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Bachelor thesis

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A systematic approach to evaluate uncertainty of a static strain measurement

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Zusammenfassung

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Thema der Bachelorthesis

Ein systematisches Verfahren zur Ermittlung der Messunsicherheit einer statischen Dehnmessung

Stichworte

Dehnungsmessung, Messunsicherheit, DMS, Winkelversatz, Anbringungsfehler, Messkorrektur, Messfeheler, K-Faktor.

Kurzzusammenfassung

Die Dehnungsmessung ist eine sehr grundlegende Methode, die ein breites Anwendungsspektrum in Forschung und Industrie hat. Und wie bei jeder Messung, sollte die Qualität der Messung möglichst genau betrachtet werden. Dies wird normalerweise anhand der Messunsicherheit ausgedrückt. Indem ein Intervall möglicher Werte definiert wird, innerhalb dessen der wahre Wert der Messung liegt.

Das Wissen über das physikalische Prinzip des Messprozesses und die Kenntnisse über die Faktoren, die das Messergebnis beeinflussen können, sind die beiden Hauptschlüsselelemente für die Bewertung einer Messunsicherheit. Daher ist ein mathematisches Modell von grundlegender Bedeutung, das diese Schlüsselelemente zusammenfasst und die Beziehung zwischen der gemessenen Größe und den Eingangsgrößen beschreibt.

Das Ziel dieser Arbeit ist es, einen Ansatz zur Bewertung der Messunsicherheit der statischen Dehnungsmessung mit einem einachsigen Dehnmessstreifen mit einer viertel Wheatstone-Brückenkonfiguration vorzustellen, um die Reproduzierbarkeit und Vergleichbarkeit von Dehnungsmessungen sicherzustellen. Die Modellierung der Messunsicherheit basiert auf dem Leitfaden zum Ausdruck der Messunsicherheit (*Guide to the Expression of Uncertainty in Measurement*). Das Modell befasst sich mit der Messkette der Dehnungsmessung und der Belastung als Ursache der Dehnung unter Berücksichtigung der Messkorrektur, der klassischen Messunsicherheit und einer erweiterten Messunsicherheit in Bezug auf externe Faktoren.

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Title of the thesis
A systematic approach to evaluate uncertainty of a static strain measurement

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Strain measurement, strain gauge, measurement uncertainty, measurement correction, GUM, angular misalignment.

Abstract

Strain measurement is a very basic and powerful method that has a wide range of applications in research and industry, and as every measurement method, a fair amount of attention should be payed to the quality of the measurement. This is usually expressed using the measurement uncertainty; an attempt to define an interval of possible values, within which the true value of the measurement lies.

The knowledge about the physical principle of the measurement process and the awareness of the factors that might influence the measurement result are the two main key elements of evaluating a measurement uncertainty. Therefore, a mathematical model that summarizes these key elements, describing the relation between the measured value and the input quantities, is of fundamental importance.

The object of this thesis is to introduce an approach to evaluate the uncertainty of static stain measurement using a single strain gauge with a quarter Wheatstone bridge configuration, to ensure the reproducibility and the comparability of strain measurements. The modelling of the measurement uncertainty is based on the *Guide to the Expression of Uncertainty in Measurement*. The model deals with the measurement chain of the strain and the load as a cause of the strain, considering the measurement correction, the classical measurement uncertainty and an extended measurement uncertainty related to external factors.

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Note:

All figures, tables and scripts included in this thesis (except for Figure 4.4-2) are generated by the author. The used programs to generate the figures are:

- Matlab
- Inkscape
- Ansys
- Microsoft Excel

1 Introduction

Since 1938 [1]; the strain measurement using a strain gauge has been a fundamental method in the engineering's toolbox that keeps evolving and finding new innovative applications. The quality of such a measurement can be expressed using its uncertainty; through which, an interval of possible values can be assigned to the strain measurement, as the exact value of the measurement is statistically not achievable.

In order to evaluate this measurement uncertainty, it is necessary to investigate all the factors that might affect the measurement. Historically, there have been multiple attempts to investigate these factors individually, which has been documented in the work and literature of well appreciated universities [1] and researchers [20] [21]. Beyond that, the characterization of strain gauges and its integration in measurement has been standardized by a scientific organization [24]. However, the author was not able to find a general model to consider several factors and describes its effects on the measurement.

The aim of this thesis is to introduce an approach to evaluate the uncertainty of a strain measurement using a bonded electric resistance strain gauge. The statistical approach of evaluating the uncertainty is based on the guide to the expression of uncertainty in measurement [2]; therefor, the historical evolution and the basics behind this guide will be introduced in the chapter Guide to the expression of uncertainty in measurement.

The chapter Strain measurement introduces the scientific fundamentals behind a strain measurement. Furthermore, an approach to evaluate the corrected strain will be proposed. This correction will be based on the factors that might affect the measurement; and are present and relevant in a practical application of strain measurement. Based on this approach of evaluating the corrected strain, a method of evaluating the measurement uncertainty will be proposed, relying on the error analysis procedure introduced in the chapter two.

In the last chapter, a study case of an actual strain measurement in the scope of stress analysis will be discussed. This stress analysis is a part of an investigation of blade bearings used in large wind turbines. To offer more context, the project and the test rig will be briefly described, and the specific conditions and settings of the measurement will be documented. Lastly, a sensitivity analysis of the strain measurement will be carried out.

2 Guide to the expression of uncertainty in measurement

As the evaluation of the strain measurement uncertainty will be based on the Guide to the expression of uncertainty in measurement (hereafter GUM) [2], it is necessary to give this guide a proper introduction. In this chapter the history of the GUM will be briefly presented. Furthermore, the systematic approach of the GUM to evaluate measurement uncertainty will be discussed, focusing on the measurement chain and the mathematical model as a foundation of the evaluation.

2.1 About the Guide

Despite the fact that the concept of the measurement uncertainty and combining measurement uncertainties has been acknowledged in the history of measurement science before the first issue of the GUM. The GUM claims to deliver a systematic, consistent and internationally approved approach to evaluate the propagation of uncertainties, which assures the reproducibility and the comparability of the evaluated uncertainties.

The first version of the GUM was published by several organizations (those are listed below) in the year 1995 to establish general rules for evaluating and expressing uncertainty in measurement; the Guide meant to be applied to a wide spectrum of measurements. It is based on the Recommendation 1 (C1-1981) of the *Comité International des Poids et Mesures (CIPM)* [3] and Recommendation INC-1 (1980) *of the Working Group on the Statement of Uncertainties* [4]. The *GUM* was produced by *Working Group 1 of the Joint Committee for Guides in Metrology (JCGM/WG 1).* The latest version of the *Guide* was published in the year 2008 under the document number *JCGM 100:2008 (GUM 1995 with minor corrections)* [2].

The following seven organizations supported the development of the *GUM*, which is published in their name:

BIPM:	Bureau International des Poids et Mesures;
IEC:	International Electrotechnical Commission;
IFCC:	International Federation of Clinical Chemistry;
ISO:	International Organization for Standardization;
IUPAC:	International Union of Pure and Applied Chemistry;
IUPAP:	International Union of Pure and Applied Physics;
OIML:	International Organization of Legal Metrology.

The purpose of the GUM is to provide a universal method for evaluating and expressing the uncertainty of the result of a measurement, allowing the method to be applicable to all kinds of measurements and input data used in the measurement. The GUM focuses also on the quality of the evaluation, considering the consistency of evaluating every uncertainty, by directly deriving it from the components that contribute to it, independently of how these components are grouped or obtained. The transferability is also assured, so it is possible to directly use the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement, that uses the first result as an input parameter [2][5].

It is important to mention that the application of the GUM has certain limitation that had to be dealt with; for this purpose, a follow-up series of documents was issued by the *JCGM*, these are:

JCGM 100	GUM (original edition of 1995 with minor corrections) [2];
JCGM 101	Propagation of distributions using a Monte Carlo method [6];
JCGM 102	Models with any number of output quantities [7];
JCGM 103	Modelling [8];
JCGM 104	Introduction to the GUM [9];
JCGM 105	Underpinning concepts and basic principles [10];
JCGM 106	The role of measurement uncertainty in conformity assessment [11];
JCGM 107	Applications of the least-squares method [12];

The German translation of *GUM (JCGM 100)* in the library of the *Deutsches Institut für Normung* are: [13]

DIN 1319-1	Grundlagen der Messtechnik [13];
DIN 1319-2	Begriffe für Messmittel [14];
DIN 1319-3	Auswertung von Messungen einer einzelnen Messgröße [15];
DIN 1319-4	Auswertung von Messunsicherheit [16].

2.2 Systematic approach

Based on the author's understanding of the GUM [2] and other reviews of the guide [5] and [17] by Sommer K and BRL Siebert, an attempt to summarise the approach of the GUM can be represented as follows:

2.2.1 Measurement chain

The focus of this step is to describe and analyse the measurement in order to define the relation between the output quantity and all input quantities, that might affect the measurement. These quantities could be linked directly to the measurement using the physical principle of the measurement, they could also be external factors due to physical components used in the chain of measurement. According to relevancy, the driving force of a change in the measured quantity could also be considered. Lastly, any correction done to the measurement carries a sure amount of uncertainty that should be taken in account.

2.2.2 Mathematical model

The second step is to describe the knowledge about all input quantities using the appropriate probability-density functions. This could be based on a normal distribution or a student's t-distribution of a repeated observation. It could also be based on an *a priori* distribution as an assumption in the case of lack of information about a certain quantity:

$$X_i = PDF_i(\overline{x_i}, \sigma_i) \qquad Eq\ 2.2\text{-}1$$

Where:

X_i the distribution describing a certain input quantity.

PDF_i the probability density function of the distribution.

$\overline{x_i}$ the median of the distribution; the estimate of X_i.

σ_i the characteristic parameter of the distribution.

According to [2] and [5] there are several possible distributions to be used:

a) Normal distribution (Figure 2.2-1):

With the probability distribution:

$$PDF = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad Eq\ 2.2\text{-}2$$

Where:
The expectation value: $x = \mu$
The standard uncertainty: $u_x = \sigma$
Supports: $x \in (-\infty, +\infty)$

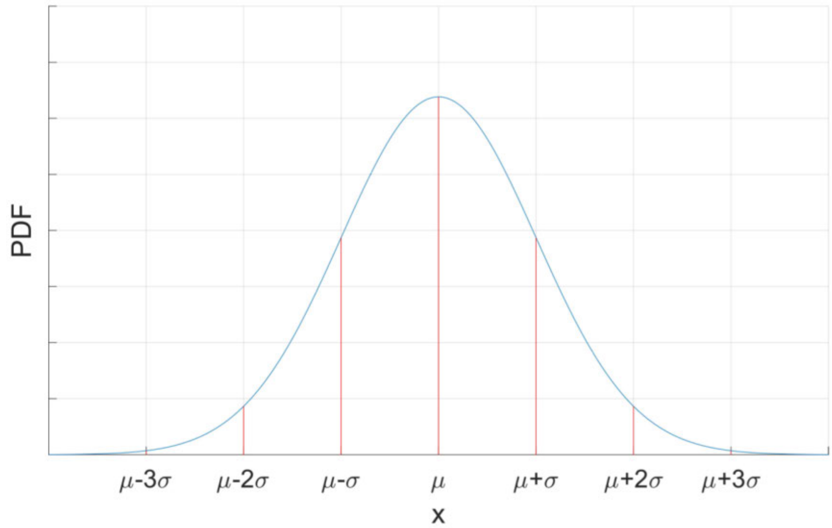

Figure 2.2-1: The probability distribution function of a normal distribution.

b) Harmonic distribution (Figure 2.2-2):
With the probability distribution:

$$PDF = a \sin(x) \qquad Eq\ 2.2\text{-}3$$

Where:
The expectation value: $x = 0$
The standard uncertainty: $u_x = a/\sqrt{2}$
Supports: $x \in (-\pi, +\pi)$

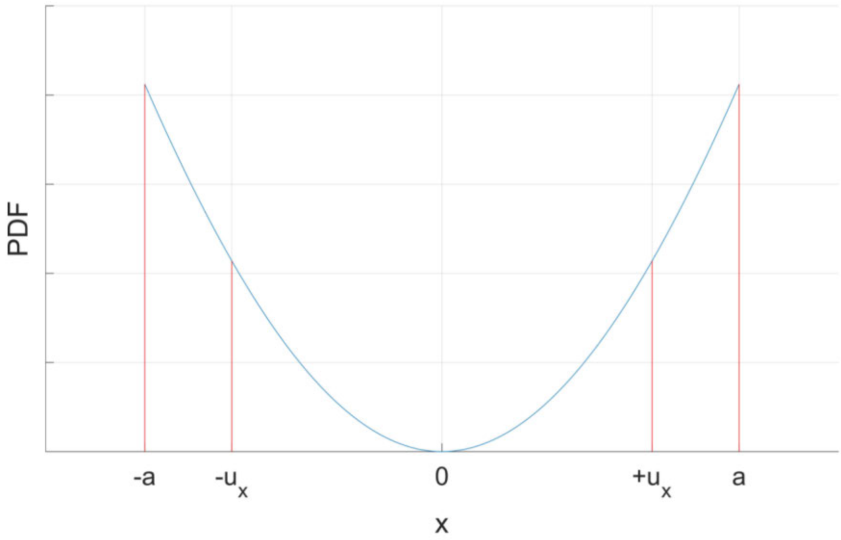

Figure 2.2-2: The probability distribution function of a harmonic distribution

c) Symmetric rectangular *a priori* distribution
With the probability distribution:

$$PDF = constant \qquad Eq\ 2.2\text{-}4$$

Where:
The expectation value: $x = \mu$
The standard uncertainty: $u_x = a/\sqrt{3}$
Supports: $x \in (\mu - a, \mu + a)$

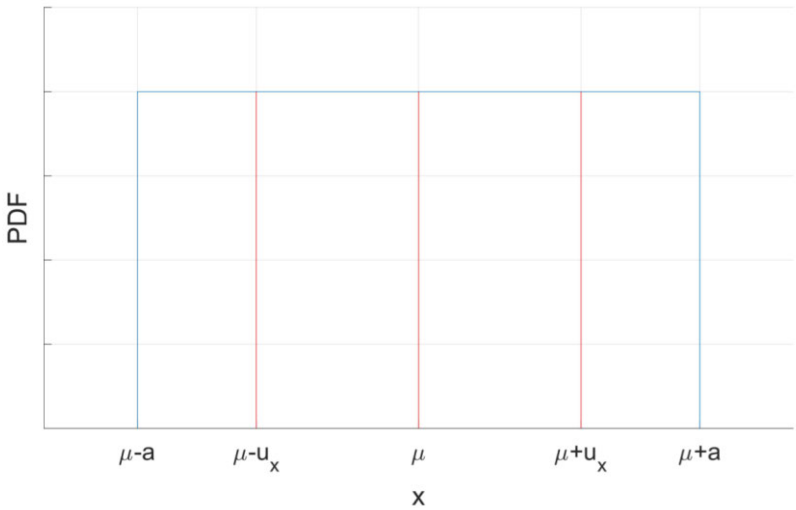

Figure 2.2-3: The probability distribution function of a rectangular a priori distribution.

d) Symmetric triangular *a priori* distribution

With the probability distribution:

$$PDF = constant \begin{cases} (x-\mu)/a, & \mu - a \le x \le \mu \\ 1-(x-\mu)/a, & \mu > x \ge \mu + a \end{cases} \qquad Eq\ 2.2\text{-}5$$

Where:

The expectation value: $x = \mu$

The standard uncertainty: $u_x = a/\sqrt{6}$

Supports: $x \in (\mu - a, \mu + a)$

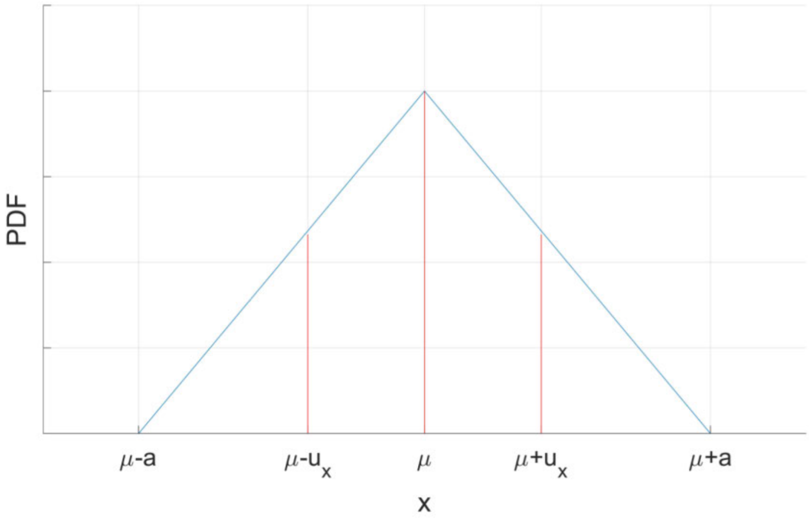

Figure 2.2-4: The probability distribution function of a triangular a priori distribution.

