}{1 - [0.15 + 0.556 + 0.14 + 0.09]} = 18.18$$

With an increase in range and a corresponding increase in fuel mass, the mass growth factor increases.

2.6 Saelman 1973

According to the definition of this paper, the mass growth factor is the ratio in which the total weight is increased to any increase in weight in the aircraft.

At a certain point the mass growth factor β is given via differentiation as follows:

$$\beta = \frac{dW_g}{dW_o} = \frac{dgross\ weight}{dfixed\ weight} \tag{2.14}$$

The fixed weight includes the payload, the weight of crew and passenger accommodation, the pressure critical cabin structure, the flight station, smaller fuselage frames etc.

To develop a mathematical term for the mass growth factor, the variation of the weights of the central aircraft components with the growth weight is determined. Finally, the total weight can be calculated as the sum of the fixed weight and combined terms that are in proportion to the take-off weight performance.

A resulting formula describes the total weight as the sum of fixed weight and combined factors. Finally, an expression for the mass growth factor is derived by implicitly differentiating the total weight according to the fixed weight.

As a result, the paper shows that the mass growth factor increases when the growth weight is increased. It should be noted that a relatively small fraction of the payload or fixed weight in the total weight results in a high mass growth factor. Wide-body commercial aircraft have growth factors of 3, whereas the SST has growth factors in the range of 6 to 7, combat aircraft have growth factors in the range of 10, and rockets and spacecraft have exceptionally high growth factors.

2.7 Howe 2000

This book shows the connection between design assumptions and mass growth. Accordingly, the analysis of design work is an iterative process. If there is sufficient justification for the initial assumptions, the process will converge to a final design with features that can be identified with sufficient accuracy. In an analysis at a later point in time it may turn out that some assumptions were not justified. A deviation of the design is possible in this case.

Figure 2.3 is a simple example of how this can be done.

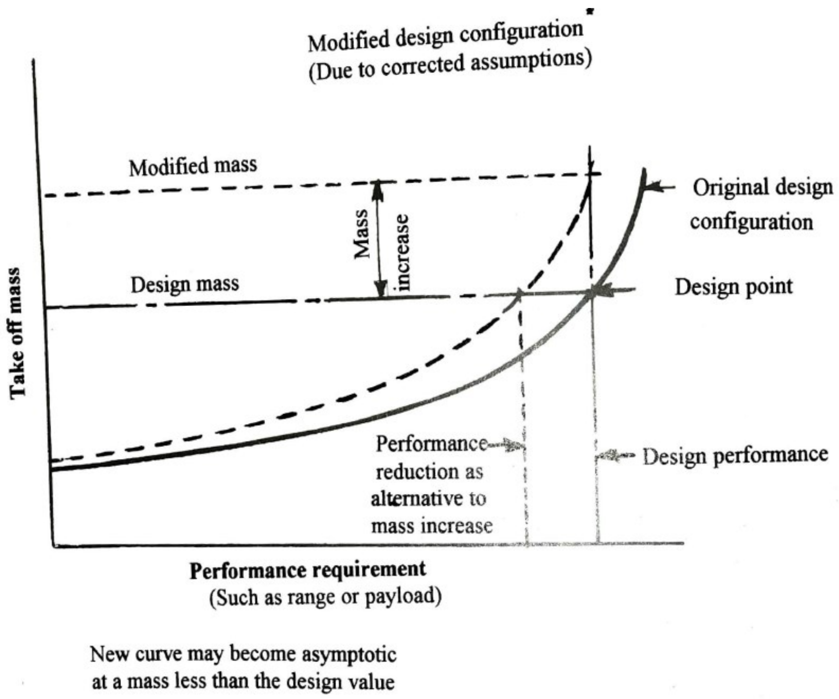

Figure 2.3 Influence of design assumptions on mass growth (according to Howe 2000,Chapter 9, p.280)

The black line represents the original design configuration. A certain design mass is assumed. If the assumptions differ, e.g. due to an increase in weight, the dashed line is generated. Then a mass increase (modified mass) occurs. However, if the mass is to correspond to the originally assumed design mass, a reduction in performance must be expected or the requirements must be reduced.

In case of deviations from the requirements, it is necessary to rethink the overall concept, possibly to investigate alternative overall configurations or to review the dominant performance requirements.

2.8 Jenkinson 1999

This book starts with an explanation of the mass snowball effect. If the aircraft specifications (e.g. take-off performance, range, etc.) are met, the structural weight increase will result in higher fuel consumption, larger engines, larger wing and empennage surfaces and stronger landing gear due to design inefficiency.

A mass growth factor of 3 is cited, which means that per kg of additional structural mass on the aircraft, the maximum take-off mass of the aircraft increases by approximately three kg. The influence of the weight increase is illustrated using two aircraft. The range of both aircraft is identical, but the number of passengers during flight is different. Table 2.1 shows the passenger numbers and the mass parameter changes of the two aircraft.

Table 2.1 Influence of aircraft size on mass parameters (according to Jenkinson 1999, Chapter 7, p.128)

Aircraft size: number of seats	300	600
Increase in structure mass (kg)	1000	1000
Increase in operational empty mass (kg)	1879	1756
Increase in fuel requirement (kg)	1255	1431
Increase in aircraft maximum TO mass (kg)	3034	3188
Weight growth factor	3.03	3.19
Increase in aircraft price ($M)	0.87	1.19

The structural mass was increased by 1000 kg for both aircraft and the aircraft were redesigned. The aircraft with 300 seats has a mass growth factor of 3.03 and the aircraft with 600 seats has a factor of 3.19. The larger aircraft therefore has a larger mass growth factor with the same increase in structural mass.

2.9 Müller 2003

In this book the problems of mass calculation and the sensitivity of mass calculation for the design are explained. The problem lies in the fact that the different weight formulas must be derived from actual masses, which in turn depend on the quality of construction and building. The sensitivity is based on the fact that mass overruns always result in losses in performance.

The problem is that the planned values are exceeded if the assumptions are too optimistic and that the aircraft becomes too large and heavy if the assumptions are too skeptical. If the wings are too large and the cell components associated with them, the slightly higher frictional resistance results in a slightly lower maximum speed.

If flight performances are kept constant, additional weights and/or higher resistances result in a growth factor in the take-off mass. It is explained that one kg of additional weight in an aircraft results in a two kg increase in take-off mass. It is also stated that with 1 daN of additional drag, the take-off mass increases by about 3.5 kg.

2.10 Roskam 1989

The book first shows the existence of a linear relation between the decadic logarithm of the operating empty mass and that of the take-off mass. The operating empty mass of different types of aircraft was plotted against the take-off mass in diagrams, thus proving the correlation. By means of a regression analysis, trend lines were then formed in the diagrams.

It becomes obvious that a logarithmic function with the parameters A and B must exist, by which said trend lines are represented in the logarithmic diagrams. By this logarithmic function the operating empty mass can be determined by the take-off mass

The mass growth factor is then discussed. For this purpose, the change in take-off mass, depending on several parameters such as payload and operating empty mass, is analyzed. First of all, the mass growth factor is specified by payload. For its determination a formula of the ratio of take-off mass by payload is used, which itself depends on different parameters. This ratio derived results in the mass growth factor by payload.

The mass growth factor by operating empty mass is obtained by using the logarithmic function with the parameters A and B. This represents the connection between the take-off mass and the operating empty mass. If the ratio between the take-off mass and the operating empty mass is derived, the mass growth factor by operating empty mass is obtained. In an example with a jet airliner that has a take-off mass of approximately 126100 lbs, this is 1.93. This means that for each additional pound of operating empty mass, the takeoff mass must be increased by 1.93 lbs to ensure consistent mission performance.

2.11 Torenbeek 1988

First of all, an explanation is given of the importance of reducing the weight of an aircraft design, mainly in large aircraft of high complexity. The increase in weight of a component is accompanied by an additional weight elsewhere. This is described as the "snowball effect of weight growth". The possibilities of weight minimization depend on the phase of the design process.

In the conceptual design at the beginning, the selected aircraft layout, the chosen geometry and the choice of detailed configuration influence the weight. The evaluation of reductions or increases in weight is usually done for a constant design performance, except when the limited engine power does not allow it. Consequently, each increase in component weight is accompanied by a higher take-off weight. However, if the increased weight of the components is caused by a design change that contributes to the performance increase (e.g. improved high-lift systems), a reduced take-off weight may result.

Table 2.2 shows the sensitivities for increases in structural weight based on some typical missions. The effects of a structure weight increase of 10% on the maximum take-off weight for a constant mission can be seen.

Table 2.2 Effect of a 10% increase in structural weight on the maximum take-off weight at constant mission (according to Torenbeek 1988, Chapter 8, p.266)

AIRPLANE CATEGORY	DESIGN RANGE	EFFECT ON MTOW
subsonic transport	250 nm	6.5 %
	1000 nm	6.9 %
	3000 nm	7.0 %
supersonic transport	3000 nm	9.4 %
transport VTOL	250 nm	6.9 %
military VTOL	250 nm	9.5 %

For a subsonic airliner with a design range of e.g. 250 NM, a 10% increase in weight of the structure leads to a 6.5% increase of the maximum take-off mass. It should also be noted that the sensitivity also depends on where the weight increase occurs.

With the help of this data from the book, a mass growth factor should now be determined. The 10% increase of the operating empty mass can be represented as:

$$\Delta m_{OE} = m_{OE} \cdot 0.1 \tag{2.15}$$

And the increase of the maximum take-off mass as

$$\Delta m_{MTO} = m_{MTO} \cdot x \tag{2.16}$$

To determine a factor, the ratio of the two mass increases is used:

$$\frac{\Delta m_{MTO}}{\Delta m_{OE}} \tag{2.17}$$

Inserting (2.15) and (2.16) into (2.17) leads to:

$$\frac{\Delta m_{MTO}}{\Delta m_{OE}} = \frac{m_{MTO} \cdot x}{m_{OE} \cdot 0.1} = \frac{\frac{x}{0.1}}{\frac{m_{OE}}{m_{MTO}}} \tag{2.18}$$

Now for x e.g. the 6.5% increase of the maximum take-off mass can be used. A value of 0.5 is assumed for the ratio of operating empty mass to maximum take-off mass. This results in a factor of:

$$\frac{\Delta m_{MTO}}{\Delta m_{OE}} = \frac{\frac{0.065}{0.1}}{0.5} = 1.3$$

Based on the data from Torenbeek 1988 and assuming that the operating empty mass fraction is 0.5, a mass growth factor of 1.3 is calculated.

3 Mass Growth Factor by Iteration

3.1 Equations

Previous studies indicate that when a local mass growth occurs, the increase in the take-off mass converges. The process of the mass snowball effect is iterative and continues until the take-off mass approaches a certain value. On the basis of this knowledge, a method for determining the mass growth factor is presented.

The maximum take-off mass m_{MTO} according to Scholz 2015 is composed of maximum payload m_{MPL}, fuel mass m_F (for the range at maximum payload) and the operating empty mass m_{OE}:

$$m_{MTO} = m_{MPL} + m_F + m_{OE} \tag{3.1}$$

The transformation of this formula results in the so-called "first law of aircraft design", which represents a relationship between the maximum take-off mass and the operating empty mass (according to Scholz 2015):

$$m_{MTO} = \frac{m_{MPL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \tag{3.2}$$

This equation is supplemented by the parameter Δm_L. This parameter contains the local mass growth. A local mass growth has the consequence that the total take-off mass growths. This global mass growth is expressed with the parameter Δm_G. This results in:

$$m_{MTO} + \Delta m_G = \frac{m_{MPL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE} + \Delta m_L}{m_{MTO}}} \tag{3.3}$$

The mass growth factor k_{MGW} finally results from the ratio of global to local mass growth:

$$k_{MGW} = \frac{\Delta m_G}{\Delta m_L} \tag{3.4}$$

The local mass growth Δm_L depends on the weight by which the operating empty mass is increased. This can therefore be assumed to be known. In our case the local mass growth is Δm_L = 1 kg.

The determination of the global mass growth Δm_G is done by iteration. First, the Equation (3.1) is supplemented by the local and global growth of mass and named $m_{MTO.0}$:

$$m_{MTO,0} = m_{MTO} + \Delta m_G = m_{MPL} + m_{OE} + \Delta m_L + m_F \tag{3.5}$$

This equation corresponds to Equation (3.3). For the first iteration step, the operating empty mass and the fuel mass are converted as mass fractions. It shall apply:

$$m_{OE} = \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO} \tag{3.6}$$

And

$$m_F = \frac{m_F}{m_{MTO}} \cdot m_{MTO} \tag{3.7}$$

With the maximum take-off mass $m_{MTO.0}$ and the mass fractions, the first iteration step is done:

$$m_{MTO,1} = m_{MPL} + \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO,0} + \Delta m_L + \frac{m_F}{m_{MTO}} \cdot m_{MTO,0} \tag{3.8}$$

The next iteration steps look like this:

$$m_{MTO,2} = m_{MPL} + \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO,1} + \Delta m_L + \frac{m_F}{m_{MTO}} \cdot m_{MTO,1} \tag{3.9}$$

$$m_{MTO,3} = m_{MPL} + \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO,2} + \Delta m_L + \frac{m_F}{m_{MTO}} \cdot m_{MTO,2} \tag{3.10}$$

$$m_{MTO,4} = m_{MPL} + \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO,3} + \Delta m_L + \frac{m_F}{m_{MTO}} \cdot m_{MTO,3} \tag{3.11}$$

The iteration is done until the maximum take-off mass converges to a value. The limit of the maximum take-off mass by increasing the operating empty mass by 1 kg is then $m_{MTO,X}$.

For the global mass growth Δm_G applies:

$$\Delta m_G = m_{MTO,X} - m_{MTO} \tag{3.12}$$

With this global mass growth Δm_G, the local mass growth, in our case of Δm_L = 1 kg, and using the formula (3.4), the mass growth factor k_{MGW} can finally be determined.

3.2 Example: Boeing 767-300

The following is an example for the calculation of the mass growth factor k_{MGW} with the above-mentioned method for the Boeing 767-300. The local mass growth Δm_L is assumed to be 1 kg.

Using Jenkinson 2019, the following masses in Table 3.1 were determined for the aircraft.

Table 3.1 Masses of the Boeing 767-300 (according to Jenkinson 2019)

Aircraft	m_{MTO} [kg]	m_{OE} [kg]	m_{MPL} [kg]
Boeing 767-300	156489	87135	39140

The fuel mass is also calculated with:

$$m_F = m_{MTO} - m_{MPL} - m_{OE} \tag{3.13}$$

The result is m_F = 30214 kg.

From these masses, the mass fractions can then be determined. These result in $\frac{m_{OE}}{m_{MTO}}$ = 0.5568123 and $\frac{m_F}{m_{MTO}}$ = 0.19307427.

First the maximum take-off mass $m_{MTO.0}$ is determined according to Equation (3.5). Inserting the values results in:

$$m_{MTO,0} = 39140\ kg + 87135\ kg + 1\ kg + 30214\ kg = 156490\ kg$$

The first iteration step according to Equation (3.8) is given by inserting:

$$m_{MTO,1} = 39140\ kg + 0.5568123 \cdot 156490\ kg + 1\ kg + 0.19307427 \cdot 156490\ kg$$

Thus $m_{MTO,1}$ = 156490.7498 kg.

Iteration is done until the difference of the new calculated and old maximum take-off mass is very small. Since the local mass growth is 1 kg, the global mass growth corresponds to the mass growth factor. The results of the further iteration steps are shown in the Table 3.2.

Table 3.2 Further iteration steps and results

Iteration step	Maximum take-off [kg]	Difference [%]	Δm_G resp. k_{MGW}
1	156490.7498	74.9887%	1.75
2	156491.3122	32.1352%	2.31
3	156491.7339	18.2372%	2.73
4	156492.0501	11.5664%	3.05
5	156492.2872	7.7743%	3.29
6	156492.4651	5.4093%	3.47
7	156492.5984	3.8482%	3.60
8	156492.6984	2.7788%	3.70
9	156492.7734	2.0274%	3.77
10	156492.8296	1.4901%	3.83
11	156492.8718	1.1010%	3.87
12	156492.9034	0.8167%	3.90
13	156492.9271	0.6074%	3.93
14	156492.9449	0.4528%	3.94
15	156492.9582	0.3380%	3.96
16	156492.9682	0.2526%	3.97
17	156492.9757	0.1889%	3.98
18	156492.9813	0.1414%	3.98
19	156492.9855	0.1059%	3.99
20	156492.9887	0.0793%	3.99
21	156492.9911	0.0594%	3.99
22	156492.9929	0.0445%	3.99
23	156492.9942	0.0334%	3.99
24	156492.9952	0.0250%	4.00
25	156492.9959	0.0188%	4.00
26	156492.9965	0.0141%	4.00
27	156492.9969	0.0105%	4.00
28	156492.9972	0.0079%	4.00
29	156492.9975	0.0059%	4.00
30	156492.9977	0.0044%	4.00
31	156492.9978	0.0033%	4.00
32	156492.9979	0.0025%	4.00
33	156492.9980	0.0019%	4.00
34	156492.9980	0.0014%	4.00
35	156492.9981	0.0011%	4.00
36	156492.9981	0.0008%	4.00
37	156492.9981	0.0006%	4.00
38	156492.9981	0.0004%	4.00
39	156492.9981	0.0003%	4.00
40	156492.9982	0.0003%	4.00
41	156492.9982	0.0002%	4.00
42	156492.9982	0.0001%	4.00

Thus the mass growth factor k_{MGW} for the Boeing 767-300 is approximately 4

3.3 Mass Growth Factor for 90% of Current Aircraft

From Robson 2019, the "World Airliner Census" is used here. This census data includes all commercial aircraft with jet and turboprop engine that are in operation worldwide or have been permanently contracted with airlines. Exceptions are those types that carry fewer than 14 passengers or equivalent cargo. This includes fleets of Western, Chinese and Russian/Ukrainian aircraft.

These aircraft figures are used to determine 90% of all current flying commercial aircraft. The evaluation can be found in Appendix A. For these aircraft the determination of the mass growth factor is required. In addition, this must be calculated for the supersonic passenger aircraft Concorde and TU-144. The calculation is done using the method described in Section 3.1 of this project.

Table 3.3 shows the results of the mass growth factor. The aircraft are arranged according to their current number. Thus, the Boeing 737-800 is the currently most operated aircraft. The two supersonic passenger aircraft Concorde and TU-144 are listed far below. In addition, the sources are listed from which the masses (maximum take-off mass, operating empty mass, maximum payload) for the respective aircraft have been taken.