After describing the input quantities with probability-density functions, the interrelation between input quantities and measurand should be expressed via a mathematical model. This is achieved by a model equation; a function that links the measurand to the input quantities:

$$\bar{y} = f_M(\overline{x_1}, \dots, \overline{x_n}) \qquad Eq\ 2.2\text{-}6$$

Where:

$\bar{y}$	the best estimate of the measurand.
f_M	the model-function.
$(\overline{x_1}, \dots, \overline{x_n})$	the medians of distributions of the input quantities.

The best estimate of the measurement will be then the result obtained from this equation using the medians of distributions describing the input quantities. However, the model equation should be linear or linearizable, in order to be able to describe the function using the first-order Taylor expansion in narrow ranges around operating points:

$$\bar{y} + \Delta y = f_M(\overline{x_1} + \Delta x_1, \dots, \overline{x_n} + \Delta x_n) \cong f_M(\overline{x_1}, \dots, \overline{x_n}) + \sum_{i=1}^{n} \frac{\partial f_M}{\partial x_i}(\Delta x_i) \qquad Eq\ 2.2\text{-}7$$

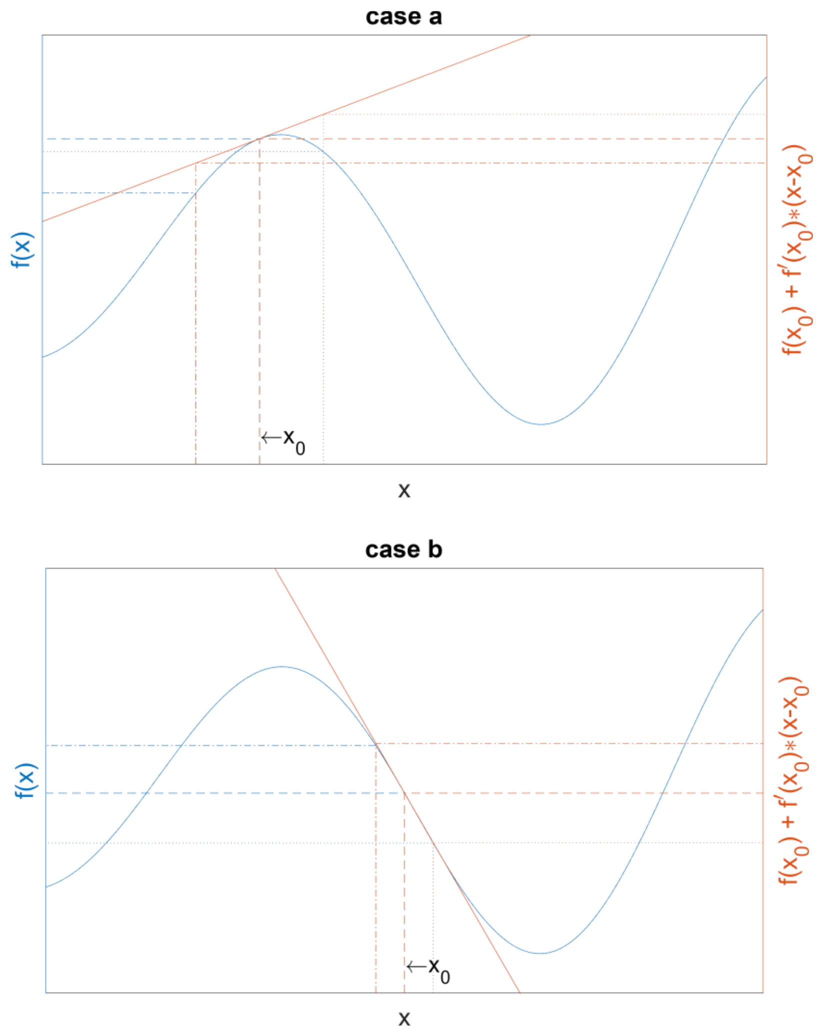

Figure 2.2-5: Example of the linearization of a function

In Figure 2.2-5 a curve of a certain function (the blue curve) and its derivative (the orange solid line) in two operating points (x_0) is shown, to illustrate the validity of the condition given with the Eq 2.2-7. In case a, the approximation of the output around the operating point as the first-order Taylor expansion is incorrect for the shown interval, because the evaluation of the first-order Taylor expansion on the limits of the shown interval differs significantly from the evaluation of the function at the same point. In case b, it is appropriate to approximate the output around the operating point with a first-order Taylor expansion instead of the function itself, as the behaviour of the function in the operating interval is almost linear. The deviation of the approximation from the output value of the function can be tolerated.

The example in Figure 2.2-5 shows a simple function with one input quantity; however, the model equation should satisfy the assumption given by the Eq 2.2-7 for each input quantity in case of a multidimensional function. A sensitivity analysis of the function's curvature is recommended to investigate the assumption given above. This analysis should be done for each measurement setting (in case of dealing with non-linear function), as the width of the interval around the operating point could vary depending on the characteristic parameter of the distribution that describes the input quantity. This analysis should also be repeated in the case of a shift of the operating point.

2.2.3 Measurement uncertainty

The uncertainty in a result of a measurement consists generally of several combined components. These components should consider every factor that might have an impact on the measurement; and should be derived according the mathematical model.

In practice, there are many possible sources of uncertainty in a measurement, the following sources are listed in the *GUM*:

a) incomplete definition of the measurand;
b) imperfect realization of the definition of the measurand;
c) nonrepresentative sampling — the sample measured may not represent the defined measurand;
d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions;
e) personal bias in reading analogue instruments;
f) finite instrument resolution or discrimination threshold;
g) inexact values of measurement standards and reference materials;
h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm;
i) approximations and assumptions incorporated in the measurement method and procedure;
j) variations in repeated observations of the measurand under apparently identical conditions.

The uncertainty components could be categorized according to the method of evaluation of their numerical values. The two categories are:

a) Type A uncertainties, which are evaluated by statistical methods.
b) Type B uncertainties, which are evaluated by means other than statistical methods.

The *GUM* notes that a simple corresponding between the classification into categories A or B and the previously used classification into “random” and “systematic” uncertainties (in error analysis) is not always distinguishable. *GUM* also discourages the use of the term “systematic uncertainty”, as it could be, in some cases, misleading.

Type A evaluation of standard uncertainty:

The uncertainties corresponding to the components in category A are characterized by the estimated variances and the number of degrees of freedom. In the case of a direct measurement of the output quantity, a normal distribution could be used to characterize the measurand and its category A uncertainty. The arithmetic mean value or average value (the median of the distribution) is the best available estimate of the expectation:

$$\bar{q} = \frac{1}{n} \sum_{k=1}^{n} q_k \qquad Eq\ 2.2\text{-}8$$

Where:
q_k a quantity that varies randomly.
n the number of independent observations.
$\bar{q}$ the arithmetic mean.

The experimental variance of the observations, which estimates the variance of the probability distribution of a given quantity, is given by the following equation:

$$s^2(q_k) = \frac{1}{n-1} \sum_{j=1}^{n} \left(q_j - \bar{q}\right)^2 \qquad Eq\ 2.2\text{-}9$$

The variance of the arithmetic mean of the observation is given by:

$$s^2(\bar{q}) = \frac{s^2(q_k)}{n} = \frac{1}{n\,(n-1)} \sum_{j=1}^{n} \left(q_j - \bar{q}\right)^2 \qquad Eq\ 2.2\text{-}10$$

This variance of the mean is the best estimate of the standard deviation of the mean value, as it quantifies how well is the mean value estimates the expectation. This value may be used as a measure of the uncertainty of the measured quantity. Although the variance of the mean is the more fundamental quantity, the standard deviation of the mean (the positive square root of the variance of the mean) is more convenient to use, as it has the same dimension as the measured quantity.

The quality of the estimation of the uncertainty is greatly dependent on the number of observations. Hence, in the case of insufficient number of observations, it is recommended to use the student *t*-distribution instead of a normal distribution; therefor, the

Type B evaluation of standard uncertainty:

According to the Bayes' theorem; the probability of an event can be described based on prior knowledge of other conditions that are related to the event [18].

As an example: For a given function: $y = f(x)$; where x is the input value and y is the output value. If the input quantity can be described as a normal distribution $X = N\ (\mu_x, \sigma_x)$; where μ_x is the median and σ_x is the standard deviation. The output quantity $Y = N\ (\mu_y, \sigma_y)$; is also a normal distribution with the median: $\mu_y = f(\mu_x)$; and the standard deviation $\sigma_y = f'(\mu_x)\sigma_x$. Illustrated in Figure 2.2-6.

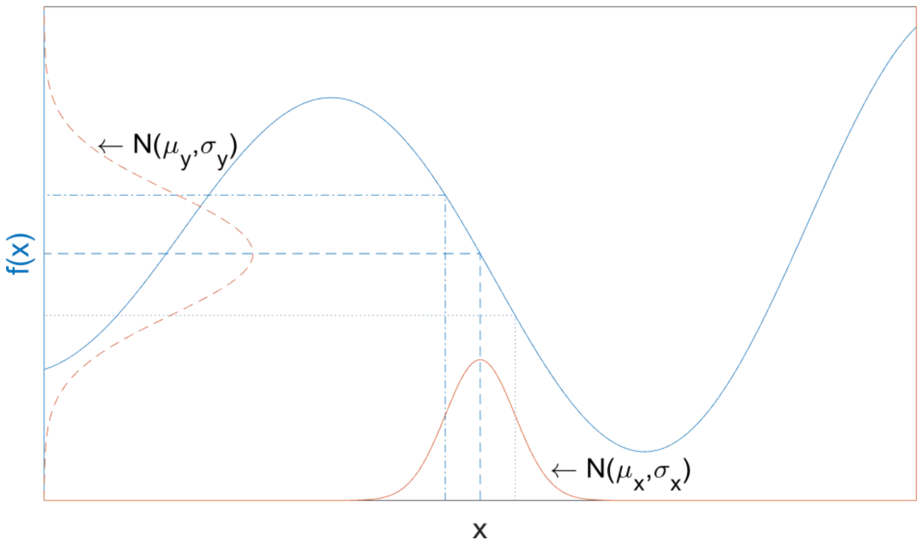

Figure 2.2-6: The output of a function can be described using a normal distribution if the input quantity is describable as a normal distribution.

The GUM simplifies this relation and characterizes the components of the category B by the product of the second power of the sensitivity function (the first derivate of the model function with respect to the examined parameter of the input quantity) and the second power of the estimated standard deviations corresponding to the input quantity as an approximation:

$$u_{x_i}^2(y) = \left(\frac{\partial f_M}{\partial x_i}\right)^2 u^2(x_i) \qquad Eq\ 2.2\text{-}11$$

Where:

u_{x_i} the standard uncertainty of category B of the quantity (y) related to the quantity (x_i)

$\frac{\partial f_M}{\partial x_i}$ the sensitivity function with respect to the quantity (x_i) evaluated at $(\overline{x_i})$

$u(x_i)$ the estimate of the standard uncertainty of the quantity (x_i)

In this approach, the calculated uncertainties of category B can carry a certain amount of deviation from the actual value, as the evaluation of the first-order Taylor expansion cannot be always identical to the evaluation of the function itself; therefor, the sensitivity analysis mentioned in 2.2.2 Mathematical model should be done.

2.2.4 Combined measurement uncertainty

For the combination of measurement uncertainty, the GUM instructs to handle the type B uncertainty components as those of type A. The combined uncertainty can be then calculated by applying the usual method of combination of variances:

$$u_c^2(y) = \sum_{i=1}^{N} u_{x_i}^2(y) \qquad Eq\ 2.2\text{-}12$$

Where:
$u_c(y)$ the combined standard uncertainty of the measurement;
$u_{x_i}(y)$ the uncertainty related to the quantity x_i.

For the evaluation of the combined standard uncertainty, the GUM differentiates between the case of correlation between input quantities and the case of independence of the input quantities. The correlation is defined in the GUM as:

> "the relationship between two or several random variables within a distribution of two or more random variables" ([2] pp:40)

The independence:

> "Two random variables are statistically independent if their joint probability distribution is the product of their individual probability distributions." ([2] pp:47)

Independent input quantities:

In the case of independent input quantities, the combined uncertainty can be added geometrically according to Eq 2.2-12

Correlated input quantities:

In the case of correlated input quantities, the combined standard uncertainty for the correlated quantities should be evaluated according to the following equation:

$$u_c^2(y) = \sum_{i=1}^{N} [c_i^2 u^2(x_i)] + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} [c_i c_j\, u(x_i)\, u(x_j) r(x_i, x_j)] \qquad Eq\ 2.2\text{-}13$$

Where:
c_{idx} the sensitivity function with respect to the quantity (x_{idx})
r the correlation coefficient
With the correlation coefficient:

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)} \qquad Eq\ 2.2\text{-}14$$

With the covariance of x_i and x_j:

$$u(x_i, x_j) = \frac{1}{n(n-1)} \sum_{k=1}^{n} (x_{i,k} - \overline{x_i})\,(x_{j,k} - \overline{x_j}) \qquad \text{Eq 2.2-15}$$

Where:
$\overline{x_i}, \overline{x_j}$ are the arithmetic mean values of the quantities x_i, x_j.
$x_{i,n}, x_{j,n}$ are individual observations of the quantities x_i, x_j.

The GUM notes that the most statistical measures of correlation are based on the degree of linear relationship. As an investigation of correlation; the GUM proposes to evaluate the covariance of quantities that are suspected to be correlated according to Eq 2.2-15. If the quantities are uncorrelated; the calculated covariance is expected to be near 0.

2.2.5 Level of confidence and expressing the measurement uncertainty:

Using measurement expectation and the combined measurement uncertainty, a normal distribution can be defined:

$$Y = N(y, u_y)$$

And the result of the measurement and its uncertainty should be expressed as:

$$Y = y \pm k_p u_y$$

Where:

y the expectation value of the distribution, which is the best estimate.

u_y the standard deviation of the mean, which is the standard uncertainty.

k_p is the coverage factor that indicates a certain level of confidence.

If the coverage factor is chosen to be $k_p = 1$, the level of confidence is approximated to be $p = 68{,}27\%$.This level of confidence is the probability of the actual measurement laying within the interval $[y - u_y\ ,\ y + u_y]$. Depending on the application and the use of the measurement; higher level of confidence might be needed, the correspondence of the level of confidence to the coverage factor for a normal distribution is given by the following table:

LEVEL OF CONFIDENCE p (%)	COVERAGE FACTOR k_p
68,27	1
90	1,645
95	1,960
95,45	2
99	2,576
99,73	3

Table 2.2-1: Coverage factor and level of confidence of normal distribution

3 Strain measurement

This chapter will deal with strain gauge measurement in general and discuss the usual factors that can affect a measurement. It will also introduce a general approach to evaluate the corrected strain and its measurement uncertainty using an axial grid strain gauge with a Wheatstone quarter-bridge.

3.1 Methods of strain measurement and applications

The strain (the ratio of change of length to the original length of an object due to a deformation of a mechanical or a thermal origin) can be measured directly or indirectly according to various methods. The direct method is to measure the length and its change directly using a length measurement tool or device (caliper, scopes or positioning systems). The indirect strain measurement is realized by measuring a physical quantity that can be linked to the deformation and/or elongation. An example of this method is performing the measurement using a strain gauge.