The fuel mass shall be determined using Equation (3.13). The mass fractions are then determined from these masses to calculate the mass growth factor.

Table 3.3 Evaluation of 90% of all current flying commercial aircraft including two supersonic aircraft

Aircraft	m_{MTO} [kg]	m_{OE} [kg]	m_{MPL} [kg]	k_{MGW}	Sources
Boeing 737-800	78220	41480	14690	**5.32**	Jenkinson 2019
A320-200	73500	42100	18633	**3.94**	Jackson 2011
A320neo	79000	44300	20000	**3.95**	Airbus 2005, Wiki 2020
A321neo	97000	50100	25500	**3.80**	Airbus 2005a, Wiki 2020
A321-200	89000	48000	22780	**3.90**	Jenkinson 2019
A319-100	64000	39200	17390	**3.68**	Jenkinson 2019
Boeing 737-700	69400	37585	11610	**5.97**	Jenkinson 2019
ATR 72-500	22500	12950	7350	**3.06**	Jackson 2011
Boeing 777-300 ER	299370	155960	68570	**4.36**	Jackson 2011
Embraer 175	37500	21810	9890	**3.79**	Jackson 2011
Boeing 787-9	244940	128850	52587	**4.65**	Boeing 2018, Wiki 2020c
A330-300	217000	118189	48400	**4.48**	Jenkinson 2019
Boeing 767-300	156489	87135	39140	**3.99**	Jenkinson 2019
A350-900	280000	142400	53300	**5.25**	Airbus 2005b, Wiki 2020a
Boeing 757-200	115900	58040	25690	**4.51**	Jenkinson 2019
A330-200	230000	120200	36400	**6.31**	Jenkinson 2019
Boeing 737-900	74389	42901	19831	**3.75**	Boeing 2013, Wiki 2020e
DHC Dash 8-400	24993	14968	7257	**3.44**	Lambert 1991
Embraer 190	50300	28080	13530	**3.71**	Jackson 2011
Bombardier CRJ900	36500	21430	10320	**3.53**	AirlinesInform 2020
A220-100	63049	35221	15127	**4.16**	Airbus 2019, Wiki 2020d
Boeing 777-200	242670	135875	54635	**4.44**	Jenkinson 2019
Embraer 145	20600	11940	5160	**3.99**	Jackson 2011
Boeing 787-8	219550	108860	45439	**4.83**	Jackson 2011
Boeing 747-400	396830	181484	61186	**6.48**	Jenkinson 2019
Boeing 737-300	56470	31869	16030	**3.52**	Jenkinson 2019
A380-841	560000	270015	90985	**6.15**	Jackson 2011
Viking Air Twin Otter 400	5670	3121	1474	**3.84**	Jackson 2011
Bombardier CRJ700	33000	19730	8530	**3.86**	AirlinesInform 2020a
Boeing 737-400	62820	33370	17740	**3.54**	Jenkinson 2019
ATR42-500	18600	11250	5450	**3.41**	Jackson 2011
A330-800neo	251000	132000	44000	**5.70**	Wiki 2020b
Boeing MD-81	58061	35330	18195	**3.19**	Jenkinson 2019
Boeing 777F	347450	145150	102000	**3.40**	Jackson 2011
A300-600R	150000	79666	33300	**4.50**	Jenkinson 2019
Saab 340 B Plus	13150	8140	3880	**3.38**	AirlinesInform 2020b
Fairchild Metro/Merlin III	6577	3963	2214	**2.97**	Wiki 2020f
Beechcraft 1900D	7765	4732	1950	**3.98**	Zimex 2020, Wiki 2019
A350-1000	316000	155000	68000	**4.64**	Jackson 2011
Embraer 195	48790	28970	12720	**3.83**	Jackson 2011
Embraer 170	35990	21140	9000	**3.99**	Jackson 2011

Boeing 787-10	254011	135500	57277	**4.43**	Boeing 2018, Wiki 2020c
Concorde (Supersonic)	187700	78700	12000	**15.64**	AirlinesInform 2020c
TU-144 (Supersonic)	180000	85000	15000	**12**	Jane 1982

4 Mass Growth Factor Beyond Iteration

4.1 Generalization of Iteration Results

By calculating the mass growth factor by iteration the following important findings can be made:

1. Larger aircraft do not necessarily have a higher mass growth factor.

The value of the factor depends primarily on the operating empty mass and the fuel mass fraction. If the maximum take-off mass is changed, but the mass fractions remain unchanged, this has no effect on the value of the mass growth factor. The size of the aircraft and thus the maximum take-off mass is not the decisive factor.

2. It does not matter how large the local mass growth is; the mass growth factor remains unaffected.

The maximum take-off mass converges against a value. The difference between the new and original take-off mass results in the global mass growth. This corresponds to a multiple (mass growth factor) of the local mass growth. Should mean:

$$\Delta m_G = k_{MGW} \cdot \Delta m_L \tag{4.1}$$

The local mass growth thus has a direct influence on the global mass growth and thus on the maximum take-off mass - but not on the mass growth factor.

3. With the mass growth factor, it is mainly a matter of increasing a fixed mass.

The fuel is not part of the fixed mass. The "first law of aircraft design" (3.2) including the growth in mass can be described as

$$m_{MTO} + \Delta m_G = m_{MPL} + m_{OE} + \Delta m_L + m_F \tag{4.2}$$

From this it can be deduced that when the aircraft takes off, it does not matter whether there is one kg more wing mass (operating empty mass) or one kg more payload on board. The mass growth factor for a growth in the operating empty mass is therefore the same factor as for a growth in the payload.

4. Values of the mass growth factor for different aircraft categories.

The mass growth factor for 90% of all current flying commercial aircraft was determined in Section 3.2 of this project. Based on these values the average mass growth factor for different aircraft categories has to be determined. The categories and the result for the mass growth factors are summarized in Table 4.1.

Table 4.1 Mass growth factor for different aircraft categories

	Aircraft categories			
	Wide-Body	Narrow-Body	Supersonic	Subsonic
k_{MGW}	4.91	3.85	4.23	13.82

Which aircraft have been grouped into which category can be seen in Appendix B.

5. Old long-range aircraft are more sensitive to local mass growth than new short-range aircraft.

To confirm this result, the aircraft from Section 3.2 are categorized into old long-range and new short-range aircraft. For the respective aircraft the mass growth factors are summarized in Figures 4.1 and 4.2.

The mass growth factor of 4.38 of the old long-range aircraft is on average higher. This is mainly due to the fact, that they fly long distances and require more fuel. This means increased fuel mass, which in turn leads to a large mass growth factor.

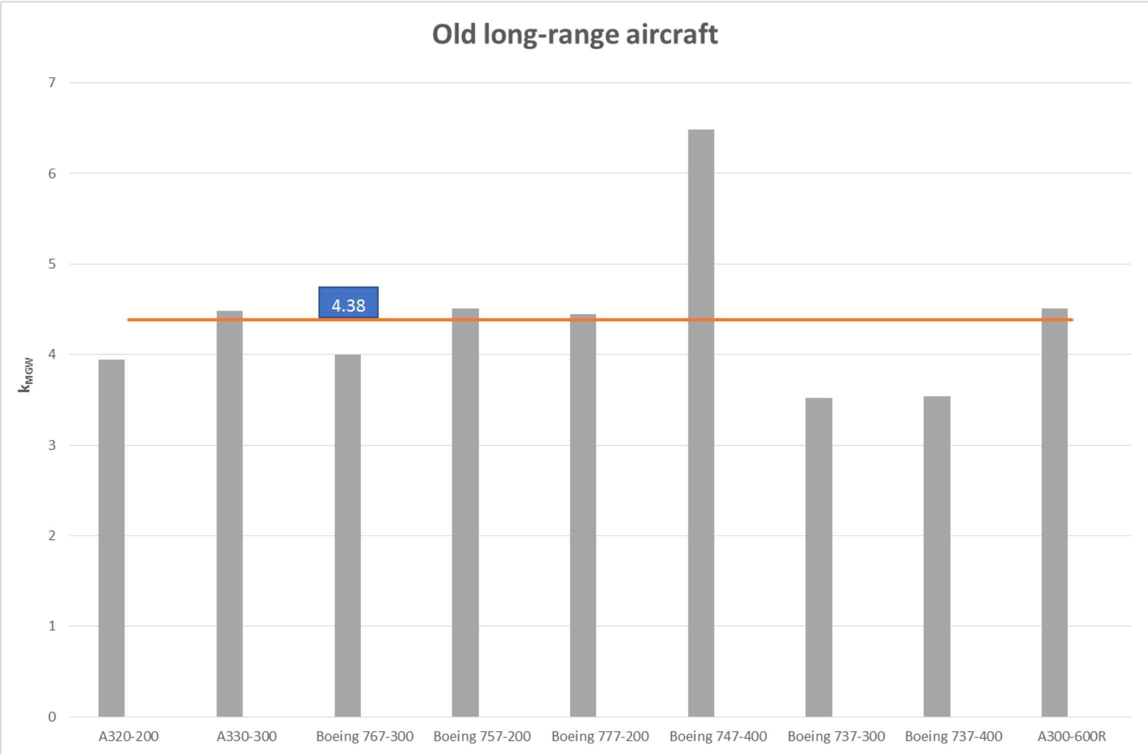

Figure 4.1 Mass growth factor of old long-range aircraft (Screenshot: Evaluation_90%_aircraft_+_category.xlsm)

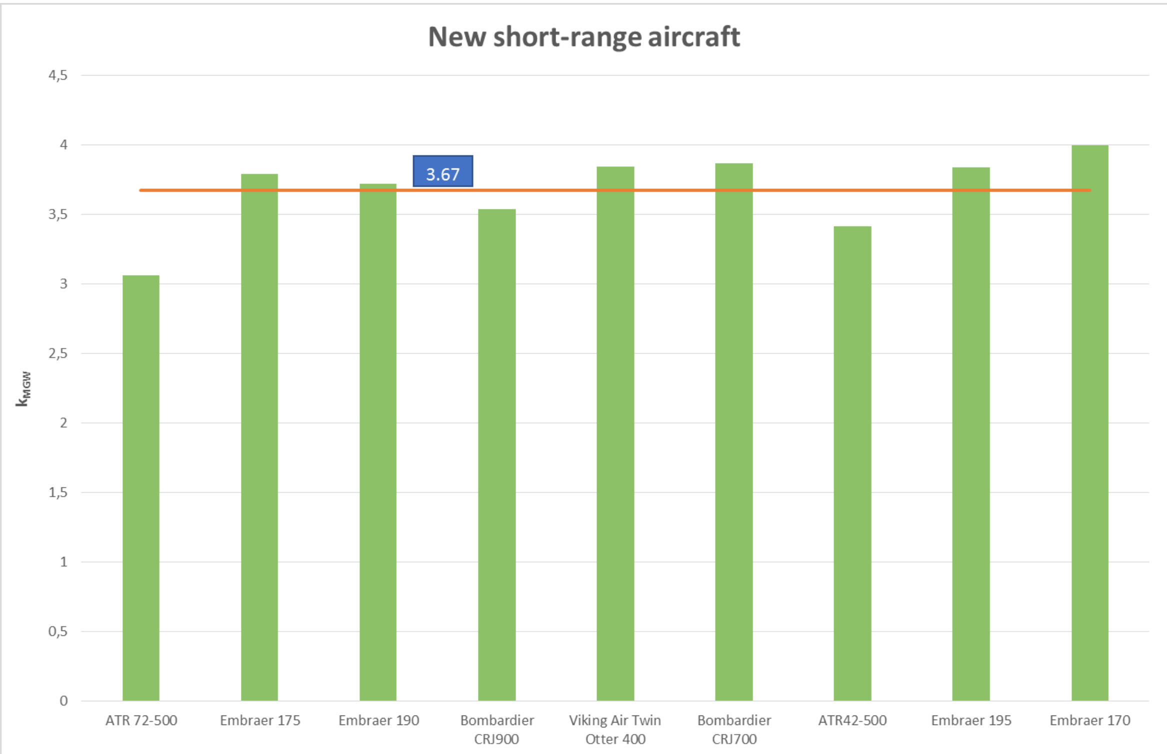

Figure 4.2 Mass growth factor of new short-range aircraft (Screenshot: Evaluation_90%_aircraft_+_category.xlsm)

4.2 Mass Growth Factor from Operating Empty Mass and Fuel Mass Fraction

The mass growth factor can be determined using the operating empty mass and fuel mass fraction of an aircraft. This knowledge shall be used to give a typical value of the mass growth factor for the three types of ranges (long-range, medium-range and short-range aircraft).

For this purpose, a typical value for the mass fractions is first determined for the respective range. Typical values for the operating empty mass fraction are taken from the Figure 4.3.

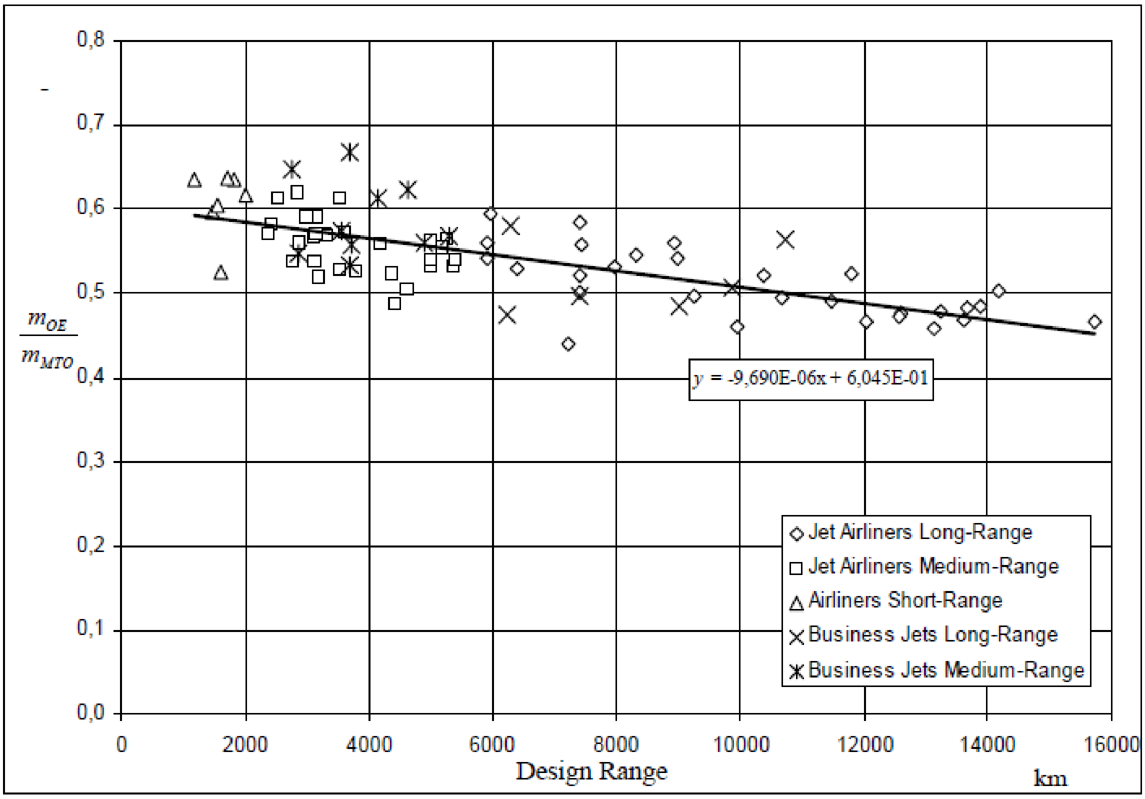

Figure 4.3 Operating empty mass fraction as a function of design range (according to Scholz 2015, Section 5, p.29)

From this, the following values are taken for the operating empty mass fraction:

- For long-range aircraft: 0.45
- For short-range aircraft: 0.6

For medium-range aircraft, the arithmetic mean of the two numbers is used. This would be 0.525.

To obtain typical values for the fuel mass fraction, Figure 4.4 is used.

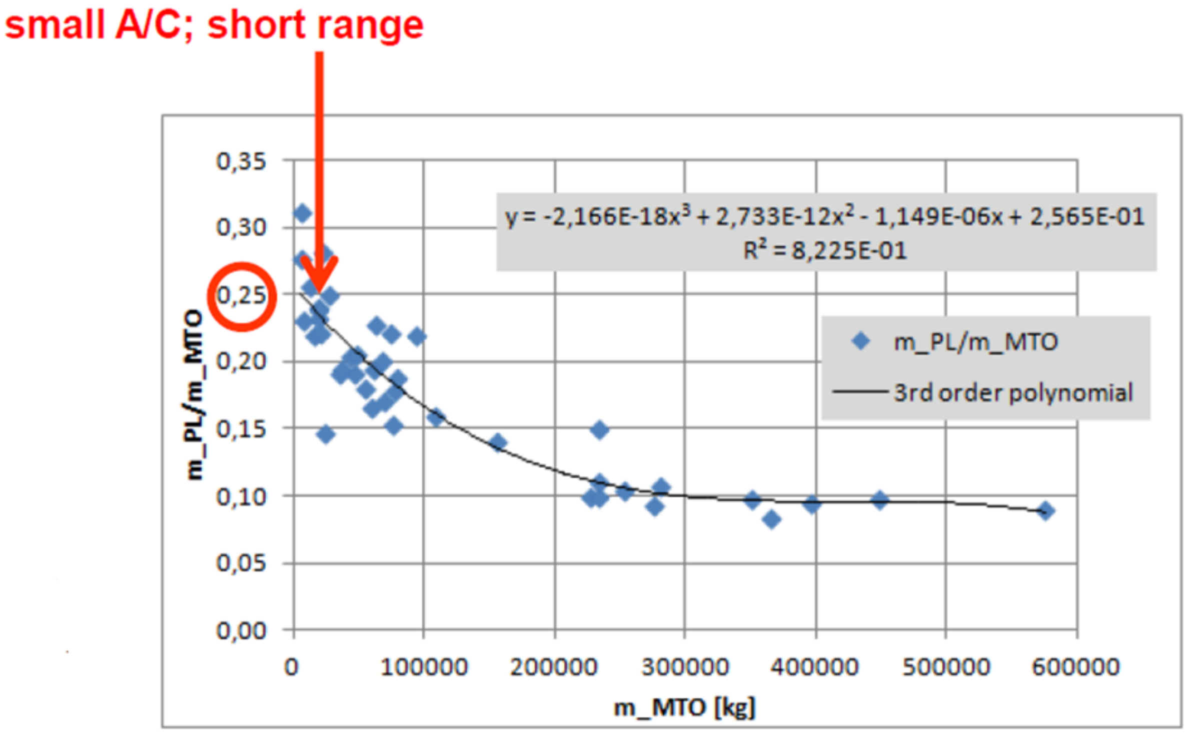

Figure 4.4 Payload fraction as a function of maximum take-off mass (according to Scholz 2018, slide 30)

In this Figure 4.4 the payload fraction is plotted over the aircraft size. For short-range aircraft the payload fraction is given as 0.25. For long-range aircraft a value of 0.1 can be read, because long-range aircraft have a large take-off mass.