The most common type of strain gauges consists of a metallic foil pattern (a grid) supported in an insulating flexible backing, that can be attached to a test object via an adhesive or by soldering it directly to the test object. The backing allows the grid to deform as the test object deforms. By doing so, the resistance of the grid will also change, this change of resistance can then be detected and used to derive the strain corresponding to it.

The method of measuring the strain using a strain gauge has a wide range of applications in research, industry and other domains, including measuring force, moment of force, pressure, stress and other physical quantities [19] [20] [21].

3.2 Physical principle of strain gauges

The physical phenomena behind a strain measurement using a strain gauge is the correspondence of the electrical resistance of a given conductor according to its shape and material properties. The relation is given by the following law:

$$R = \rho \frac{l}{A} \qquad Eq\ 3.2\text{-}1$$

Where:

R the resistance of the conductor.
ρ the electrical resistivity.
l the length of the conductor, parallel to current flow.
A the cross-sectional area of the conductor, perpendicular to current flow.

While measuring with a strain gauge; the grid of the strain gauge deforms as the test object deforms, which causes a change in resistance of the grid of the strain gauge. The change in length will cause a change in the cross-sectional area as well. However, the behaviour of the ratio of the length to the cross-sectional area is predictable. Under the assumption that the material of the gauge grid is isotopically elastic this ratio can be expressed using the Poisson's ratio.

The change in electrical resistance is related to the strain by the quantity known as the gauge factor or the strain sensitivity of strain gauge, also "*k*-factor". In a uniaxial stress field, in which the principal strain is intended to be measured, the relation between the strain, the gauge-factor and the ratio of change in resistance to the resistance of the strain gauge is given by the following equation:

$$k = \frac{\Delta R/R}{\varepsilon} \qquad Eq\ 3.2\text{-}2$$

Where:

k the gauge-factor.
ΔR the measured change in resistance under deformation.
R the resistance of the gauge grid with no deformation.
ε the strain.

The gauge factor is the key element to define the behaviour of a strain gauge, but it is not a constant value and it is not the only characteristic of a strain gauge. Other factors and characteristic will be discussed in Characteristics of strain gauge 3.5.

3.3 Strain measurement with Wheatstone Bridge

Because strain gauges measure the overall strain occurring along the area covered with the strain gauge, the dimensions of strain gauges are kept as small as possible to avoid a high deviation of the captured mean strain from the actual strain in the center of the measurement area, as the objective is to measure this center strain. Therefore, the change of resistance as a result of the deformation of the grid is also relatively small. Making the accuracy of a direct measurement of the change of resistance with a conventional ohmmeter (following the principle of a simple voltage divider) inadequate. Instead, a Wheatstone bridge circuit is almost always used to detect the resistance change.

A Wheatstone bridge circuit (invented by Samuel Hunter Christie and improved by Sir Charles Wheatstone) [22] is an electrical circuit used to measure an unknown value of resistance by balancing it to a configuration of other known resistances Figure 3.3-1. It can also be used to provide an accurate determination of relative changes in resistance.

One of the primary benefits of the use of a Wheatstone bridge circuit, other than the high accuracy, is the immediate proportional response to an excitation with no significant delay or dead time, owing to the linear behavior of all components of the circuit. Which makes the measurement with a strain gauge and a Wheatstone bridge very suitable for dynamic, cyclic and transient measurements.

According to the purpose of the measurement and the number of measured components of the strain, there are various of configurations and techniques to use a Wheatstone bridge circuit. These configurations might use one or multiple strain gauges that are connected to the Wheatstone bridge. The main four configurations are:

Quarter bridge:

Which supports one strain gauge to measure the strain in one direction.

Half bridge:

This configuration supports two strain gauges and is usually used to measure the strain in two different direction (perpendicular to each other) on the same surface, this configuration is also called the Poisson half bridge configuration. It could also be used to measure the strain in the same direction on two opposite parallel surfaces of the same test object. The third possible way is to use an active strain gauge with a redundant strain gauge to compensate the thermal deviation.

Double quarter (also known as diagonal bridge):

This configuration uses two strain gauges that are installed on opposite surfaces of the test object. The strain gauges are connected diagonally to the Wheatstone bridge (hence diagonal bridge). This technique is used to measure the normal strain independently of the bending strain.

Full Bridge:

This configuration uses four active strain gauges and can be used with various techniques like:

a) Double Poisson half bridge (higher gain, improved precision).
b) One direction, opposite sides
c) 45° to the main axis configuration (rosettes)
d) 45° perpendicular to the main axis (stacked rosettes)

The focus of this thesis is on the measurement using a Wheatstone quarter bridge circuit with one strain gauge; shown in Figure 3.3-1.

The balance equation of this configuration is given by the equation:

$$R_{SG} = \frac{R_4 U_{excitation} - (R_3 + R_4) U_{measured}}{R_3 U_{excitation} + (R_3 + R_4) U_{measured}} R_2 \qquad Eq\ 3.3\text{-}1$$

Where:

R_{SG}	the resistance of the strain gauge.
R_2, R_3 and R_4	the Wheatstone resistance.
$U_{excitation}$	the voltage of the excitation current.
$U_{measured}$	the voltage across the bridge.

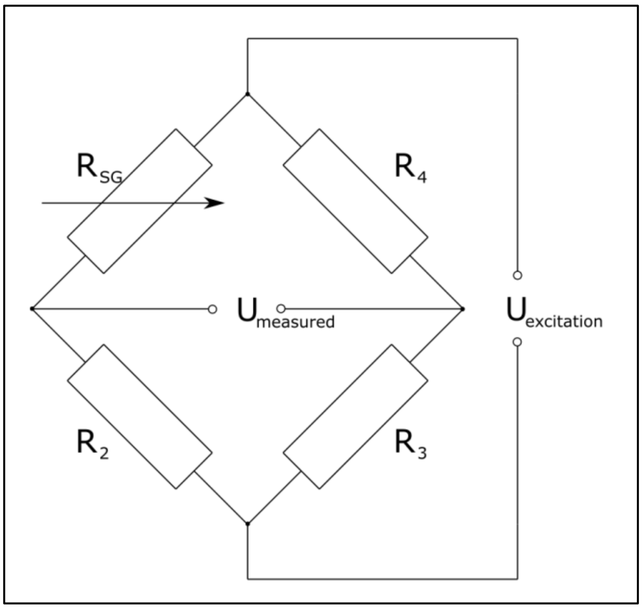

Figure 3.3-1: a Wheatstone quarter bridge circuit with one strain gauge.

3.4 Measurement chain

In the modern metrology, the term measurement chain usually refers to the path (that consists of the physical components) taken by the measurement's signal from generation till evaluation [23]. The chain of a strain measurement using a Wheatstone quarter bridge with one gauge consists of the following four main components:

a) The sensor.
   The grid strain gauge used in the measurement.

b) The measurement module.
   Contains the Wheatstone bridge circuit and an electrical source for the excitation of the sensor.

c) Control and DAQs module.
   This component might contain an analogue-to-digital converter. This converter can also be integrated in the measurement module. The main purposes of this component are to control the output current of the excitation and the communication with the server hosting the databank of the measurement.

d) The wiring.
   To connect the three components listed above with each other.

The path of the analogue signal starts with generation of the excitation current, which can be a direct or an alternating current. The current then flows through the components of the Wheatstone bridge circuit including the wiring between the measurement module and the strain gauge. The voltage drop across the bridge is then measured, and the resistance of the strain gauge is calculated; typically, via an arrangement of logic gates. The analogue signal is converted to a numerical value via an analogue-to-digital converter. Lastly, the numerical value is written to a data base using the DAQs module.

3.5 Characteristics of strain gauge

According to the strain gauge standard VDI/VDE 2635 "Bonded electric resistance strain gauges, characteristics and testing conditions" [24], the following characteristics of a strain gauge should be determined and given:

a) The gauge factor (k)

That is introduced inPhysical principle of strain gauges 3.2 and Eq 3.2-2.

b) The initial resistance (R).

The resistance of the unstressed gauge (with no deformation).

c)Coefficient of the temperature dependence of the k-factor (α_k).

This coefficient should be used to consider the temperature dependence of the electrical sensitivity of the gauge grid. This coefficient is defined as:

$$\alpha_k(T - T_R) = \frac{\Delta k(T)}{k_{Rk}} = \frac{k(T) - k_{RK}}{k_{RK}} \qquad Eq\ 3.5\text{-}1$$

Where

$k(T)$ of the k-factor at the temperature (T).
α_k the coefficient of the temperature dependence of the k-factor.
k_{Rk} of the k-factor at the reference temperature.
T_R the reference temperature. $T_R = (23 + 273.15)K$.
T the temperature at the measurement.

Thus, the corrected K-factor is given by:

$$k(T) = k_{RK}(1 + \alpha_k(T - T_R)) \qquad Eq\ 3.5\text{-}2$$

d) The thermal response of the strain gauge

Will be discussed in 3.5.1 Thermal response of strain gauge below.

e) The transverse sensitivity

Will be discussed in 3.5.2 Transverse sensitivity of strain gaugebelow.

3.5.1 Thermal response of strain gauge

According to VDI/VDE 2635 [24] the thermal behaviour of the strain gauge should be characterised using a function of temperature to evaluate the apparent strain (the strain of a thermal origins). This should be done by fitting a polynomial function to a series of measurements; nonetheless, this investigation does not focus on an individual thermal effect that would affect the measurement; instead, it treats the strain gauge as a black box and attempts to describe the apparent strain in relation to the variation of temperature. Possible reasons of the apparent strain are:

a) The inhomogeneous thermal expansion of the test object and the gauge.
   As a result of the deference between the thermal expansion of the test object and the thermal expansion of the strain gauge. Here, it is noteworthy that the thermal expansion of the test object can be described using a linear thermal expansion coefficient; on the opposite side, the thermal expansion behaviour of the strain gauge cannot be expressed using such a linear relation, due to the fact that a strain gauge consists of several layers of different material.

b) The creep of the gauge grid.
   The movement of the grid of a strain gauge relative to the test object. Although this can be minimised by improving the material of the backing of strain gauge and enhancing the process of manufacturing it; it cannot be eliminated, especially whilst dealing with a long-term strain measurement with the strain gauge being stressed for extended periods of time.

c) Reasons related to the adhesive or the soldering used to attach the strain gauge.

The apparent strain function of temperature can be given as follows:

$$\varepsilon_s(T) = c_0 + c_1 T + c_2 T^2 + c_3 T^3 + c_4 T^4 \pm \varepsilon_0 \qquad Eq\ 3.5\text{-}3$$

Where
ε_s the apparent strain at the temperature (T).
T the temperature.
c_{idx} the coefficients of the polynomial function.

The VDI/VDE 2635 [24] recommends also a graphical description of the investigated thermal behaviour of the strain gauge. An example is shown in Figure 3.5-1, where the horizontal axis represents the temperature, while the vertical axis represents the apparent strain. The VDI/VDE 2635 demands to document the thermal expansion coefficient of the test object used in this investigation. It also instructs that the mounting of the tested strain gauge should happen with the strain gauge and the test object having the room temperature.

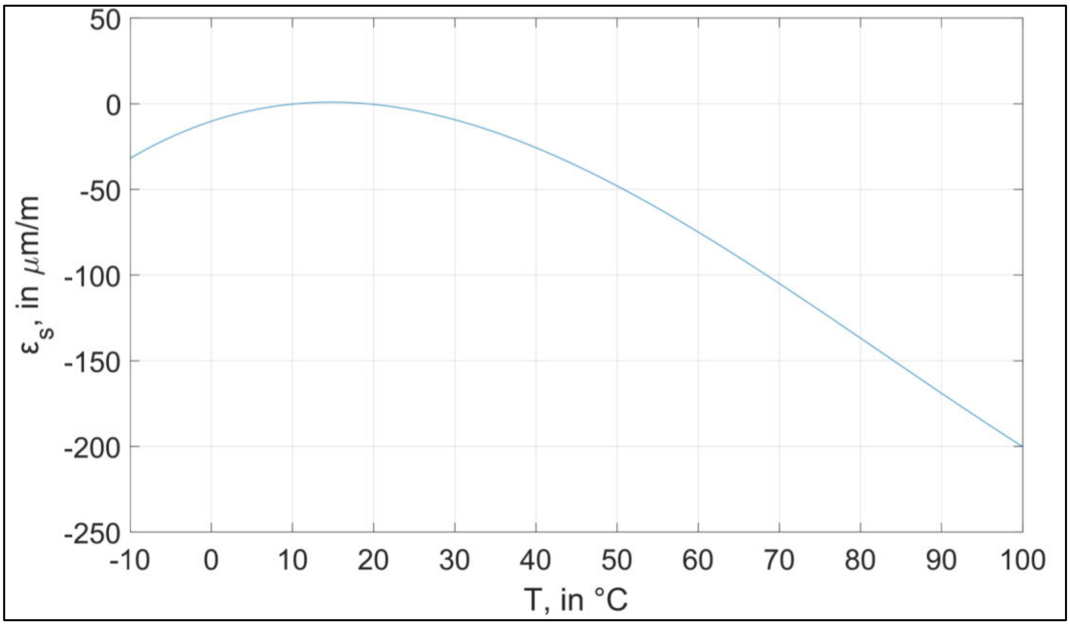

Figure 3.5-1 The apparent strain as a function of temperature.

3.5.2 Transverse sensitivity of strain gauge

Although the name of uniaxial strain gauges implies that the strain is measured in one direction, they are in fact sensitive to both the longitudinal strain in the direction of measurement and to the strain perpendicular to the direction of measurement [20]. Accordingly, the change of resistance corresponds to the strain in both directions:

$$\frac{\Delta R}{R} = k_l \varepsilon_l + k_q \varepsilon_q \qquad Eq\ 3.5\text{-}4$$

Where:

ε_l the longitudinal strain in the direction of measurement.
ε_q the strain perpendicular to the direction of measurement.
k_l the longitudinal strain sensitivity.
k_q the transverse strain sensitivity.

Figure 3.5-2 illustrates a strain gauge in both stressed and unstressed states, where l and h are the dimensions of the strain gauge, ε_q and ε_l are the longitudinal and transverse strains. The measured strain in this case is $\varepsilon_{sg} = \Delta R/(Rk) = k_l \varepsilon_l + k_q \varepsilon_q$. Usually k_q is distinctly smaller than k_l (in most cases, $k_q = 5 \cdot 10^{-3} k_l$); thereupon, there is a tendency to disregard the transverse strain sensitivity, particularly if the ratio $\varepsilon_q/\varepsilon_l$ is small as well [20]. If the transverse strain is not assumed to be negligible, a correction of the measured strain can be done using the coeffect of transverse sensitivity.

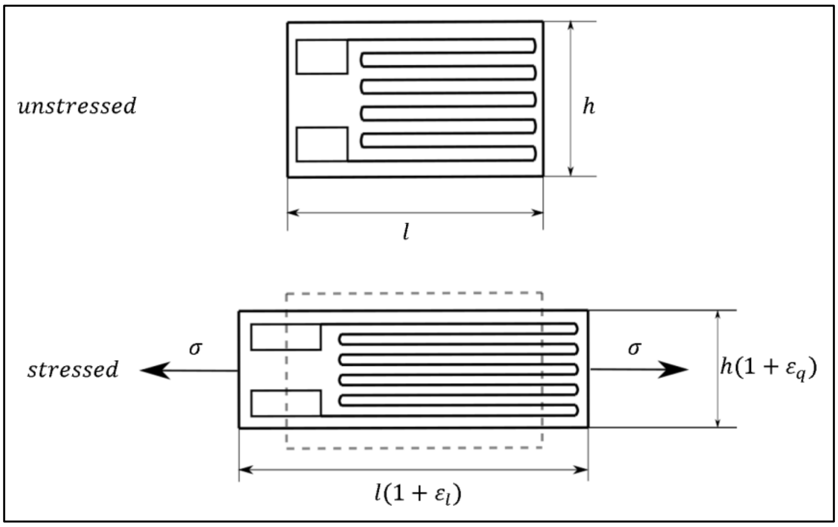

Figure 3.5-2: Longitudinal and transverse sensitivity of strain gauge.

The transverse sensitivity of a stain gauge is defined as the ratio of the strain sensitivity transverse to the direction of measurement and the strain sensitivity along the direction of measurement [24]:

$$q = \frac{k_t}{k_l} \qquad Eq\ 3.5\text{-}5$$

Where:
q The transverse sensitivity.