Using the Equation (4.3):

$$\frac{m_F}{m_{MTO}} = 1 - \frac{m_{PL}}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}} \tag{4.3}$$

And with the values for the operating empty mass and payload fraction for short and long-range, the fuel mass fraction is obtained:

- For long-range aircraft: 0.45
- For short-range aircraft: 0.15

For medium-range aircraft, the arithmetic mean of the two numbers is used. This would be 0.3.

The typical values for both mass fractions are determined. Now the mass growth factor is calculated for these values. The results are shown in Table 4.2.

Table 4.2 Mass growth factor for typical mass fractions

	Range type		
	Short-range	Medium-range	Long-range
m_{OE}/m_{MTO}	0,6	0,525	0,45
m_F/m_{MTO}	0,15	0,3	0,45
$\boldsymbol{k_{MGW}}$	**4**	**5,71**	**10**

With these results it can be determined that the greater the mass growth factor, the greater the range. This relationship is examined in more detail in Section 4.4.

4.3 Mass Growth Factor from Payload Fraction

When determining the mass growth factor for current aircraft, a correlation between the mass growth factor and the payload fraction m_{MPL}/m_{MTO} is noticeable.

As calculated in Section 3.1.1, the mass growth factor for the Boeing 767-300 is approximately 4. The payload fraction of this aircraft is 0.25. The reciprocal value of the payload fraction thus corresponds to the mass growth factor. This picture appears for all examined aircraft.

It is therefore assumed that the mass growth factor is calculated as follows:

$$k_{MGW} = \frac{1}{\frac{m_{MPL}}{m_{MTO}}} \tag{4.4}$$

An equation for direct calculation of the mass growth factor is derived and an attempt is made to confirm Equation (4.4).

The Equation (3.1) can be paraphrased as

$$1 = \frac{m_{MPL}}{m_{MTO}} + \frac{m_{OE}}{m_{MTO}} + \frac{m_F}{m_{MTO}} \tag{4.5}$$

The operating empty mass and fuel mass fraction are summarized as "zero passenger weight" m_{ZP}:

$$\frac{m_{ZP}}{m_{MTO}} = \frac{m_{OE}}{m_{MTO}} + \frac{m_F}{m_{MTO}} \tag{4.6}$$

By including m_{ZP}/m_{MTO} in Equation (3.1) follows:

$$m_{MTO} = m_{MPL} + \frac{m_{ZP}}{m_{MTO}} \cdot m_{MTO} \tag{4.7}$$

Taking into account the local and global growth in masses, the result is

$$m_{MTO} + \Delta m_G = m_{MPL} + \frac{m_{ZP}}{m_{MTO}} \cdot (m_{MTO} + \Delta m_G) + \Delta m_L \tag{4.8}$$

Now the context (4.4) is derived instead of iterating as in Section 3.1. For this purpose, first of all, the transformation is done so that the maximum payload is on one side:

$$m_{MTO} + \Delta m_G - \frac{m_{ZP}}{m_{MTO}} \cdot (m_{MTO} + \Delta m_G) = m_{MPL} + \Delta m_L \tag{4.9}$$

This can be described as:

$$(m_{MTO} + \Delta m_G) \cdot (1 - \frac{m_{ZP}}{m_{MTO}}) = m_{MPL} + \Delta m_L \tag{4.10}$$

The maximum payload is given as payload fraction. To do this, the entire equation is divided by m_{MTO}. The expression $1 - m_{ZP}/\, m_{MTO}$ remains unchanged:

$$(1 + \frac{\Delta m_G}{m_{MTO}}) \cdot (1 - \frac{m_{ZP}}{m_{MTO}}) = \frac{m_{MPL}}{m_{MTO}} + \frac{\Delta m_L}{m_{MTO}} \tag{4.11}$$

Forming and dividing by $\Delta m_L / m_{MTO}$ results:

$$\frac{\frac{\Delta m_G}{m_{MTO}}}{\frac{\Delta m_L}{m_{MTO}}} = \frac{\frac{\frac{m_{MPL}}{m_{MTO}} + \frac{\Delta m_L}{m_{MTO}}}{1 - \frac{m_{ZP}}{m_{MTO}}} - 1}{\frac{\Delta m_L}{m_{MTO}}} \tag{4.12}$$

The ratio on the left side of the equation corresponds to the mass growth factor k_{MGW}. The right side of this equation is simplified.

The mass growth factor is thus obtained according to the Equation (4.4) assumed above:

$$k_{MGW} = \frac{1}{\frac{m_{MPL}}{m_{MTO}}}$$

The simplification of Equation (4.12) to Equation (4.4) is given in Appendix C.

This has confirmed the assumption that the reciprocal value of the maximum payload fraction corresponds to the mass growth factor. In addition, a further simple equation is given in which the mass growth factor can be calculated directly by applying the operating empty mass and fuel fraction.

The Equation (4.4) can be paraphrased as:

$$k_{MGW} = \frac{m_{MTO}}{m_{MPL}} \tag{4.13}$$

Forming from the "first law of aircraft design" (3.2) to $m_{MTO}/\ m_{MPL}$ results:

$$k_{MGW} = \frac{m_{MTO}}{m_{MPL}} = \frac{1}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \tag{4.14}$$

Depending on the available mass fraction, the mass growth factor can be calculated directly using Equation (4.4) or (4.14).

4.4 Mass Growth Factor Depending on Technology and Range Requirements

In this section, the extent to which the mass growth factor is influenced by the technology and the requirement will be examined. The Breguet factor B is considered to check the dependence on the technology. This includes both, the maximum glide ratio E as well as the specific fuel consumption c as technology parameters. One of the important requirements is the range R.

In the previous section it was derived that the mass growth factor can be determined by the mass fractions. Equation (4.14) is used for the sensitivity test. Initially, functions of the operating empty mass and fuel mass fraction are determined depending on the technology parameter B and requirement R.

The project work from Lehnert (2018) provides several statistically based formulas for the determination of the operating empty mass fraction. One formula represents the operating empty mass fraction as a function of the range:

$$\frac{m_{OE}}{m_{MTO}} = 0.5967 - 0.00000166 \cdot R \tag{4.15}$$

A function is to be derived for the fuel mass fraction, which is dependent on both, the Breguet factor and the range. The "Breguet range equation" according to Scholz 2012 is used for the derivation:

$$R = \frac{E \cdot v}{c \cdot g} \ln \frac{m_1}{m_2} \tag{4.16}$$

In this case, m_1 is the take-off mass. For the landing mass m_2 applies:

$$m_2 = m_1 - m_F \tag{4.17}$$

The part before the logarithm of Equation (4.16) is the so-called Breguet factor B:

$$B = \frac{E \cdot V}{c \cdot g} \tag{4.18}$$

Thus Equation (4.16) can be paraphrased as:

$$R = B \ln \frac{m_1}{m_1 - m_F} \tag{4.19}$$

With $m_1 = m_{MTO}$ results:

$$R = B \ln \frac{1}{1 - \frac{m_F}{m_{MTO}}} \tag{4.20}$$

Dissolution according to the fuel mass fraction:

$$\frac{m_F}{m_{MTO}} = 1 - e^{-\frac{R}{B}} \tag{4.21}$$

There is now a function for the operating empty mass fraction, depending on range, and a function for the fuel mass fraction, depending on range and the Breguet factor. Inserting these two functions into Equation (4.14) gives:

$$k_{MGW} = \frac{1}{1 - (0.5967 - 0.00000166 \cdot R) - \left(1 - e^{-\frac{R}{B}}\right)} \tag{4.22}$$

This formula is used for the sensitivity test. The aircraft used is the A320-200. For this aircraft the following values from Table 4.3 are required.

Table 4.3 Sensitivity Test - Parameters for the A320-200

Parameters		Value (unit)	Source
Cruising speed	V_{cr}	230 m/s	De Grave 2017
Range (at maximum payload)	R	1600 NM resp. 2963200 m	Jackson 2011
Glide ratio in cruise flight	E	17.91	De Grave 2017
Specific fuel consumption	c	1.63 E-05 kg/N/s	De Grave 2017

4.5 Mass Growth Factor Sensitivity with Range

When checking the dependence of the mass growth factor on the range, the Breguet factor is calculated using the values in Table 4.3 and is not changed thereafter. For the A320-200, this gives a Breguet factor value according to Equation (4.18) of B = 25761242.8 m.

At a range of 1600 NM and using Equation (4.22), a mass growth factor of k_{MGW} = 3.3636 results. In Section 3.2 a factor of 3.9446 was determined for the A320-200. This was done using the masses that were researched for the finished aircraft.