The relation between the measured strain and the strain field of measurement is given by the following equation:

$$\varepsilon_{sg} = \frac{\varepsilon_l + q\varepsilon_q}{1 - \nu_0 q} \qquad Eq\ 3.5\text{-}6$$

Where:
ε_{sg} the measured strain.
ε_l the longitudinal strain in the direction of measurement.
ε_q the strain perpendicular to the direction of measurement.
ν_0 Poisson's ratio of the strain gauge grid.

The equation Eq 3.5-6 is an approximation based on assumptions given Eq 3.5-7 in and Eq 3.5-8:

$$\varepsilon_{0,q} = \nu_0 \varepsilon_{0,l} \qquad Eq\ 3.5\text{-}7$$

Where $\varepsilon_{0,q}$ and $\varepsilon_{0,l}$ are the strains of an uniaxial stress field in the direction of $\varepsilon_{0,l}$, which describes the conditions of testing the strain gauge and evaluating q.

$$k \approx k_l \qquad Eq\ 3.5\text{-}8$$

The approximation in $Eq\ 3.5\text{-}6$ deviates from the actual measurement, which causes an error that can be approximated as follows [20]:

$$f_{\varepsilon_q} = \frac{q\left(\nu_0 + \frac{\varepsilon_q}{\varepsilon_l}\right)}{1 - \nu_0 q} \qquad Eq\ 3.5\text{-}9$$

3.6 Strain gauge misalignment.

A small angular misalignment with respect to the intended measurement direction is very probable while mounting a strain gauge to the surface of the test object. As a result, a measurement deviation will be caused, that can be corrected if the misalignment is measurable or can be approximated. Notwithstanding the existence of such a correction, a component of a measurement uncertainty can be used to consider the uncertainty about the alignment of the strain gauge with respect to the direction of the strain that is intended to be measured.

For the special case where the measured strain is the main strain in a uniaxial stress field, [21] and [20] give an equivalent of the following equation, to correct the measurement of the strain:

$$\varepsilon_c = \frac{2\ \varepsilon_{sg}}{\cos 2\phi\,(1+\nu) + 1 - \nu} \qquad Eq\ 3.6\text{-}1$$

Where:

ε_{sg} the measured strain;
ε_c the corrected strain;
ϕ the angle of misalignment;
ν Poisson’s ratio of the test object.

However, the relation given by $Eq\ 3.6\text{-}1$ is only valid if the strain intended to be measured is the first principal strain as a result of a uniaxial stress field under the following assumptions:

a) Isotropic material of the test object.
b) Linear elasticity of the material of the test object (Constant Young’s modulus).
c) The transverse sensitivity is negligible.

As shown in 3.5.2 the choice of neglecting the transverse sensitivity is well-founded if the ratios k_q/k_l and $\varepsilon_q/\varepsilon_l$ allow such an assumption. The assumptions of a linear elasticity and an isotropy of the test object’s material are also accurate for certain materials. The condition of a uniaxial stress field, on the other hand, is more of a special case that is not always given. Therefore, the author has developed a general approach to correct the strain measurement error due to misalignment, that is valid for every stress field and introduced in 3.6.1.

3.6.1 An approach to correct the measurement error due to strain gauge misalignment

To correct a strain measurement error due to strain gauge misalignment, a relation between the measured strain, that includes the error, and the strain intended to be measured, which is the measurement subject, must be established. For this purpose, the tensor of the principal stresses and its principal planes must be known. Such a knowledge can be obtained experimentally or using simulation's results. This approach is valid for any stress field under the following assumptions:

a) Isotropic material of the test object.
b) Linear elasticity of the material of the test object (Constant Young's modulus).

To maintain the generality of the approach, it is fundamental to start with the tensor of the principal stresses in the measurement position:

$$\boldsymbol{\sigma_p} = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \qquad Eq\ 3.6\text{-}2$$

Where:

$\boldsymbol{\sigma_p}$ the tensor of the principal stresses.
$\sigma_I, \sigma_{II}, \sigma_{III}$ the principle stresses.

The principal planes are required to be known, since they are essential to perform the next step of this approach, which is to transform the stress tensor with an appropriate rotation-matrix to obtain a stress tensor with an orientation that fulfils the following criteria:

a) Two normal stresses of the tensor should define the plane containing the measurement's surface or define a plane tangential to the measurement's surface at the measurement's point. In consequence the third normal stress will be perpendicular to the surface of measurement at the measurement's point.
b) One of the normal stresses should have the same direction of the strain intended to be measured.

Such a transformation is required in order to define the strain to be measured using the transformed stress tensor. This transformation is illustrated in Figure 3.6-1 can be done according to Cauchy's transformation rule and shall be called hereafter **the first transformation tensor:**

$$\boldsymbol{\sigma} = \boldsymbol{R_0}\boldsymbol{\sigma_p}\boldsymbol{R_0^T} \qquad Eq\ 3.6\text{-}3$$

Where:

$\boldsymbol{\sigma}$ the first transformation tensor.
$\boldsymbol{\sigma_p}$ the tensor of the principle stresses.
$\boldsymbol{R_0}$ the general rotation-matrix.

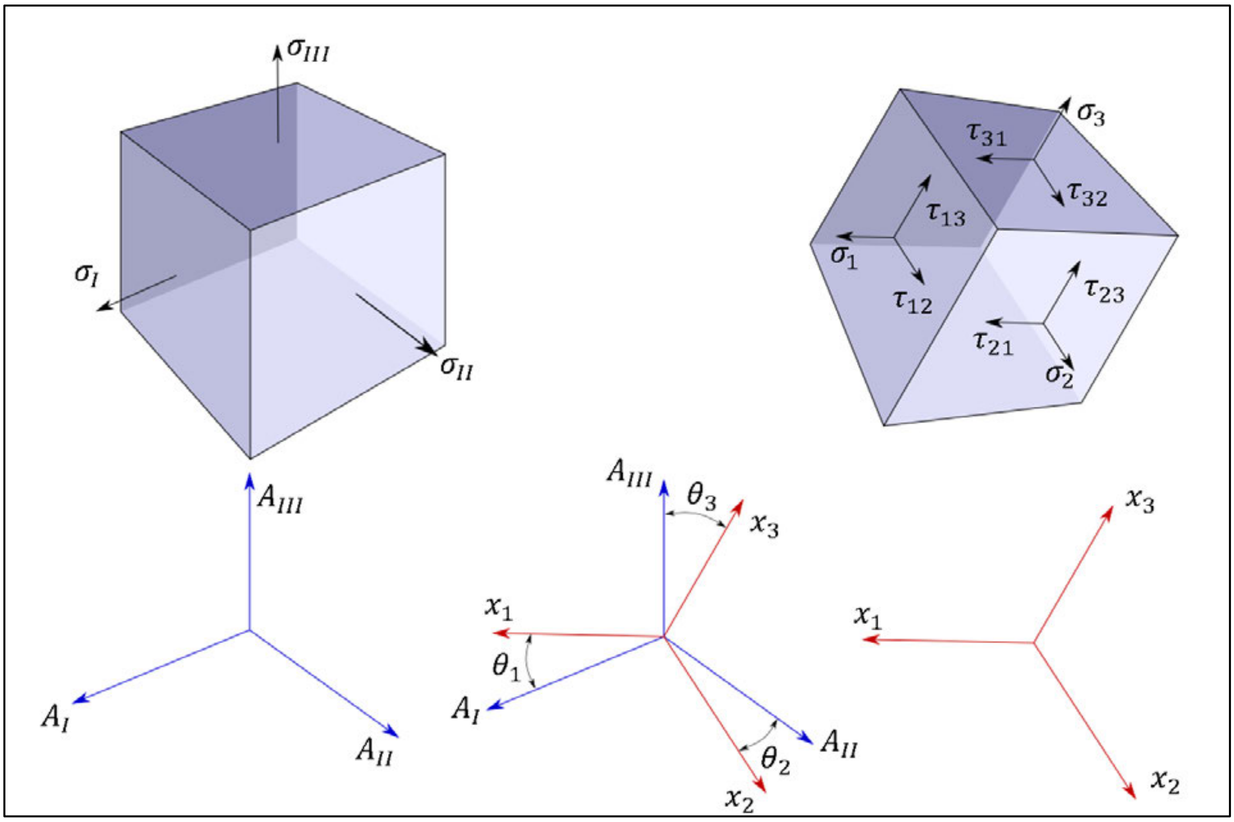

Figure 3.6-1: A rotation of a principal stress tensor

A general rotation-matrix is the product of three individual rotation-matrices used to describe the rotation about the individual Cartesian axis:

$$\boldsymbol{R_0} = \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix} \qquad Eq\ 3.6\text{-}4$$

Where α, β and γ are the angles of the rotation about the Cartesian axis that describe the orientation of the principle stresses. It is important to distinguish that the angles $\alpha, \beta,\ \gamma$ in Eq 3.6-4 are the rotation angles around the axis A_1,A_2, A_3(the axis of the principle stresses, that are shown in Figure 3.6-1). Where the angles θ_1, θ_2 and θ_3 are the angles between axis of the original coordinate system defined by the axis A_1,A_2 *and* A_3 and the transformed coordinate system defined by the axis x_1,x_2, x_3 (the axis of the normal stresses of the first transformation tensor). The following three equations describe the relation between α, β, γ and θ_1, θ_2, θ_3:

$$\cos\theta_1 = \cos\alpha\cos\beta \qquad Eq\ 3.6\text{-}5$$

$$\cos\theta_2 = \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma \qquad Eq\ 3.6\text{-}6$$

$$\cos\theta_3 = \cos\beta\cos\gamma \qquad Eq\ 3.6\text{-}7$$

After performing the first transformation, a second transformation of **the first transformation tensor** is needed (shown in Figure 3.6-2 A transformation of a stress tensor with the rotation angle of misalignmentFigure 3.6-2). This rotation should be done around the axis perpendicular to the measurement's surface with a rotation angle equals to the angle of misalignment. The tensor obtained from this transformation will then provide the normal stresses with an orientation that will facilitate the calculation of the actual measured strain (with the error due to angular misalignment), in order to correct the strain and consider its component of measurement uncertainty. The second transformation shall be performed according to Cauchy's transformation rule as well, and shall be called hereafter **the second transformation tensor**:

$$\boldsymbol{\sigma}' = \boldsymbol{R\sigma R^T} \qquad Eq\ 3.6\text{-}8$$

With the rotation-matrix:

$$\boldsymbol{R}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Eq\ 3.6\text{-}9$$

Where ϕ represents the angular misalignment.
By combining Eq 3.6-8 and Eq 3.6-9 the following equation can be obtained:

$$\begin{bmatrix} \sigma'_1 & \tau'_{12} & \tau'_{13} \\ \tau'_{12} & \sigma'_2 & \tau'_{23} \\ \tau'_{13} & \tau'_{23} & \sigma'_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Eq\ 3.6\text{-}10$$

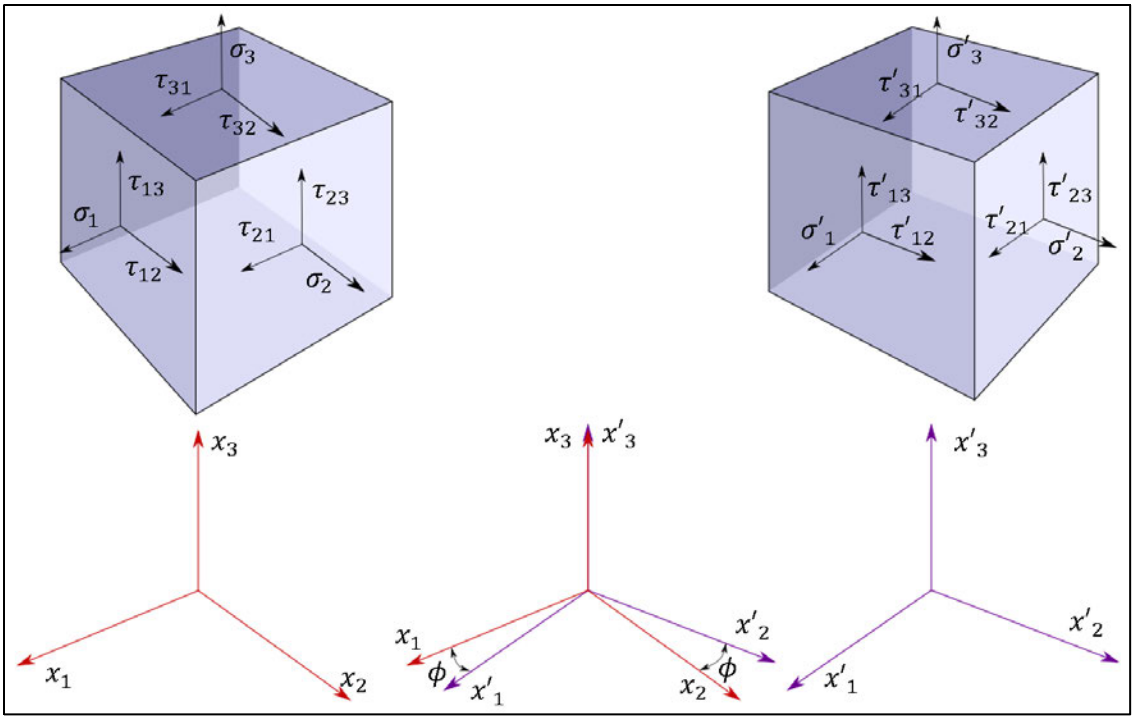

Figure 3.6-2 A transformation of a stress tensor with the rotation angle of misalignment

Since the rotation done around the axis x_3, the error due to angular misalignment can be evaluated for the strains ε_1 end ε_2, which are present in the plane defined by the axis x_1 and x_2 (shown in Figure 3.6-3)

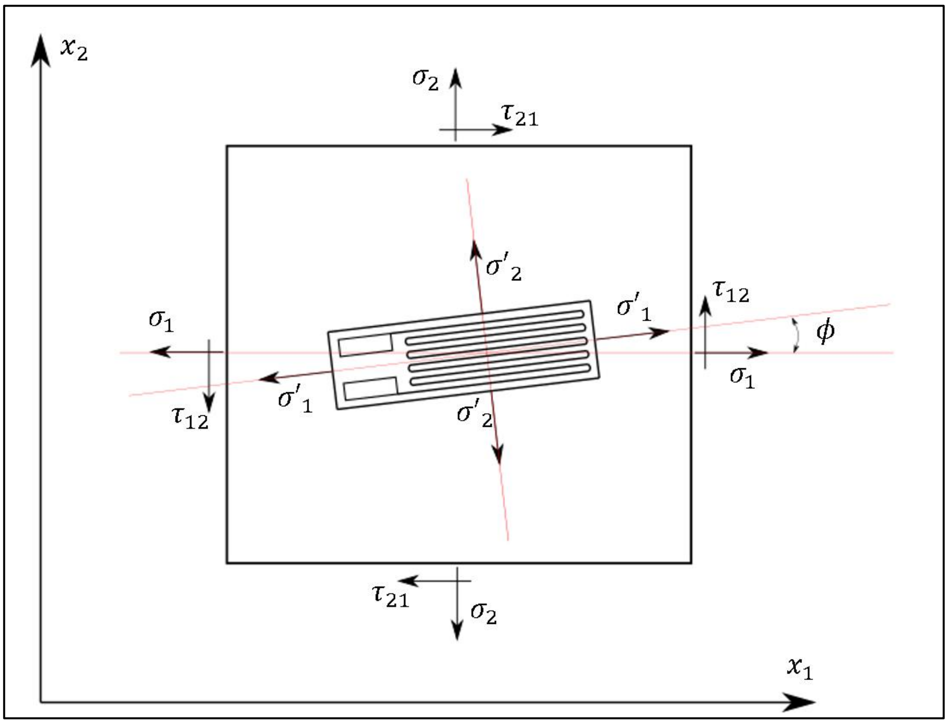

Figure 3.6-3 Angular misalignment of a strain gauge

The relations between the normal stress components of the **second transformation tensor** and the stress components of the **first transformation tensor** can be derived from the Eq 2.2-12, and can be explicitly given by:

$$\sigma'_1 = \sigma_1 \cos^2\phi + \sigma_2 \sin^2\phi - 2\cos\phi \sin\phi\, \tau_{12} \qquad Eq\ 3.6\text{-}11$$

$$\sigma'_2 = \sigma_1 \sin^2\phi + \sigma_2 \cos^2\phi + 2\cos\phi \sin\phi\, \tau_{12} \qquad Eq\ 3.6\text{-}12$$

$$\sigma'_3 = +\sigma_3 \qquad Eq\ 3.6\text{-}13$$

The strain components in the measurement position can be then expressed according to Hook's law using the stresses of the transformed tensor. The strains intended to be measured in relation to the normal stresses of **the first transformation tensor** are given by the equations:

$$\varepsilon_1 E = \sigma_1 - \nu(\sigma_2 + \sigma_3) \qquad Eq\ 3.6\text{-}14$$

$$\varepsilon_2 E = \sigma_2 - \nu(\sigma_1 + \sigma_3) \qquad Eq\ 3.6\text{-}15$$

The strain in the direction of the misalignment can be calculated using the normal stresses of **the second transformation tensor**:

$$\varepsilon'_1 E = \sigma'_1 - \nu(\sigma'_2 + \sigma'_3) \qquad Eq\ 3.6\text{-}16$$

After replacing the stresses from the second transformed tensor with those from the first transformation, the following equation can be obtained:

$$\varepsilon'_1 E = \sigma_1 \cos^2\phi - 2\tau_{12} \cos\phi \sin\phi + \sigma_2 \sin^2\phi \\ - \nu\,(+\sigma_1 \sin^2\phi + 2\tau_{12} \sin\phi \cos\phi + \sigma_2 \cos^2\phi + \sigma_3) \qquad Eq\ 3.6\text{-}17$$

Respectively for the second strain:

$$\varepsilon'_2 E = +\sigma_1 \sin^2\phi + 2\tau_{12} \cos\phi \sin\phi + \sigma_2 \cos^2\phi \\ - \nu\,(\sigma_3 + \sigma_1 \sin^2\phi - 2\tau_{12} \cos\phi \sin\phi + \sigma_2 \cos^2\phi) \qquad Eq\ 3.6\text{-}18$$

If a strain measurement in the direction of $x_1$is to be considered, then the measured strain is a combination of the longitudinal strain (in the direction of x'_1) and the transverse strain (in the direction of x'_2) according to Eq 3.5-6:

$$\varepsilon_{sg1} = \frac{\varepsilon'_1 + q\,\varepsilon'_2}{1 - \nu_0 q} \qquad Eq\ 3.6\text{-}19$$

The strain measurement in the direction of x'_2 is analogously given by:

$$\varepsilon_{sg2} = \frac{\varepsilon'_2 + q\,\varepsilon'_1}{1 - \nu_0 q} \qquad Eq\ 3.6\text{-}20$$

For the sake of ease of handling the equations Eq 3.6-14 till Eq 3.6-20, the following functions can be defined:

$$\varepsilon_1 = f_{\varepsilon_1}(\sigma_1, \sigma_2, \sigma_3, \nu)/E \qquad Eq\ 3.6\text{-}21$$

$$\varepsilon_2 = f_{\varepsilon_2}(\sigma_1, \sigma_2, \sigma_3, \nu)/E \qquad Eq\ 3.6\text{-}22$$

$$\varepsilon'_1 = f_{\varepsilon'_1}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi)/E \qquad Eq\ 3.6\text{-}23$$

$$\varepsilon'_2 = f_{\varepsilon'_2}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi)/E \qquad Eq\ 3.6\text{-}24$$

$$\varepsilon_{sg1} = f_{\varepsilon_{sg1}}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi, q, \nu_0)/E \qquad Eq\ 3.6\text{-}25$$

$$\varepsilon_{sg2} = f_{\varepsilon_{sg2}}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi, q, \nu_0)/E \qquad Eq\ 3.6\text{-}26$$

The relations between the measured strains ε_{sg1} & ε_{sg2} and the strains intended to be measured ε_1 & ε_2 can be established by dividing the equations Eq 3.6-25 & Eq 3.6-26 respectively be the equations Eq 3.6-21 & Eq 3.6-22:

$$\frac{\varepsilon_{sg1}}{\varepsilon_1} = \frac{f_{\varepsilon_{sg1}}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi, q, \nu_0)}{f_{\varepsilon_1}(\sigma_1, \sigma_2, \sigma_3, \nu)} \qquad Eq\ 3.6\text{-}27$$

$$\frac{\varepsilon_{sg2}}{\varepsilon_2} = \frac{f_{\varepsilon_{sg2}}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi, q, \nu_0)}{f_{\varepsilon_2}(\sigma_1, \sigma_2, \sigma_3, \nu)} \qquad Eq\ 3.6\text{-}28$$

Based on the relations given by Eq 3.6-27 and Eq 3.6-28 the following functions can be defined:

$$\varepsilon_{1,corrected} = \varepsilon_{sg1} f_{\varepsilon_{1,corrected}}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi, q, \nu_0) \qquad Eq\ 3.6\text{-}29$$

$$\varepsilon_{2,corrected} = \varepsilon_{sg2} f_{\varepsilon_{2,corrected}}(\sigma_1, \sigma_2, \sigma_3, \nu, \tau_{12}, \phi, q, \nu_0) \qquad Eq\ 3.6\text{-}30$$

Where:

$\varepsilon_{1,corrected}$ & $\varepsilon_{2,corrected}$	the corrected strains.
ε_{sg1} & ε_{sg2}	the measured strains.

It should be noted that the stresses $\sigma_1, \sigma_2, \sigma_3, \tau_{12}$of the first transformation tensor can be expressed using the principle stresses $\sigma_I, \sigma_{II}, \sigma_{III}$ and the angles of transformation α, β, γ. The exact numeric values of stresses $\sigma_1, \sigma_2, \sigma_3, \tau_{12}$ are not necessarily required in order to correct the measurement error of the angular misalignment. Instead, linearity coefficients between the principle stresses can be approximated:

$$\sigma_2 = c_{\sigma_2}\sigma_1 \qquad Eq\ 3.6\text{-}31$$

$$\sigma_3 = c_{\sigma_3}\sigma_1 \qquad Eq\ 3.6\text{-}32$$

$$\tau_{12} = c_{\tau}\sigma_1 \qquad Eq\ 3.6\text{-}33$$

By replacing the stresses σ_2, σ_3 and τ_{12} respectively with the terms $c_{\sigma_2}\sigma_1,\ c_{\sigma_3}\sigma_1$ and $c_\tau\sigma_1$ in the equations Eq 3.6-29 and Eq 3.6-30, the stress σ_1 can be eliminated from the equation, as the functions $f_{\varepsilon_1}, f_{\varepsilon_2}, f_{\varepsilon_{sg1}}$and $f_{\varepsilon_{sg1}}$can be represented using sums as follows:

$$f_{\varepsilon_{idx}} = \sum_{i=1}^{n} c_{i_{idx}}\sigma_I = \sigma_I \sum_{i=1}^{n} c_i \qquad Eq\ 3.6\text{-}34$$

Where the functions $f_{\varepsilon_{1,corrected}}$ and $f_{\varepsilon_{2,corrected}}$ are ratios of $f_{\varepsilon_{sg1}}$and $f_{\varepsilon_{sg1}}$respectively to f_{ε_1} and f_{ε_2}.

And the functions Eq 3.6-29 and Eq 3.6-30 can be written as:

$$\varepsilon_{1,corrected} = \varepsilon_{sg1}\, f_{\varepsilon_{1,corrected,n}}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right) \qquad Eq\ 3.6\text{-}35$$

$$\varepsilon_{2,corrected} = \varepsilon_{sg2}\, f_{\varepsilon_{2,corrected,n}}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right) \qquad Eq\ 3.6\text{-}36$$

This means that the correction of the measurement depends on the following factors:

a) The stress tensor at the measurement position and its orientation.
b) The misalignment angle ϕ.
c) The transverse sensitivity q.
d) Poisson's ratios of the strain gauge grid ν_0 and the material of the test object ν.

Based on the dependencies listed above, it is essential to assess the correction function for each setting of measurement.

The correction given by Eq 3.6-29 and Eq 3.6-30 deals with a strain measurement error due to angular misalignment of the gauge based on a general principal stress tensor in the measurement position; however, in several special cases, for example where the stress field is uniaxial, special cases can be derived.

Special case 1 and 2 in a uniaxial stress field:

A uniaxial stress field can be described using the following stress tensor:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad Eq\ 3.6\text{-}37$$

The strain tensor is then:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \qquad Eq\ 3.6\text{-}38$$

Case 1 is defined by:

- Uniaxial stress field, which has the direction of the stress intended to be measured.

Case 2 is defined by:

- Uniaxial stress field, which has the direction perpendicular to the stress intended to be measured.
- The direction of the stress is parallel to the measurement plan.

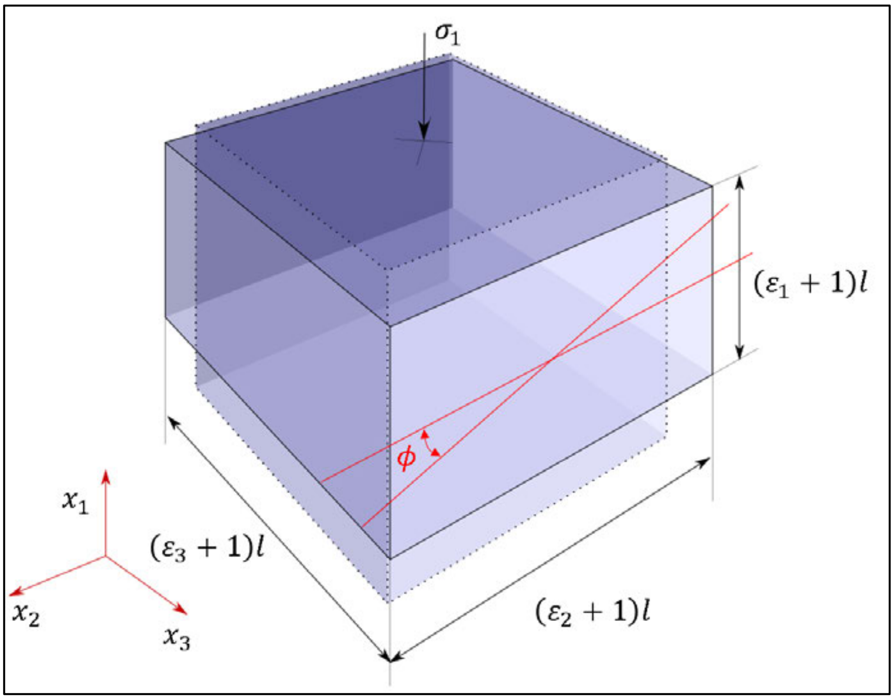

Figure 3.6-4 Special cases 1 and 2 of correcting the misalignment error

Figure 3.6-4 shows a uniaxial stress field with the only stress component σ_1 .In such an example, case 1 applies to ε_1, while case 2 applies to ε_2 .

For these special cases, the strains of the first transformation tensor will simplify to:

$$\varepsilon_1 E = \sigma_1 \qquad Eq\ 3.6\text{-}39$$

$$\varepsilon_2 E = -\nu\sigma_1 \qquad Eq\ 3.6\text{-}40$$

The stresses of the second transformation tensor are then:

$$\sigma'_1 = \sigma_1 \cos^2\phi \qquad Eq\ 3.6\text{-}41$$

$$\sigma'_2 = \sigma_1 \sin^2\phi \qquad Eq\ 3.6\text{-}42$$

$$\sigma'_3 = 0 \qquad Eq\ 3.6\text{-}43$$

The strains of this tensor are:

$$\varepsilon'_1 E = \sigma_1 (\cos^2\phi \ - \nu\ \sin^2\phi\ ) \qquad Eq\ 3.6\text{-}44$$

$$\varepsilon'_2 E = \ \sigma_1 (\sin^2\phi - \nu\cos^2\phi) \qquad Eq\ 3.6\text{-}45$$

The strain measured by the strain gauge

$$\varepsilon_{sg1} E = \frac{\sigma_1 \left((\cos^2\phi \ - \nu\ \sin^2\phi\ ) + q\,(\sin^2\phi - \nu\cos^2\phi)\right)}{1 - \nu_0 q} \qquad Eq\ 3.6\text{-}46$$

$$\varepsilon_{sg2} E = \frac{\sigma_1 \left((\sin^2\phi - \nu\cos^2\phi) + q(\cos^2\phi \ - \nu\ \sin^2\phi\ )\right)}{1 - \nu_0 q} \qquad Eq\ 3.6\text{-}47$$

And the correction functions are:

$$\frac{\varepsilon_{sg1}}{\varepsilon_1} = \frac{\left((\cos^2\phi \ - \nu\ \sin^2\phi\ ) + q\,(\sin^2\phi - \nu\cos^2\phi)\right)}{1 - \nu_0 q} \qquad Eq\ 3.6\text{-}48$$

$$\frac{\varepsilon_{sg2}}{\varepsilon_2} = \frac{\left((\sin^2\phi - \nu\cos^2\phi) + q(\cos^2\phi \ - \nu\ \sin^2\phi\ )\right)}{-\nu(1 - \nu_0 q)} \qquad Eq\ 3.6\text{-}49$$

If the transverse sensitivity is chosen to be ignored, the equations will reduce to:

$$\frac{\varepsilon_{sg1}}{\varepsilon_1} = \cos^2\phi \ - \nu\ \sin^2\phi \qquad Eq\ 3.6\text{-}50$$

$$\frac{\varepsilon_{sg2}}{\varepsilon_2} = \sin^2\phi - \nu\cos^2\phi \qquad Eq\ 3.6\text{-}51$$

After performing the substitutions of the trigonometric identities:

$$\cos^2\phi = \frac{\cos 2\phi + 1}{2} \qquad Eq\ 3.6\text{-}52$$

$$\sin^2 \phi = \frac{-\cos 2\phi + 1}{2} \qquad Eq\ 3.6\text{-}53$$

The equations Eq 3.6-50 and Eq 3.6-51 will become:

$$\frac{\varepsilon_{sg1}}{\varepsilon_1} = \frac{\cos 2\phi\,(1+\nu) + 1 - \nu}{2} \qquad Eq\ 3.6\text{-}54$$

$$\frac{\varepsilon_{sg2}}{\varepsilon_2} = \frac{-\cos 2\phi\,(1+\nu) + 1 + \nu}{2} \qquad Eq\ 3.6\text{-}55$$

The equation Eq 3.6-55 is equivalent to the equation Eq 3.6-1, but is usually given in textbooks as:

$$\frac{\varepsilon_{sg1} - \varepsilon_1}{\varepsilon_1} = \frac{(\cos 2\phi - 1)(1+\nu)}{2} \qquad Eq\ 3.6\text{-}56$$

Special case 3 in a uniaxial stress field:

The uniaxial stress field for this case can be described using the following stress tensor:

$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \qquad Eq\ 3.6\text{-}57$$

The strain tensor is then:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \qquad Eq\ 3.6\text{-}58$$

Case 3 is illustrated in figure and defined by:

- Uniaxial stress field, which has the direction perpendicular to the stress intended to be measured.
- The direction of the stress is perpendicular to the measurement plane.

In such a case the strain intended to be measured is $\varepsilon_1$or ε_2, and both strains are given by:

$$\varepsilon_1 E = \varepsilon_2 E = -\nu\sigma_3 \qquad Eq\ 3.6\text{-}59$$

The stresses of the second transformation tensor after the rotation around the angle of misalignment are:

$$\sigma'_1 = \sigma'_2 = 0 \qquad Eq\ 3.6\text{-}60$$

$$\sigma'_3 = +\sigma_3 \qquad Eq\ 3.6\text{-}61$$

As a result, the strains components of this tensor are given b:

$$\varepsilon_1' E = \varepsilon_2' E = -\nu \sigma'_3 = -\nu \sigma_3 \qquad Eq\ 3.6\text{-}62$$

By comparing the equations $Eq\ 3.6\text{-}59$ and $Eq\ 3.6\text{-}62$, it can be concluded that any angular misalignment of the strain gauge does not affect the measurement in this special case.