In this case, however, the mass fractions are determined with a statistically based formula and by using dimensioning approaches with estimated values for the glide ratio and specific fuel consumption. The deviating mass growth factor is not critical in this context, as it is mainly a matter of sensitivity.

For the sensitivity test the range of 1600 NM is increased and the change of the mass growth factor is recorded. The results are summarized in Table 4.4.

Table 4.4 Results of sensitivity test depending on the requirement

Range [NM]	Change in range [%]	k_{MGW}	Effect on k_{MGW} [%]	Effect on original additional kg [%]
1600	0	3.36	0.00	
1616	1	3.37	0.34	1.13
1632	2	3.38	0.68	2.27
1760	10	3.47	3.45	11.62
1920	20	3.60	7.11	23.92

If the range is changed by 1%, the mass growth factor changes by 0.34%. If the range growths by 10%, the factor growths by 3.45%. It should be noted, however, that the sensitivity depends very much on the initial range at maximum payload. If the range is twice as large, the sensitivities will be higher.

In the right column, the effect is shown on the original additional kg. This shows by how much more the aircraft is loaded. A local mass growth of 1 kg (original additional kg) results in a factor of 3.3636. This means that the local mass growth of 1 kg leads to a global mass growth of 3.3636 kg according to Equation (4.1).

By turning the range adjustment screw, the mass growth factor changes. In this case to 3.3749, if the range is increased by 1%. This again means a global mass growth of 3.3749 kg.

If one look at how much the global mass growth has risen, you can see:

$$3.3749 \text{ kg} - 3.3636 \text{ kg} = 0.0113 \text{ kg}$$

This value in relation to the original additional kg corresponds to 1.13%.

Furthermore, it can be concluded from this study, that the more the range growths in size, the greater the mass growth factor. Aircraft with such a longer range transport more fuel, which growths its fraction and thus the mass growth factor is correspondingly high. The fraction of fuel growths with the range, and the operating empty mass fraction decreases slightly. In sum, the two mass fractions go towards 1.

Equation (4.14) can also be paraphrased as:

$$k_{MGW} = \frac{1}{1 - \left(\frac{m_F}{m_{MTO}} + \frac{m_{OE}}{m_{MTO}}\right)} \tag{4.23}$$

The more the sum of the two mass fractions approaches 1, the more the mass growth factor growths. A further range can only be reached up to the point where $k_{MGW} = \infty$ is. Figure 4.5 shows how the mass fractions and the mass growth factor behave with increasing range.

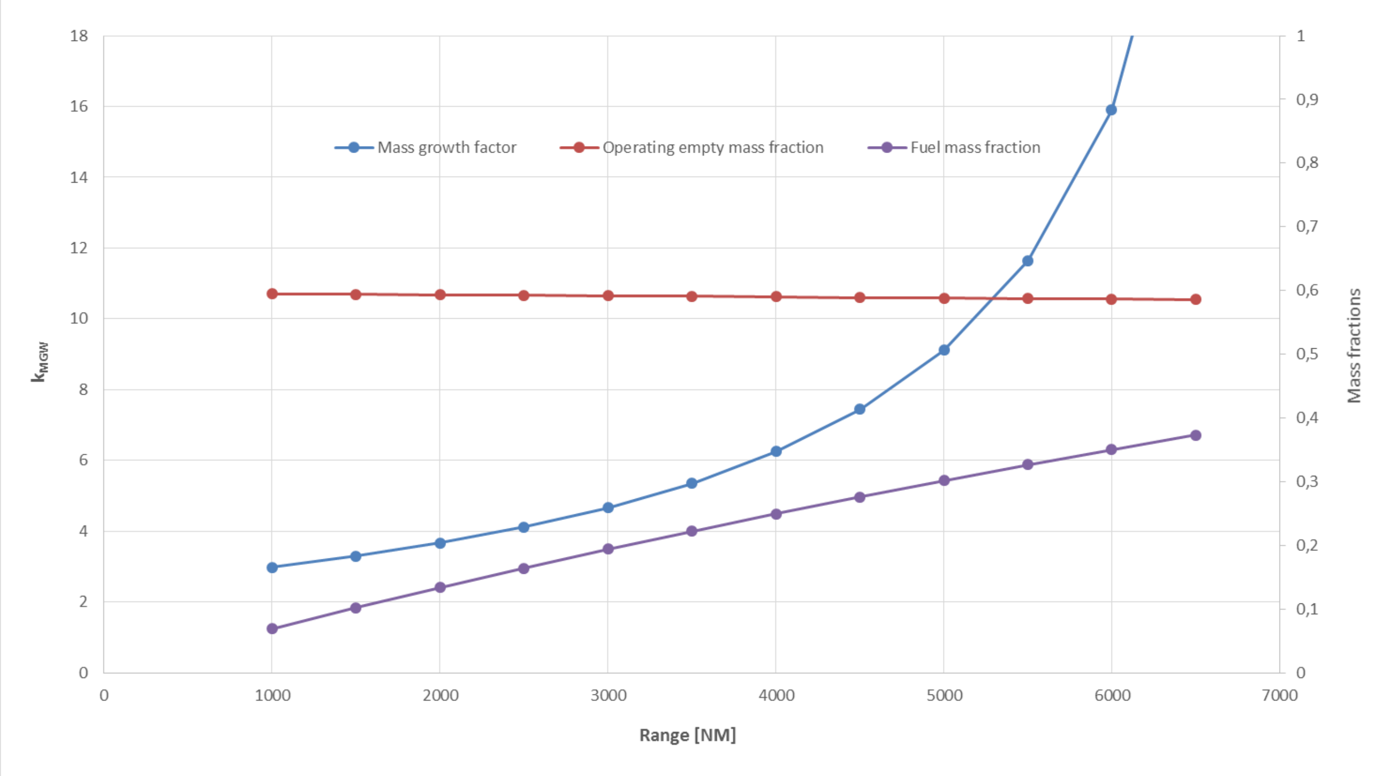

Figure 4.5 Mass growth factor and mass fraction over the range (Screenshot: Sensitivity_with_Range.xlsm)

4.6 Mass Growth Factor Sensitivity with Technology

When checking the dependence of the mass growth factor on technology, the range is kept constant and the Breguet factor is changed. The range in this case is 1600 NM. We begin with the Breguet factor of B = 25761242.8 m. The mass growth factor therefore has a value of k_{MGW} = 3.3636.

By changing the Breguet factor and the resulting effect on the mass growth factor, the sensitivity of this factor as a function of the glide ratio E or the specific fuel consumption c can be closed. If the Breguet factor is increased by 1%, this corresponds to a 1% growth in glide ratio or a 1% reduction in specific fuel consumption.

For the sensitivity test, the Breguet factor of B = 25761242.8 m is increased and the change in the mass growth factor was recorded. The results are summarized in Table 4.5.

Table 4.5 Results of sensitivity test depending on the technology

Breguet factor [m]	Change in Breguet factor [%]	k_{MGW}	Effect on k_{MGW} [%]	Effect on original additional kg [%]
25761242.75	0	3.36	0.00	
26018855.18	1	3.35	0.34	1.15
26276467.61	2	3.34	0.67	2.26
28337367.03	10	3.26	3.06	10.28
30913491.30	20	3.17	5.48	18.45

If the Breguet factor is changed by 1%, the mass growth factor changes by 0.34%. For a 10% conversion, it is 3.06%. The right-hand column again shows the effect on the original additional kg.

If the Breguet factor is increased, the mass growth factor becomes smaller. A growth in the Breguet factor means an improvement and thus a removal from the critical state. The sum of the operating empty mass and fuel mass fraction in Equation (4.23) thus moves further and further away from 1.

A deterioration or reduction in the Breguet factor can be obtained by increasing the range, which in turn will lead to a growth in the mass growth factor. The Table 4.6 shows the results obtained by reducing the Breguet factor.

Table 4.6 Results of sensitivity test depending on technology - with reduction

Breguet factor [m]	Change in Breguet factor [%]	k_{MGW}	Effect on k_{MGW} [%]	Effect on original additional kg [%]
25761242.75	0	3.36	0.00	
25503630.33	1	3.37	0.35	1.18
25246017.90	2	3.38	0.71	2.38
23185118.48	10	3.49	3.96	13.31
20608994.20	20	3.67	9.29	31.24

Comparing the effects in terms of increasing the mass growth factor and the original additional kg, it is striking that the Breguet factor has a greater effect than increasing the range, at least on this aircraft. In any case, it can be said that the mass growth factor is greater with a higher specific fuel consumption and a lower glide ratio.

4.7 Mass Growth Factor as an Economical Indicator

The mass growth factor as the reciprocal of the ratio of maximum payload to maximum take-off mass can also be considered as a kind of economic efficiency factor.

According to Scholz 2015, the Direct Operation Costs (DOC) are composed of:

$$C_{DOC} = C_{DEP} + C_{INT} + C_{INS} + C_F + C_M + C_C + C_{FEE} \tag{4.24}$$

The take-off mass includes the material that makes up the aircraft price and fuel. The depreciation is related to the price and this in turn is related to the kg aircraft. Therefore the operating empty mass corresponds to the depreciation. The fuel mass is the same as the fuel and the requirement is included as payload.