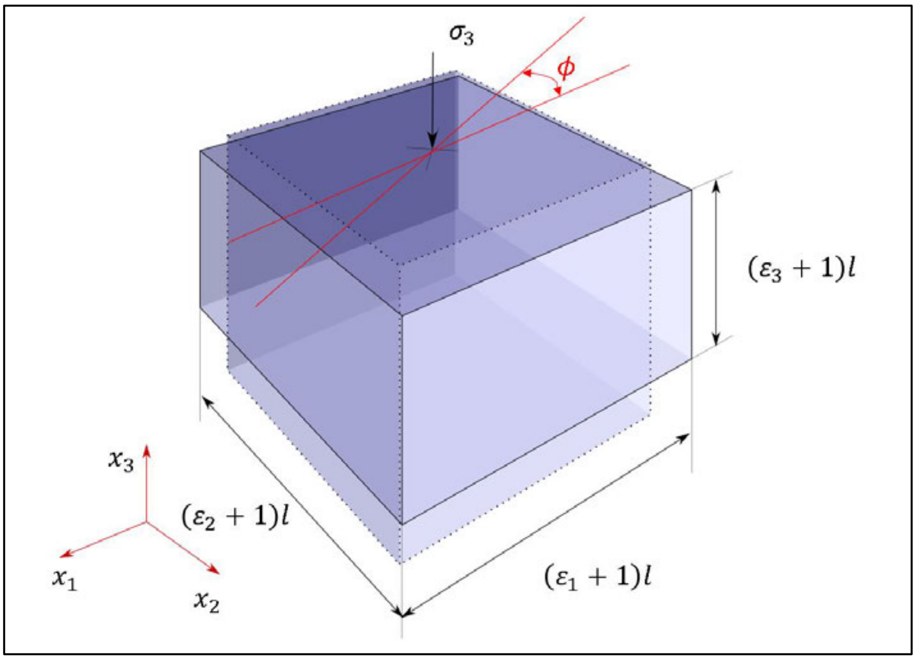

Figure 3.6-5: Special case 3 of correcting the misalignment error

Note:

This approach relies on comparing the actual strain of mechanical origin with the measured strain of mechanical origin as well. Since both apparent strain and the mechanical strain are measured by the strain gauge, the correction of the apparent strain must be done before correcting the error due to transverse sensitivity and angular misalignment.

To validate this mathematical approach (of correcting the angular misalignment) and its implementation in a tool-script, a comparison between values generated using a simulation of multiple stress field-cases and the correction according to this approach was done. This investigation and its results are documented in Appendix C.

A correction of a translational misalignment similar to the correction of the angular misalignment based on simulation data is also possible if the gradient of the stress along the translational position is high enough to cause a significant measurement deviation.

3.7 Measurement deviation related to the operating modules:

Although the measurement module and the DAQ module are the two main parts of the measurement chain, the Wheatstone bridge circuit, the electrical source of the excitation and the analogue-to-digital converter are the actual sources of uncertainties in these modules. It should be pointed to the fact that these three elements (the electrical source of the excitation, the Wheatstone bridge circuit and the analogue-to-digital converter) can be analysed in depth if adequate information about their design and components are available. But most manufacturers of such modules deliver the quality of its performance in terms of percentages of accuracy, repeatability, linearity error, drift and temperature sensitivity. Hence the best way to approximate the measurement uncertainties related to the operating modules is the use of the approximations given by the manufacturer of the modules [25].

The components of the uncertainties are typically related to the following issues:

a) Irregularity of the excitation current.
b) Inaccuracy of the voltage-drop measurement.
c) Uncertainty about the numerical values of the resistances of the Wheatstone bridge.
d) Inaccuracy and resolution of the timestamp of the chip controlling the analogue-to-digital converter.
e) Rough discretisation while converting the signal from analogue to digital; as the conversion is usually done by integrating the function of signal-to-time with respect to time within a specified interval and then dividing the integrated value by the width of the interval to obtain an effective mean value.
f) Temperature, pressure and humidity dependence.

3.8 Measurement deviation related to wiring:

There are two main methods to connect a strain gauge to a quarter bridge circuit:

Two-wire quarter bridge installation

As the name implies, the strain gauge will be connected to the Wheatstone bridge using two leads (shown in Figure 3.8-1). These two leads have surly an electrical resistance that can be calculated using the dimensions of the wires and the electrical resistivity according to Eq 3.2-1. As a result, the bridge arm resistance of the strain gauge will be the sum of resistances of the strain gauge and the two leads in series. Hence, a bridge imbalance will be induced, which must be compensated using the bridge balance adjustment. This is usually done by changing the resistance values of the other three bridge arms.

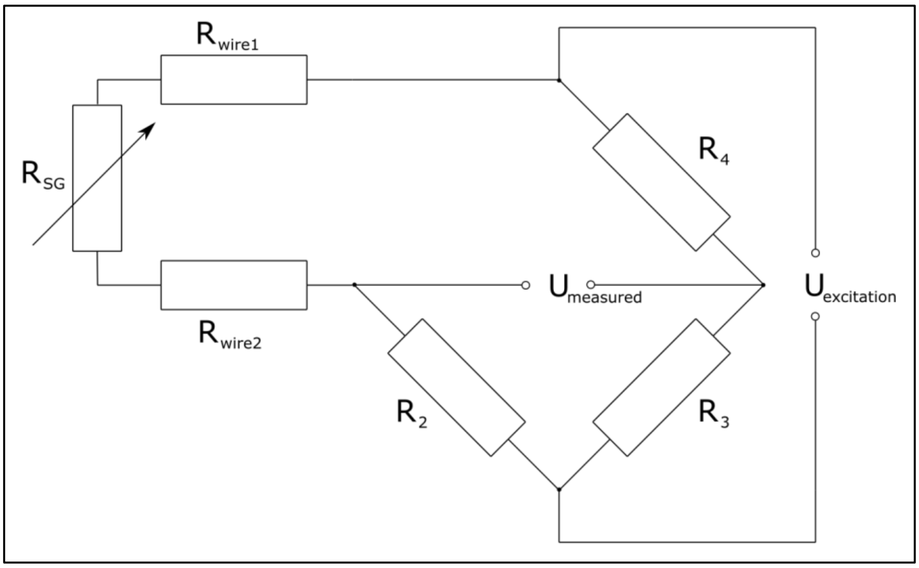

Figure 3.8-1: Two-wire quarter bridge installation

It should also be noted that this adjustment is temperature dependent, as the wires' resistance temperature is dependent as well.

Another disadvantage of this installation is the gauge factor desensitization, which is the reduction of the sensitivity of the gauge to the strain, since the wires' resistance is inactive (not sensitive to strain) but still introduced to the quarter bridge arm of strain gauge resistance.

Three-wire quarter bridge installation

This installation uses three identical leads to connect the strain gauge to the Wheatstone bridge as shown in Figure 3.8-2. By doing so, the equal lead resistance of the wires will be placed in two adjacent arms of the bridge, which maintain the bridge balance with respect to the lead wire.

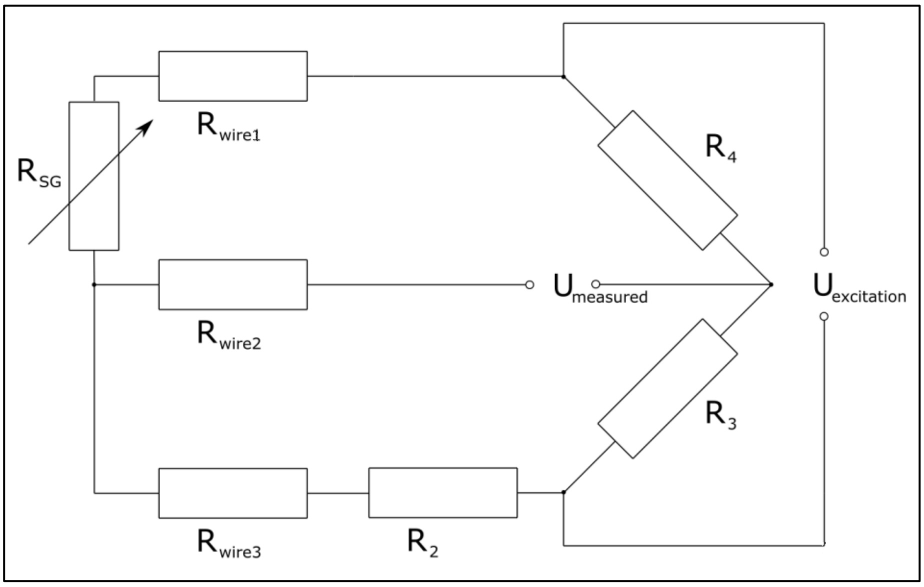

Figure 3.8-2: Three-wire quarter bridge installation

A great advantage of this setup is the cancelation of effect of the temperature-induced resistance change of the wire on the bridge, as this resistance change does not affect the balance of the bridge. Which omits any potential apparent strain readings, if is the temperature is not constant during a test.

It is noteworthy that the signal (the current flowing through the wires) is sensitive to magnetic fields. Consequently, this effect must be considered as a potential source of a measurement deviation and should be investigated. As a protective measure, shielded wires can be used to prevent any source of distortion of the signal due to magnetic fields. Although the use of shielded wires might introduce another source of measurement deviation, represented in an inductive effect; this effect remains almost unnoticeable, since the used currents are usually not significantly high. Nonetheless, this should be considered according to the individual setup of a measurement.

3.9 Mathematical Model of strain measurement

As mentioned in chapter 2, the second step of evaluating a measurement uncertainty is to develop a mathematical model that describes the relations between all input quantities and the output quantity of the measurement. To apply this approach to a strain measurement using a strain gauge with a Wheatstone quarter bridge, the author has chosen to break down the model of evaluating the measurement into several subfunctions; for the ease of handling and introducing the model, and more importantly to facilitate the implementation of the model in a program that automatizes the procedure of evaluating the uncertainty of a static strain measurement. For this evaluation, two different cases are distinguishable, which are the one-point measurement and the two-point measurement.

3.9.1 One-point Measurement

This method is used to measure the strain at a given stress-condition relative to a zero stress-condition of the strain gauge; for that reason, this method is also known as a zero-point related measurement. The zero stress-condition is represented by the measured stress before attaching the strain gauge to the test object, which is zero. The drawback of this method is introducing the problem of potentially measuring an apparent strain due to stress applied to the gauge while mounting it, which will be present in each measurement as a permanent offset. This apparent strain may be significant if mounting the strain gauge was carried out poorly.

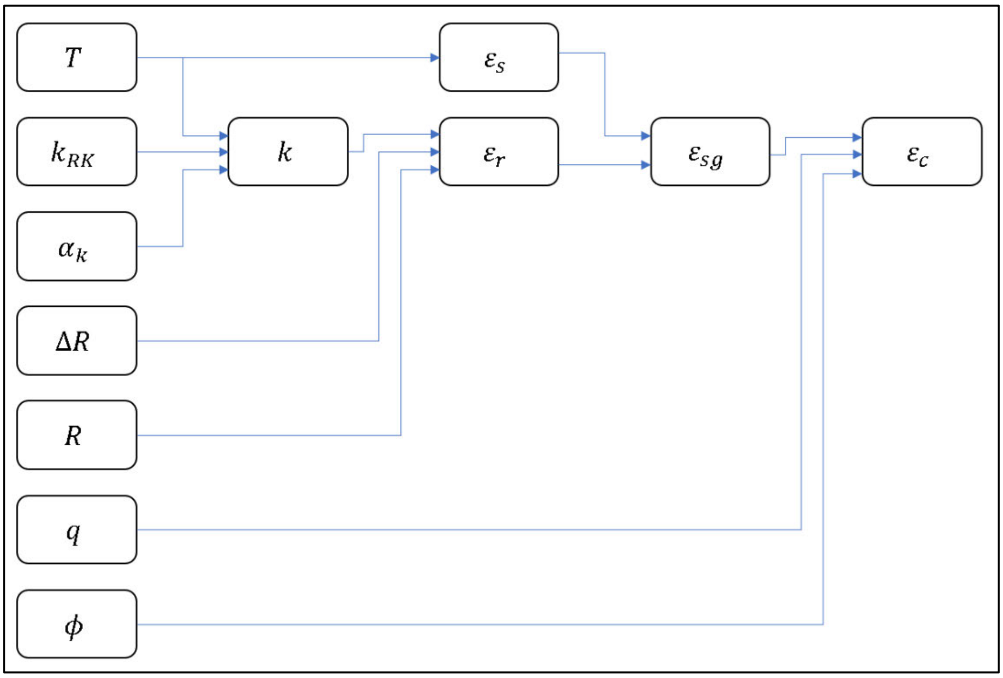

Figure 3.9-1: Flowchart of evaluating a strain measurement

The approach of evaluating a one-point strain measurement is illustrated in Figure 3.9-1, and starts with the correction of the gauge-factor error due to temperature-dependence of the resistance of the strain gauge grid, which is given by:

$$k = k_{RK}(1 + \alpha_k(T - T_R)) \qquad Eq\ 3.9\text{-}1$$

With the input quantities:

α_k the coefficient of temperature-dependence of the gauge factor.
k_{Rk} the gauge-factor at the reference temperature.
T the temperature at the measurement.

The reference temperature $T_R = (23 + 273.15)K$ is assumed here to be a constant value and not a variable.

The strain measured by the gauge is then:

$$\varepsilon_r = \frac{\Delta R}{Rk} \qquad \text{Eq } 3.9\text{-}2$$

With the input quantities:

k the corrected gauge-factor.
ΔR the measured change in resistance under deformation.
R the resistance of the gauge grid with no deformation.

To compensate for the apparent strain, the following equation is used to evaluate the apparent strain at first:

$$\varepsilon_s = c_0 + c_1 T + c_2 T^2 + c_3 T^3 + c_4 T^4 \qquad Eq\ 3.9\text{-}3$$

With the input quantities:
T the temperature.
c_{idx} the coefficients of the polynomial function.

The measured strain after compensating for the apparent strain is then:

$$\varepsilon_{sg} = \varepsilon_s + \varepsilon_r \qquad Eq\ 3.9\text{-}4$$

With the input quantities:
ε_s the apparent strain.
ε_r the strain gauge reading.

The correction of the error due to transverse sensitivity and the angular misalignment of the strain gauge is given by:

$$\varepsilon_c = \varepsilon_{sg}\ f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right) \qquad \text{Eq 3.9-5}$$

With the input quantities:
ε_{sg} the measured strain after compensating for the apparent strain;
$f_{\varepsilon,corrected}$ the correction function with the input quantities;
$c_{\sigma_2} c_{\sigma_3}, c_\tau$ the linearity coefficients of the strain tensor;
ϕ the misalignment angle;
q the transverse sensitivity;
ν_0 Poisson's ratios of the strain gauge grid;
ν Poisson's ratios of the material of the test object ν.

With

$$f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right) = \varepsilon_{object}/\varepsilon_{sg,app} \qquad \text{Eq 3.9-6}$$

With
ε_{object} the object of the strain measurement;
$\varepsilon_{sg,app}$ the approximation of the measured strain by the strain gauge.

With

$$\varepsilon_{object}E = \sigma_1 - \nu(\sigma_2 + \sigma_3) \qquad \text{Eq 3.9-7}$$

Where
$\sigma_1, \sigma_2\ and\ \sigma_3$ are the component of the stress tensor with no angular misalignment

With

$$\varepsilon_{sg,app}E = \frac{\varepsilon'_1 E + q\ \varepsilon'_2 E}{1 - \nu_0 q} \qquad \text{Eq 3.9-8}$$

Where

$$\begin{aligned}\varepsilon'_1 E = {} & \sigma_1 \cos^2\phi - 2\tau_{12} \cos\phi \sin\phi + \sigma_2 \sin^2\phi \\ & - \nu\left(+\sigma_1 \sin^2\phi + 2\tau_{12} \sin\phi\cos\phi + \sigma_2 \cos^2\phi + \sigma_3\right)\end{aligned} \qquad Eq\ 3.9\text{-}9$$

And

$$\begin{aligned}\varepsilon'_2 E = {} & +\sigma_1 \sin^2\phi + 2\tau_{12} \cos\phi \sin\phi + \sigma_2 \cos^2\phi \\ & - \nu\left(+\sigma_1 \cos^2\phi - 2\tau_{12} \cos\phi\sin\phi + \sigma_2 \sin^2\phi + \sigma_3\right)\end{aligned} \qquad Eq\ 3.9\text{-}10$$

3.9.2 Two-point Measurement

This method is called a non-zero-point related measurement; and is applied when measuring a strain difference induced between two stress states. As an example, measuring the strain-change due to change of a load asserted on a test object in a short period of time. In this method the first measurement represents the reference strain, where the second measurement carries the information about the strain-change in relation to the load.