Maintenance costs are not considered. Thus, both fixed and variable costs are included. Thus the maximum take-off mass can be assumed approximately as a substitute value for the DOC:

$$DOC \approx m_{MTO} = m_{OE} + m_F + m_{PL} \tag{4.25}$$

According to Scholz 2015, the DOC can be viewed in different ways. The DOC can be considered as costs "which are incurred for an aircraft (aircraft, index: a/c) of a fleet within one year (annual costs, index: a). They are annual aircraft annual costs". (Scholz 2015, Section 14, p.18):

$$C_{DOC} = C_{a/c,a} \tag{4.26}$$

According to Scholz 2015, the aircraft trip costs $C_{a/c,t}$ are obtained, when the DOC are calculated for a defined flight (flight,cycle or trip, index: t):

$$C_{a/c,t} = \frac{C_{a/c,a}}{R \cdot n_{t,a}} \tag{4.27}$$

Here is $n_{t,a}$ the number of flights an aircraft can perform in a flight mission (number of flights per year) and R is the range.

The seat-mile costs are the result, when the DOC are based on "the range R and the number of seats n_s or the maximum number of passengers n_{PAX}." (Scholz 2015, Section 14, p.18):

$$C_{s,m} = \frac{C_{a/c,t}}{R \cdot n_{PAX}} = \frac{C_{a/c,a}}{R \cdot n_s \cdot n_{t,a}} \tag{4.28}$$

Except for the cargo, the payload is proportional to the number of seats. The number of flights depends on how efficiently the aircraft can be operated. The range is given by the specification, among other things. The denominator therefore acts as a substitute value for the maximum payload. The denominator also contains the DOC, which is almost equal to the maximum take-off mass.

Thus follows:

$$\frac{DOC}{n_{t,a}} \approx \frac{m_{MTO}}{m_{PL}} = \frac{1}{\frac{m_{PL}}{m_{MTO}}} \tag{4.29}$$

The mass growth factor is thus related to the DOC per seat (seat costs). If the aircraft is able to transport many people who share the costs, the aircraft has a large payload. Therefore it has a small mass growth factor and is efficient.

If one wants to use the mass growth factor to consider a comparison of aircraft with regard to their efficiency effect, it should be noted that they must have the same range and the same cruising speed. The cruising speed has a direct effect on the number of flights per year. This is explained in the next section.

4.8 The Problem of a Large Mass Growth Factor

At this point it is interesting to look at the Concorde. This one has a mass growth factor of nearly 15. With this aircraft one found out that there is a DOC problem. However, it was assumed that due to its speed more tickets are sold according to the following motto: If the aircraft flies twice as fast, it can also sell twice as many tickets.

There were two approaches to solving the transport problem at that time. Either they built a large aircraft, the Boeing 747, or a fast aircraft, the Concorde. In the beginning it was not obvious that the Boeing 747 was more economical. It was assumed that both would be equally profitable and that the Concorde would also be much faster. For this, however, the same mass growth factors would have had to be available. The Concorde has a much larger mass growth factor than the Boeing 747 and is therefore able to transport only a few passengers.

The very high speed of the Concorde had a negative effect in the end. To estimate the seat-mile-costs after Equation (4.28) one needs the number of flights per year. This gives it after Scholz 2015 with:

$$n_{t,a} = \frac{U_{a,f}}{t_f} \tag{4.30}$$

Here is $U_{a,f}$ the annual aircraft usage and t_f the defined aircraft time, which according to Scholz 2015 results from the ratio of range to cruising speed:

$$t_f = \frac{R}{V_{CR}} \tag{4.31}$$

So if you fly faster, the defined aircraft time is smaller. A smaller flight time means a high number of flights per year. This in turn means that the seat-mile costs are significantly lower when flying faster for the same cost. The costs will also growth significantly, because the flight continues at higher speed in the same available time (variable costs), whereas the fixed costs (depreciation, insurance, etc.) are independent of the route and are better distributed over the increased number of kilometers when the flight continues.

On the basis of the Concorde, it can thus be stated that an excessively large mass growth factor is disastrous. To support this statement a quotation is quoted, which has dealt with the Concorde in detail.

„What happened is that it [Concorde] had to be redesigned twice, perhaps two and half times, completely because of weight growth, and in the course of redesigning it, more expensive methods of construction had to be adopted and more expensive materials, and the result was that the production cost turned out to be about four times as high as had been anticipated. The economics of it went completely. They had never been very good, but it became obvious from the mid-1960s to anybody that this was economically disastrous, even there were doubts about whether it was technically possible which still had to be resolved." (Glancey 2015).

5 Discussion

Various definitions of the mass growth factor are given. One of these definitions accurately reflects the mass growth factor. Accordingly, the factor quantifies how much the take-off mass is increased, caused by a local mass growth. In the present work two methods for the determination of the mass growth factor were derived.

The occurrence of a mass snowball effect can be clearly shown by the first method by iteration. A mass growth iteration is shown. Thus a convergence of the take-off mass increase takes place when the mass is increased locally. It has to be considered, that the length of the iteration can vary from aircraft to aircraft until the maximum take-off mass reaches the limit.

This iteration method shows, that there is no dependence of the mass growth factor on the size of the aircraft. So there is a contradiction to the opinion of the book Jenkinson 1999, which made the assumption, that a bigger aircraft has a higher factor. The comparison of two Boeing aircraft of Section 3.2 confirms the findings of this work. While the mass growth factor of the Boeing 737-700 is 6, the much larger Boeing 787-9 has a factor of 4.6.

The factor size is bound to the mass fractions. Aircraft with a larger operating empty mass and fuel mass fraction and consequently a lower payload fraction have a higher mass growth factor. From the SAWE paper Saelman 1973 it is already evident that a high mass growth factor exists with a small payload fraction. The second method for determining the mass growth factor confirms this correlation.

By the second method a simple Equation (4.4) resp. (4.14) is provided to calculate the mass growth factor directly without iteration. Therefore the factor can be determined easily by the payload fraction or the operating empty mass and fuel mass fraction. Comparable formulas can already be found in Ballhaus 1954, Fürst 1999 and SAWE 2019. The formulas from the literature also include an additional subdivision of the operating empty mass. A formula derivation is not available for Fürst 1999 and SAWE 2019. The derivation cannot be fully understood in Ballhaus 1954, which is not the case in this project work.

At Roskam 1989, the mass growth factor for an increase in operating empty mass and payload is looked at. However, the mass growth factor for an increase in payload is the same factor as for an increase in operating empty mass. The decisive factor is the increase of a fixed mass. A distinction whether one kg of additional weight is in the wing mass (operating empty mass) or in the payload is not important for aircraft take-off. The two SAWE papers Ballhaus 1954 and Saelman 1973 already contain this statement.

The different literature sources also contain values for the mass growth factor. Exactly which aircraft were used to determine these values remains unknown. Furthermore it can be seen that there is a difference between the values of the factor in the literature and the values calculated in this work. The results of this work do not confirm the values at Saelman 1973, Fürst 1999 and Müller 2003. For an aircraft with an operating empty mass of 0.5, a factor value of 1.3 was calculated with the data from Torenbeek 1988. For numerous aircraft mentioned in Chapter 3.2, the operating empty mass fraction is approximately 0.5, but the mass growth factor is not as low for any of these aircraft.

In the factor analysis of technology and requirement the following can be determined: Aircraft with higher specific fuel consumption and lower glide ratio have a higher mass growth factor. With a longer range, the mass growth factor increases significantly. Already in the paper Ballhaus 1954 it was shown how much the factor depends on the range. Of great importance is the fact that at a certain range the mass growth factor develops towards infinity. The reason for this is again that at a long-range and consequently a large fuel mass, almost nothing is left of the 1 in the denominator of formula (4.14).

Furthermore, the mass growth factor provides information about the DOC per seat (seat costs). An aircraft with a low mass growth factor has a higher efficiency and economy. The aircraft can therefore transport a large number of people who together bear the costs. The ATR 72 has a factor of only 3.1 and convinces with its good performance among the aircraft sold in large numbers. In contrast, the Concorde, which has a factor of almost 15, can be described as an "economic disaster".

6 Summary and Recommendations

The mass snowball effect is the iterative interaction between the take-off mass on the one hand and the component masses of the system, structure or drive on the other. If the operating empty mass of the aircraft increases, fuel consumption also increases. As a result, larger fuel tanks become necessary, which primarily leads to an increase in the maximum take-off mass, and structural component adjustments are also necessary. The resulting increase in component masses results in a higher fuel mass and thus a higher take-off mass. This results in an iterative process.

The so-called mass growth factor quantifies this effect. It quantifies the extent of the increase in take-off mass that results from a local increase in mass. In the course of some phases of aircraft design, the mass increases rather than decreases compared to the original estimates. Therefore the factor is called mass growth factor. From a mathematical point of view, however, there is no difference to a mass reduction factor.

Based on literature research, a presentation of first findings as well as investigations on mass growth and mass growth factor is made. If the operating empty mass has to be increased, the design draft also has to be revised, which in turn has an effect on the take-off mass. If the take-off mass varies, it must be accepted that the aircraft performance will be restricted. In the course of the literature research, an explanation of the methods and formulas is also provided. The values of the mass growth factor are also given; this is used to determine a reference value for the factor.

By means of the "first law of aircraft design" and by iteration, a method for determining the mass growth factor is developed. Here the mass growth factor represents the ratio of the increase of the global mass (take-off mass) by the arbitrary increase of the local mass (empty mass). In this method an iteration of the mass growth is represented, where there is a convergence of the take-off mass increase with local mass increase. The lowest mass growth factor that is possible is 1. From this it follows that in case of an empty mass increase there is no need for further measures such as structural strengthening.