The appropriate approach to evaluate the strain using this method is to consider the measurement to be two separate measurements and evaluate both measurements according to the approach given above for a one-point measurement. The strain difference can be obtained by subtracting the first measurement from the second measurement:

$$\varepsilon_{1,2} = \varepsilon_{c2} - \varepsilon_{c1} \qquad Eq\ 3.9\text{-}11$$

The approach is to evaluate the first strain measurement with neglecting the apparent strain; and then to use the negative value of the first measurement as the apparent strain to evaluate the second measurement is generally invalid. Since a clear differentiation between the apparent strain and the strain of mechanical origin is required, in order to be able to correct the error due to transverse sensitivity and angular misalignment according to the approach introduced in 3.6.1.

3.10 Evaluation of one-point measurement uncertainty:

The evaluation of the measurement will be based on the mathematical model introducer in 3.9 and is described in the flowchart in Figure 3.10-1

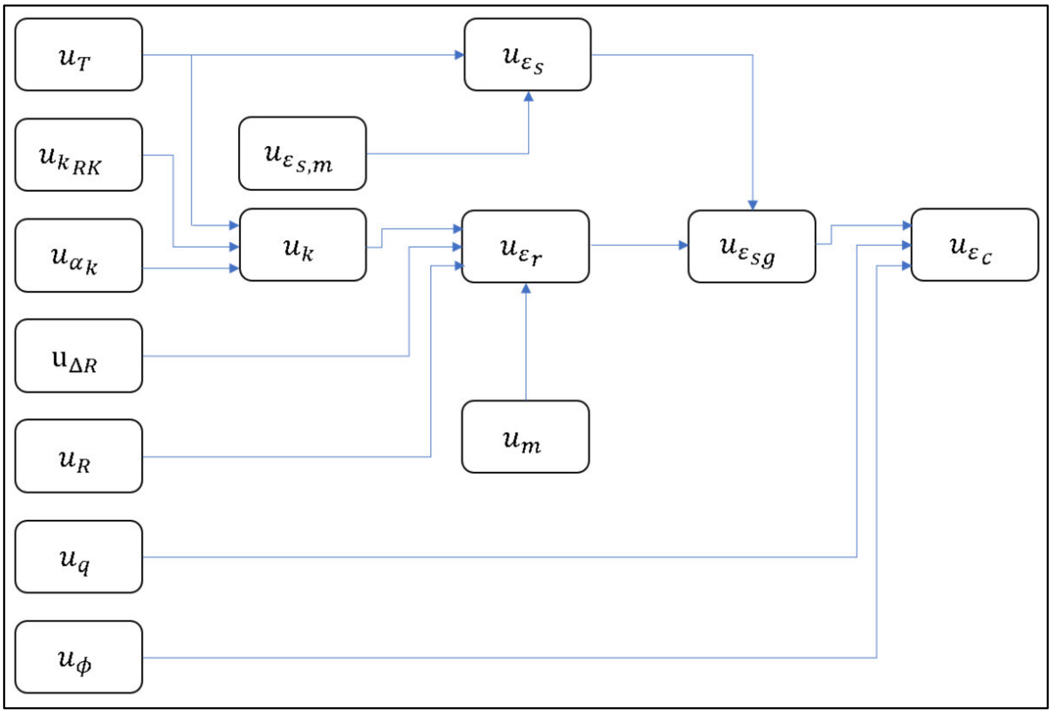

Figure 3.10-1: Flowchart of evaluating the strain measurement uncertainty

The uncertainty of the gauge factor is given by:

$$u_k^2 = u_T^2 c_{k,T}^2 + u_{k_{RK}}^2 c_{k,k_{RK}}^2 + u_{\alpha_k}^2 c_{k,\alpha_k}^2 \qquad Eq\ 3.10\text{-}1$$

Where:
u_T the uncertainty of the temperature measurement;
$u_{k_{RK}}$ the uncertainty of the reference gauge factor;
u_{α_k} the uncertainty of the coefficient of temperature-dependence of the gauge factor;
$c_{k,T}$ the sensitivity function of the gauge factor with respect to the quantity T;
$c_{k,k_{RK}}$ the sensitivity function of the gauge factor with respect to the quantity k_{RK};
c_{k,α_k} the sensitivity function of the gauge factor with respect to the quantity α_k;

With

$$c_{k,T} = \frac{\partial k}{\partial T} = k_{RK}\alpha_k \qquad Eq\ 3.10\text{-}2$$

$$c_{k,k_{RK}} = \frac{\partial k}{\partial k_{RK}} = 1 + \alpha_k(T - T_R) \qquad Eq\ 3.10\text{-}3$$

$$c_{k,\alpha_k} = \frac{\partial k}{\partial \alpha_k} = k_{RK}(T - T_R) \qquad Eq\ 3.10\text{-}4$$

The uncertainty of the apparent strain is given by:

$$u_{\varepsilon_s}^2 = u_T^2 c_{\varepsilon_s,T}^2 + u_{\varepsilon_{s;m}}^2 \qquad Eq\ 3.10\text{-}5$$

Where
$c_{\varepsilon_s,T}$ the sensitivity function of the apparent strain with respect to the quantity T;
$u_{\varepsilon_{s;m}}$ the standard uncertainty of the apparent strain function (given by manufacturer).

With

$$c_{\varepsilon_s,T} = \frac{\partial \varepsilon_s}{\partial T} \qquad Eq\ 3.10\text{-}6$$

The uncertainty of the measured strain is:

$$u_{\varepsilon_r}^2 = u_{\Delta R}^2 c_{\varepsilon_r,\Delta R}^2 + u_k^2 c_{\varepsilon_r,k}^2 + u_R^2 c_{\varepsilon_r,R}^2 + u_m^2 \qquad Eq\ 3.10\text{-}7$$

Where:
$u_{\Delta R}$ the uncertainty of the resistance change;
u_k the uncertainty of the gauge factor;
u_R the uncertainty of the resistance of the unstressed gauge;
u_m the uncertainty of the measured strain related to operating modules (given by manufacturer);
$c_{\varepsilon_r,\Delta R}$ the sensitivity function of the measured strain with respect to the quantity ΔR;
$c_{\varepsilon_r,k}$ the sensitivity function of the measured strain with respect to the quantity k;

$c_{\varepsilon_r,R}$ the sensitivity function of the measured strain with respect to the quantity R;

With

$$c_{\varepsilon_r,\Delta R} = \frac{\partial \varepsilon_r}{\partial \Delta R} = \frac{1}{Rk} \qquad Eq\ 3.10\text{-}8$$

$$c_{\varepsilon_r,k} = \frac{\partial \varepsilon_r}{\partial k} = -\frac{\Delta R}{Rk^2} \qquad Eq\ 3.10\text{-}9$$

$$c_{\varepsilon_r,R} = \frac{\partial \varepsilon_r}{\partial R} = -\frac{\Delta R}{R^2 k} \qquad Eq\ 3.10\text{-}10$$

And the uncertainty of the measured strain related to operating modules:

$$u_m^2 = \varepsilon_r^2 \cdot \left(u_{lin}^2 + u_{rep}^2 + u_{acc}^2\right) \qquad Eq\ 3.10\text{-}11$$

Where:
u_{lin} the uncertainty related to linearity;
u_{rep} the uncertainty related to repeatability;
u_{acc} the uncertainty related to accuracy.

The uncertainty of the measured strain after compensating for the apparent strain is:

$$u_{\varepsilon_{sg}}^2 = u_{\varepsilon_r}^2 c_{\varepsilon_{sg},\varepsilon_r}^2 + u_{\varepsilon_s}^2 c_{\varepsilon_{sg},\varepsilon_s}^2 \qquad Eq\ 3.10\text{-}12$$

Where:
u_{ε_r} the uncertainty of the measured strain;
u_{ε_s} the uncertainty of the apparent strain;
$c_{\varepsilon_{sg},\varepsilon_r}$ the sensitivity function with respect to the quantity ε_r;
$c_{\varepsilon_{sg},\varepsilon_s}$ the sensitivity function with respect to the quantity ε_s;

With

$$c_{\varepsilon_{sg},\varepsilon_r} = c_{\varepsilon_{sg},\varepsilon_s} = \frac{\partial \varepsilon_{sg}}{\partial \varepsilon_r} = \frac{\partial \varepsilon_{sg}}{\partial \varepsilon_s} = 1 \qquad Eq\ 3.10\text{-}13$$

The uncertainty of the final corrected strain is:

$$u_{\varepsilon_c}^2 = u_{\varepsilon_{sg}}^2 c_{\varepsilon_c,\varepsilon_{sg}}^2 + u_\phi^2 c_{\varepsilon_c,\phi}^2 + u_q^2 c_{\varepsilon_c,q}^2 \qquad Eq\ 3.10\text{-}14$$

Where:
$u_{\varepsilon_{sg}}$ the uncertainty of the measured strain after compensating for the apparent strain;
u_ϕ the uncertainty of misalignment angle;
u_q the uncertainty of the transverse sensitivity compensation;
$c_{\varepsilon_c,\varepsilon_{sg}}$ the sensitivity function of the corrected strain with respect to the quantity ε_{sg};
$c_{\varepsilon_c,\phi}$ the sensitivity function of the corrected strain with respect to the quantity ϕ;
$c_{\varepsilon_c,q}$ the sensitivity function of the corrected strain with respect to the quantity q.

With:

$$c_{\varepsilon_c,\varepsilon_{sg}} = \frac{\partial \varepsilon_c}{\partial \varepsilon_{sg}} = f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right) \qquad Eq\ 3.10\text{-}15$$

$$c_{\varepsilon_c,\phi} = \frac{\partial \varepsilon_c}{\partial \phi} = \varepsilon_{sg} \frac{\partial\, f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial \phi} \qquad Eq\ 3.10\text{-}16$$

With

$$\frac{\partial\, f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial \phi} = \varepsilon_{object} \frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial \phi} \qquad Eq\ 3.10\text{-}17$$

And:

$$\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial \phi} = (1 - \nu_0 q) \frac{\partial((\varepsilon'_1 + q\ \varepsilon'_2)^{-1})}{\partial \phi} \qquad Eq\ 3.10\text{-}18$$

With

$$\frac{\partial((\varepsilon'_1 + q\ \varepsilon'_2)^{-1})}{\partial \phi} = -\ \frac{\partial(\varepsilon'_1 + q\ \varepsilon'_2)}{\partial \phi} (\varepsilon'_1 + q\ \varepsilon'_2)^{-2} \qquad Eq\ 3.10\text{-}19$$

With

$$\frac{\partial(\varepsilon'_1 + q\ \varepsilon'_2)}{\partial \phi} = \frac{\partial \varepsilon'_1}{\partial \phi} + q \frac{\partial\ \varepsilon'_2}{\partial \phi}$$

Where

$$\begin{aligned}\varepsilon'_1 E = \sigma_1\ \cos^2\phi\ - 2\tau_{12}\ \cos\phi \sin\phi + \sigma_2\ \sin^2\phi \\ - \nu\,(+\sigma_1\ \sin^2\phi + 2\tau_{12}\ \sin\phi\cos\phi + \sigma_2\ \cos^2\phi \\ + \sigma_3)\end{aligned} \qquad Eq\ 3.10\text{-}20$$

And

$$\begin{aligned}\varepsilon'_2 E = +\sigma_1\ \sin^2\phi + 2\tau_{12}\ \cos\phi \sin\phi + \sigma_2\ \cos^2\phi \\ - \nu\,(+\sigma_1\ \cos^2\phi - 2\tau_{12}\ \cos\phi\sin\phi + \sigma_2\ \sin^2\phi \\ + \sigma_3)\end{aligned} \qquad Eq\ 3.10\text{-}21$$

With

$$\begin{aligned}E\frac{\partial \varepsilon'_1}{\partial \phi} = 2\cos\phi\ \sin\phi\,(\,\sigma_2 -\ \sigma_1) + 2\,\tau(\,\sin^2\phi -\ \cos^2\phi) \\ - \nu\left(2\,\tau(\cos^2\phi\ -\ \sin^2\phi) \right. \\ \left. + 2\cos\phi\ \sin\phi\,(\,\sigma_1 -\ \sigma_2)\right)\end{aligned} \qquad Eq\ 3.10\text{-}22$$

And

$$\begin{aligned}E\frac{\partial \varepsilon'_2}{\partial \phi} = 2\cos\phi\ \sin\phi\,(\,\sigma_1 -\ \sigma_2) + 2\,\tau(\,\cos^2\phi -\ \sin^2\phi) \\ + \nu\left(2\,\tau(\cos^2\phi\ -\ \sin^2\phi) \right. \\ \left. + 2\cos\phi\ \sin\phi\,(\,\sigma_1 -\ \sigma_2)\right)\end{aligned} \qquad Eq\ 3.10\text{-}23$$

The sensitivity function of the corrected strain with respect to the transverse sensitivity is:

$$c_{\varepsilon_c,q} = \frac{\partial \varepsilon_c}{\partial q} = \varepsilon_{sg} \frac{\partial\, f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial q} \qquad Eq\ 3.10\text{-}24$$

With

$$\frac{\partial\, f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial q} = \varepsilon_{object} \frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial q} \qquad Eq\ 3.10\text{-}25$$

And:

$$\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial q} = \frac{\partial(-\nu_0 q(\varepsilon'_1 + q\ \varepsilon'_2)^{-1})}{\partial q} \qquad Eq\ 3.10\text{-}26$$

$$\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial q} = \frac{-\nu_0}{(\varepsilon'_1 + q\ \varepsilon'_2)} \qquad Eq\ 3.10\text{-}27$$

However, the transverse sensitivity is usually given by the manufacturer of the strain gauge as a constant with no estimate of a standard uncertainty. Furthermore, the correction of the transverse sensitivity compared to the correction of the angular misalignment is in most cases negligible. As a result, the uncertainty component related to the transverse sensitivity is negligible as well.

3.11 Evaluation of two-point measurement uncertainty:

This evaluation is based on the mathematical approach given in 3.9.2 and is given by:

$$u_{\varepsilon_{1,2}}^2 = u_{\varepsilon_{c2}}^2 + u_{\varepsilon_{c1}}^2 \qquad Eq\ 3.11\text{-}1$$

Where

$u_{\varepsilon_{1,2}}$ the uncertainty of the two-point measurement;

$u_{\varepsilon_{c1}}$ the uncertainty of the first measurement;

$u_{\varepsilon_{c2}}$ the uncertainty of the second measurement;

3.12 Load-Strain measurement uncertainty

Depending on the object of the strain measurement, the load-strain relation might be of a great interest. In such a case, the uncertainty about quantifying the load as a driving force of the change in strain might be considered. To realize this, the load-strain relation should be defined, to which a good starting point is the stress-strain relation:

$$\varepsilon = \frac{\sigma}{E} \qquad Eq\ 3.12\text{-}1$$

Where

ε the strain

σ the stress

E Young's modulus

The stress can be replaced according to its origin (normal stress and bending stress) with:

$$\sigma = \sum \frac{F}{A} + \sum \frac{M}{I}\, e \qquad Eq\ 3.12\text{-}2$$

Where

F the force

A the cross-sectional Area, to which the force is perpendicular

M bending moment

I moment of inertia around the bending axis

e the distance between the measurement position and the bending axis

A new function to describe the stress can be defined:

$$\sigma = \sum \frac{F_{eff}}{A_{eff}} \qquad Eq\ 3.12\text{-}3$$

Where

F_{eff} an effective force

A_{eff} an effective cross-sectional area

For a normal stress:

$$F_{eff} = F$$

$$A_{eff} = A$$

For a bending stress:

$$F_{eff} = \frac{M}{e}$$

$$A_{eff} = \frac{I}{e^2}$$

A new function to describe the stress can be defined:

$$\sigma = f_\sigma(F_{eff,1}, A_{eff,1}, F_{eff,2}, A_{eff,2}, \dots F_{eff,n}, A_{eff,n}) \qquad Eq\ 3.12\text{-}4$$

By combining Eq 3.12-1 and Eq 3.12-4 , the following equation can be obtained:

$$\varepsilon = \frac{f_\sigma\left(F_{eff,1}, A_{eff,1}, F_{eff,2}, A_{eff,2}, \dots F_{eff,n}, A_{eff,n}\right)}{E} \qquad Eq\ 3.12\text{-}5$$

If a strain difference due to load change is to be considered, the following equation can be written:

$$\Delta\varepsilon = \frac{f_\sigma\left(\Delta F_{eff,1}, A_{eff,1}, \Delta F_{eff,2}, A_{eff,2}, \dots \Delta F_{eff,n}, A_{eff,n}\right)}{E} \qquad Eq\ 3.12\text{-}6$$

With known expectation values and standard deviations of the load components, the uncertainty about the measured strain is then:

$$u_{\Delta\varepsilon,L}^2 = \left[\sum_{i=1}^{n}\left(\frac{\partial f_\sigma}{\partial \Delta F_{eff,i}}\, u_{\Delta F_{eff,i}}\right)^2\right] / E^2 \qquad Eq\ 3.12\text{-}7$$

Here, it should be pointed to the fact that the stiffness of the test object is not necessarily a constant value, more often the stiffness of the test object is also load dependent. Accordingly, the nonlinear behavior of the load-strain should be investigated. The Young's modulus on the other hand is safe to consider constant in most cases.