The mass growth factor is calculated with this method for such a large number of renowned passenger aircraft that it covers 90% of the passenger aircraft currently flying. If these aircraft are categorized, the value of the mass growth factor can be summarized as follows: Subsonic passenger aircraft have an average mass growth factor of 4.2, narrow-body planes of 3.9 and wide-body planes (whose routes tend to be longer) of 4.9, whereas the factor for supersonic passenger aircraft is approximately 14.

It can be stated that the mass growth factor for some aircraft is the same as the reciprocal of the payload fraction. This picture recurs for all aircraft. With this knowledge the derivation of an equation is done, with which the factor can be calculated directly (without iteration).

The mass growth factor is analyzed in more detail using the two methods of determination - iteration method and equation. It can be seen that the value of the mass growth factor does not depend on the size of the aircraft, but on the fractions of the masses. Aircraft with a larger operating empty mass and fuel mass fraction and a smaller payload fraction have a higher mass growth factor.

The decisive factor is the increase of a fixed mass, which makes it the same for the payload and the operating empty mass. If one examines to what extent the mass growth factor depends on requirements and technology, it can be seen that a higher specific fuel consumption and a lower glide ratio is accompanied by a larger factor. With increasing range, the factor also increases considerably.

Furthermore, there is a correlation between the mass growth factor and the DOC per seat. If the aircraft is able to transport a large number of people, who together bear the costs, the payload fraction of this aircraft is high. The mass growth factor of this aircraft is therefore low and therefore the aircraft is efficient. The negative example of the Concorde shows, how a too high mass growth factor can have a considerable effect on the economic efficiency.

Since the present project work focused on the determination of the mass growth factor, there was no room for a more detailed investigation of the location of the local mass increase. However, this aspect could be a relevant approach for further investigations.

The decisive factor is where on the aircraft the mass growth takes place. If the mass increases at the wing, this is not of high importance, because there the mass is carried equally. Engines on the wing can counteract the increase of the bending moment. However, this is different if the mass is increased locally in the front area of the aircraft. In this case, assuming the completion of the aircraft construction, an increase in the production of downforce must take place. Therefore a variation of the factor with the weight increase location along the fuselage station is possible.

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Appendix A Aircraft Making Up 90% of the World Fleet

Table A.1 90% of current aircraft (according to Robson 2019)

Aircraft Type	Number in Operation	Percent of total	Sum of most aircraft types in percent
Boeing 737-800	4804	16.83%	16.83%
A320	4135	14.48%	31.31%
A320neo	658	2.30%	33.61%
A321neo	160	0.56%	34.18%
A321	1650	5.78%	39.95%
Boeing 737 Max 8		0.00%	39.95%
A319	1249	4.37%	44.33%
Boeing 737-700	1005	3.52%	47.85%
ATR72	775	2.71%	50.56%
Boeing 777-300(ER)	829	2.90%	53.47%
Embraer 175	595	2.08%	55.55%
Boeing 787-9	451	1.58%	57.13%
A330-300	707	2.48%	59.61%
Boeing 767-300	622	2.18%	61.79%
A350-900	261	0.91%	62.70%
Boeing 757-200	600	2.10%	64.80%
A330-200	547	1.92%	66.72%
Boeing 737-900	550	1.93%	68.64%
De Havilland Canada Dash 8-400	502	1.76%	70.40%
Embraer 190	495	1.73%	72.14%
Boeing 737 Max TBD		0.00%	72.14%
Bombardier CRJ900	444	1.56%	73.69%
A220	77	0.27%	73.96%
Boeing 777-200	431	1.51%	75.47%
Embraer ERJ-145	422	1.48%	76.95%
Boeing 737 Max 10		0.00%	76.95%
Boeing 787-8	328	1.15%	78.10%
Boeing 747-400	331	1.16%	79.26%
Boeing 737-300	297	1.04%	80.30%
A380	233	0.82%	81.11%
Viking Air Twin Otter Total	263	0.92%	82.04%
Bombardier CRJ700	266	0.93%	82.97%
Boeing 737-400	261	0.91%	83.88%
ATR42	213	0.75%	84.63%
A330neo	16	0.06%	84.68%
Boeing MD-80	222	0.78%	85.46%
Boeing 777F	164	0.57%	86.04%
A300	202	0.71%	86.74%
Saab 340	202	0.71%	87.45%
Fairchild Metro/Merlin	196	0.69%	88.14%
Beechcraft 1900D	184	0.64%	88.78%
A350-1000	21	0.07%	88.85%
Embraer 195	160	0.56%	89.42%
Embraer 170	160	0.56%	89.98%
Boeing 787-10	29	0.10%	90.08%
Aircraft total	**28550**		

Appendix B Mass Growth Factor for Four Aircraft Categories

Table B.1 Categorization of current aircraft (Screenshot: Evaluation_90%_aircraft_+_category.xlsm)

	Wide-Body	Narrow-Body	Subsonic	Supersonic
	Boeing 777-300 ER	Boeing 737-800	Boeing 737-800	Concorde
	Boeing 787-9	A320-200	A320-200	TU-144
	A330-300	A320neo	A320neo	
	Boeing 767-300	A321neo	A321neo	
	A350-900	A321-200	A321-200	
	A330-200	A319-100	A319-100	
	Boeing 777-200	Boeing 737-700	Boeing 737-700	
	Boeing 787-8	ATR 72-500	ATR 72-500	
	Boeing 747-100	Embraer 175	Boeing 777-300 ER	
	A380-841	Boeing 757-200	Embraer 175	
	A330-800neo	Boeing 737-900	Boeing 787-9	
	Boeing 777F	De Havilland Canada Dash 8-400	A330-300	
	A300-600R	Embraer 190	Boeing 767-300	
	A350-1000	Bombardier CRJ900	A350-900	
	Boeing 787-10	A220-100	Boeing 757-200	
		Embraer 145 ER	A330-200	
		Boeing 737-300	Boeing 737-900	
		Viking Air Twin Otter Total 400	De Havilland Canada Dash 8-400	
		Bombardier CRJ700	Embraer 190	
		Boeing 737-400	Bombardier CRJ900	
		ATR42	A220-100	
		Boeing MD-81	Boeing 777-200	
		Saab 340 B Plus	Embraer 145 ER	
		Fairchild Metro/Merlin	Boeing 787-8	
		Beechcraft 1900D	Boeing 747-400	
		Embraer 195	Boeing 737-300	
		Embraer 170	A380 [A380-841]	
			Viking Air Twin Otter Total 400	
			Bombardier CRJ700	
			Boeing 737-400	
			ATR42	
			A330-800neo	
			Boeing MD-81	
			Boeing 777F	
			A300-600R	
			Saab 340 B Plus	
			Fairchild Metro/Merlin	
			Beechcraft 1900D	
			A350-1000	
			Embraer 195	
			Embraer 170	
			Boeing 787-10	
k_{MGW}	**4.912566593**	**3.856368633**	**4.23358219**	**13.82083333**

Appendix C Derivation: Mass Growth Factor – Inverse of Payload Fraction

It starts with the Equation (4.12):

$$\frac{\frac{\Delta m_G}{m_{MTO}}}{\frac{\Delta m_L}{m_{MTO}}} = \frac{\frac{\frac{m_{MPL}}{m_{MTO}} + \frac{\Delta m_L}{m_{MTO}}}{1 - \frac{m_{ZP}}{m_{MTO}}} - 1}{\frac{\Delta m_L}{m_{MTO}}}$$

To simplify matters, the following terms are replaced as follows:

$$\frac{\Delta m_L}{m_{MTO}} = y \tag{C.1}$$

$$\frac{m_{MPL}}{m_{MTO}} = x \tag{C.2}$$

Transforming the Equation (4.5) according to m_{MPL}/m_{MTO} leads to:

$$\frac{m_{MPL}}{m_{MTO}} = 1 - \frac{m_{OE}}{m_{MTO}} - \frac{m_F}{m_{MTO}} \tag{C.3}$$

Thus the Equation (4.6) can be rewritten as:

$$\frac{m_{ZP}}{m_{MTO}} = 1 - \frac{m_{MPL}}{m_{MTO}} \tag{C.4}$$

By inserting the simplification (C.2) it follows:

$$\frac{m_{ZP}}{m_{MTO}} = 1 - x \tag{C.5}$$

The left side of Equation (4.12) corresponds to the mass growth factor. Inserting (C.1), (C.2) and (C.5) into Equation (4.12) leads to a clear presentation:

$$k_{MGW} = \frac{\frac{x + y}{1 - (1 - x)} - 1}{y} \tag{C.6}$$

This equation is simplified in the following steps. Dissolving the bracket leads to:

$$k_{MGW} = \frac{\frac{x+y}{x} - 1}{y} \tag{C.7}$$

This matches with:

$$k_{MGW} = \frac{1 + \frac{y}{x} - 1}{y} \tag{C.8}$$

Omission of ones:

$$k_{MGW} = \frac{\frac{y}{x}}{y} \tag{C.9}$$

By shortening it finally follows:

$$k_{MGW} = \frac{1}{x} \tag{C.10}$$

Resetting (C.2) leads to the formula (4.4) for the direct calculation of the mass growth factor:

$$k_{MGW} = \frac{1}{\frac{m_{MPL}}{m_{MTO}}}$$