A way to approximate the load-strain function is to fit value pairs of load-strain cases to an appropriate polynomial function using either simulation or experimental data.

For the special case of assuming the linearity of load-strain relation, with only one load component:

$$\varepsilon_{sp} = \frac{F_{eff_{sp}}}{E\ A_{eff_{sp}}} \qquad Eq\ 3.12\text{-}8$$

The strain deference between the two states can be expressed as:

$$\Delta\varepsilon_{sp} = \frac{\Delta F_{eff_{sp}}}{E\ A_{eff_{sp}}} \qquad Eq\ 3.12\text{-}9$$

With known expectation values and standard deviations of the load, the uncertainty about the measured strain that is related to the uncertainty about the load can be evaluated according to:

$$u_{\Delta\varepsilon_{sp}}^2 = \left(\frac{\partial \Delta\varepsilon_{sp}}{\partial \Delta F_{eff_{sp}}}\ u_{\Delta F_{eff_{sp}}}\right)^2 = \left(\frac{u_{\Delta F_{eff_{sp}}}}{E\ A_{eff_{sp}}}\right)^2 \qquad Eq\ 3.12\text{-}10$$

Which can be written as:

$$u_{\Delta\varepsilon_{sp}}^2 = \left(\frac{u_{\Delta F_{eff_{sp}}}}{E\ A_{eff_{sp}}}\right)^2 \left(\frac{\Delta F_{eff_{sp}}}{\Delta F_{eff_{sp}}}\right)^2 \qquad Eq\ 3.12\text{-}11$$

And after rearranging the equation:

$$u_{\Delta\varepsilon_{sp}}^2 = \left(\frac{\Delta F_{eff_{sp}}}{E\ A_{eff_{sp}}}\right)^2 \left(\frac{u_{\Delta F_{eff_{sp}}}}{\Delta F_{eff_{sp}}}\right)^2 \qquad Eq\ 3.12\text{-}12$$

Which is equivalent to:

$$u_{\Delta\varepsilon_{sp}}^2 = \Delta\varepsilon_{sp}{}^2 \left(\frac{u_{\Delta F_{eff_{sp}}}}{\Delta F_{eff_{sp}}}\right)^2 \qquad Eq\ 3.12\text{-}13$$

4 Study case

In this chapter a strain measurement performed on a large bearing as a part of stress analysis will be introduced. Furthermore, the evaluation of the strain and its uncertainty according to the approach proposed in this thesis will be discussed. As the test analysis is performed in the scope of a project called ‘highly accelerated pitch bearing test’; this project will be briefly introduced. The test bench, on which the experiments are done, will be introduced as well.

4.1 Highly Accelerated Pitch Bearing Test

Highly Accelerated Pitch Bearing Test is the name of a project on the Fraunhofer Institute for Wind Energy Systems IWES, in which the blade bearings of wind turbines are tested. The aim of the project is to better understand the damage mechanisms and other factors that might affect the life of the blade bearings, considering the specific operating circumstances in a wind turbine; such as the loads applied on the blade bearing and the behavior of the stiffness profile of the structure surrounding the blade bearing in an actual wind turbine. The goal is to improve the dimensioning methods and reduce the risk of premature failure of blade bearings [26] [27].

The necessity of such an investigation stems from the fact, that the usual methods of dimensioning the blade bearing are largely founded on empirical knowledge, that is based on its turn on data achieved by experiments; that typically fulfill certain operating conditions, but lacks the adequate description of the operating conditions of a wind turbine. In addition, new methods of load reduction and new bearing designs might play a rule of characterizing the operating condition, which requires a new assessment of the followed approach of calculating the fatigue life of the blade bearings.

This project deploys multiple methods and test benches; but the test rig relevant to this study case is the Bearing Endurance and Acceptance Test 6.1 which will be introduced in the following section.

4.2 Bearing Endurance and Acceptance Test Rig 6.1

The name of the test rig is abbreviated with BEAT 6.1. It is a full-scale bearing test rig that operates since spring 2019 and can test two bearings simultaneously with a diameter from $3\ m$ up to $6.5\ m$ (see Figure 4.2-1). The maximum achievable static load is a bending moment of $50\ MNm$, while the dynamic maximum bending moment can reach $25\ MNm$ with a frequency of $0.7\ Hz$. The concept of accelerated testing allows the load simulation of a wind turbine lifetime (typically 20 years of operation) within few months (around 6 months). The measurement system of the bench cooperates with a data acquisitions system, that is capable of deploying 500 high-resolution measurement channels.

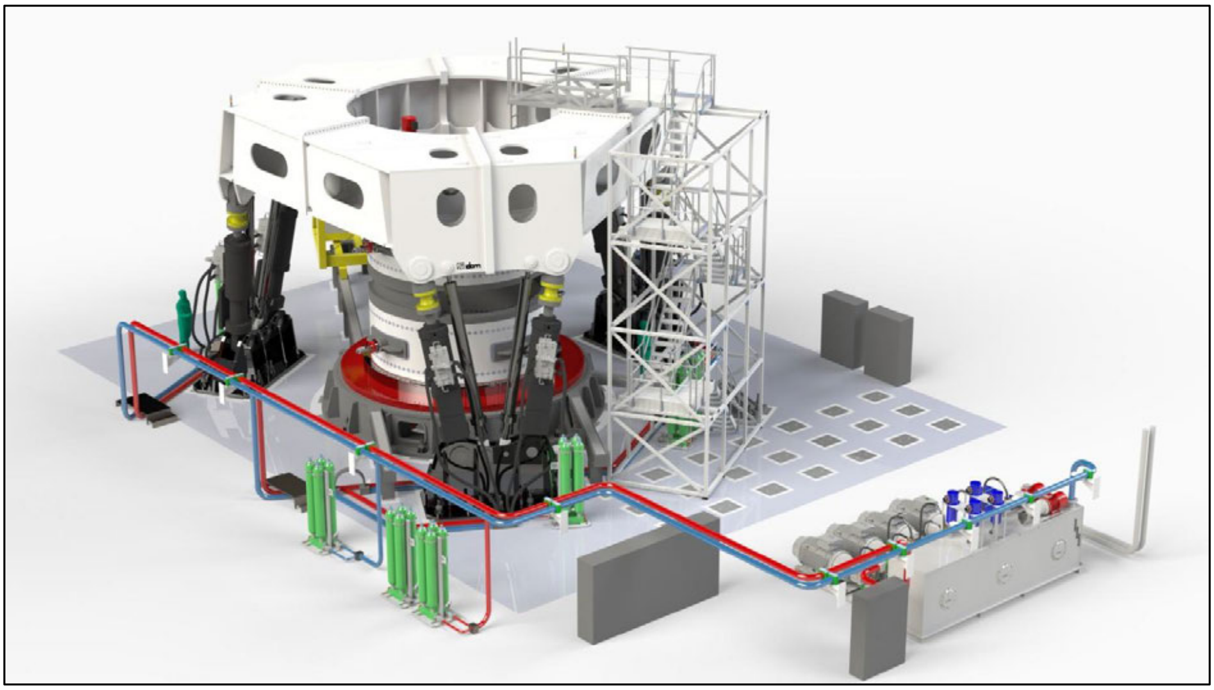

Figure 4.2-1: Bearing endurance and acceptance test rig 6.1 (source: https://www.iwes.fraunhofer.de; retrieved: 13/08/2020 12:00)

The test rig consists of 5 main elements, these are:

- The reaction frame: the base of the test rig.
- The lower hub adapter: a steel structure that connects the lower bearing to be tested with the reaction frame. The purpose of this element is to emulate the stiffness profile of the hub.
- The force transmitting element: which connects the two tested bearings. This element represents the blade. Optionally, it can also emulate the deformation of the blade in a certain operating condition.
- The upper hub adapter: similar to the lower hub adapter.
- The load platform: a hexagon-prism shaped steel structure that is connected to the upper hub adapter with a bolt connection. This element is also connected to six load cylinders for the purpose of applying a given load to the bearings to be tested.

The load application system, which consists of the hexapod of the load platform and the six hydraulic cylinders, can realize a static or a dynamic load in six degrees of freedom, which enables the rig to perform realistic tests, that describe various operating conditions of blade bearing in an actual wind turbine accurately.

The data acquisition system collects amongst others:

- the measurement of the load application systems
- the deformation of the tested bearings and other elements of the rig in various chosen positions using strain gauges and laser distance sensors
- The position and orientation of the tested bearings.
- The temperature and humidity in the test hall.

4.3 Specific conditions of strain measurement

The strain is measured using a single bonded electric resistance strain gauge with a quarter Wheatstone bridge installation in several positions. These positions are usually circumferential to the outer ring and the inner ring of the bearing as shown in Figure 4.3-1.

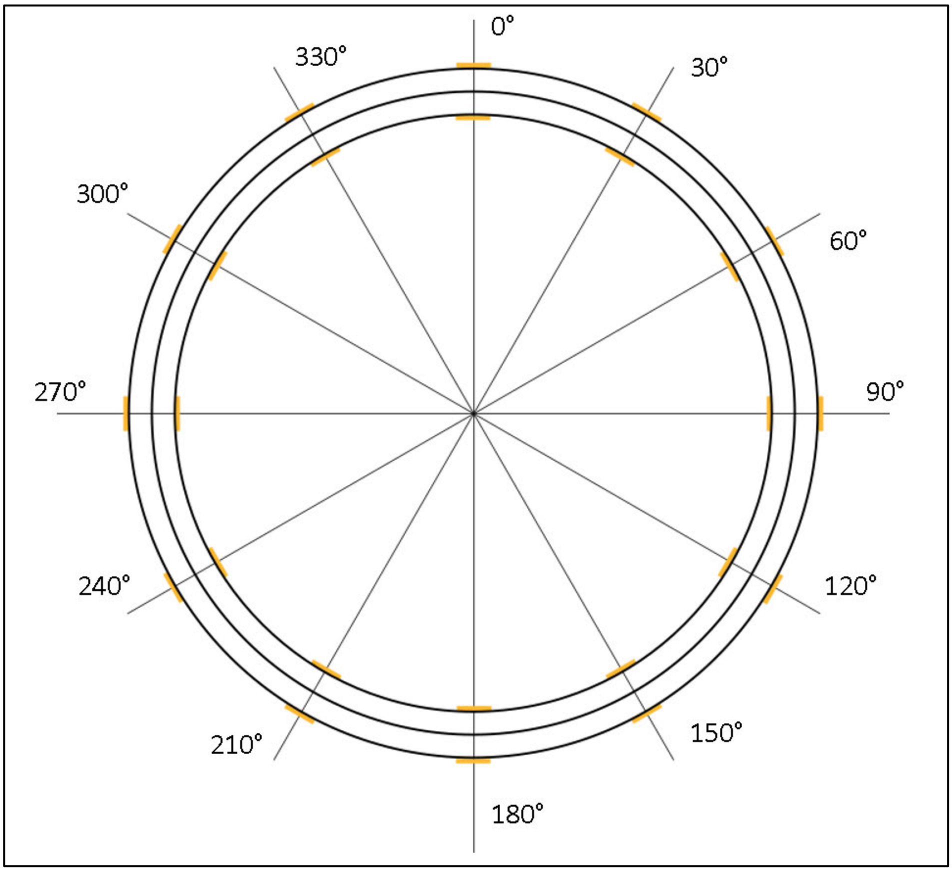

Figure 4.3-1: The positions of the bearing's strain measurement.

Two separate measurement modules are used to generate the excitation current of the strain gauges, to house the quarter Wheatstone bridge that detects the change of resistance of the strain gauges under deformation and to convert the signal into digital output.

These measurement settings are used to detect the strain introduced to the bearing under a static load. The temperature is assumed to be constant during the test, as the duration of the repetition of a static test usually does not exceed one day. Here; it should be pointed to the fact that the test rig including the bearings to be tested have enough mass to ensure a thermal inertia against the minimal fluctuation of the temperature of the ambient (in the test hall). Therefor any measurement deviation related to the change of temperature of the test object is negligible. Any change or fluctuation of the strain gauge temperature due to direct radiation exposure of the gauge is normally overruled or tolerated.

The characteristic values of the strain gauges are retrieved form the specification paper provided by the manufacturer of the strain gauges. The characteristic values of the measurement modules are also given by their manufacturer. The change of resistance of the strain gauges and the ambient temperature in the test hall, which is assumed to be in equilibrium with the test object's temperature, are measured with a frequency of $200\ Hz$. The load of each individual cylinder is measured using load cells, the sum of the load introduced to the bearing is then derived from the load cells readings. The frequency of this measurement is also $200\ Hz$. The stress tensors and its orientation in the measurement positions are obtained from the result of a simulation of the test.

Quantity	Symbol	Approximation	Source
Resistance of the unstressed strain gauge	R	Normally distributed	Manufacturer given
Gauge factor	k_{RK}	Normally distributed	Manufacturer given
Temperature compensation coefficient of the strain gauge	α_k	Normally distributed	Manufacturer given
Apparent strain function	c_0, c_1, c_2, c_3, c_4	Normally distributed	Manufacturer given
Linearity of measurement module	u_{lin}	Constant	Manufacturer given
Repeatability of measurement module	u_{rep}	Constant	Manufacturer given
Accuracy of measurement module	u_{acc}	Constant	Manufacturer given
Resistance-change	ΔR	Normally distributed	Measured
Temperature	T	Normally distributed	Measured
Temperature referance	T_{RK}	Constant	Manufacturer given
The load	L	Normally distributed	Measured
Stress tensor and orientation	$\boldsymbol{\sigma}$	Constant	Simulated
Poisson's ration of the bearing's material	ν	Constant	Manufacturer given
Transverse sensitivity of the strain gauge	q	Constant	Manufacturer given
Angular misalignment	α	Normally distributed	Measured
Poisson's ration of the strain gauge	ν_0	Constant	Manufacturer given

Table 4.3-1: The input values of the evaluation of strain measurement and its uncertainty

Table 4.3-1 lists all the input values of the evaluation of strain measurement and its uncertainty based on the specific settings of the strain measurement.

The author has documented an evaluation of a strain measurement and its uncertainty in Appendix B. The evaluation in this appendix is based on hypothetical input quantities that realistically describe possible measurement settings and specifications.

In Appendix A the source script of the evaluation tool is documented in addition to an example of the output quantities and a documentation of the evaluation. Because the simulation's information and its results for specific tests are confidential, the author has simulated a simplified case for the sake of completeness. In order to do so, a geometrical description of a hypothetical bearing with an external diameter of $6\ m$ were used Figure 4.3-2.

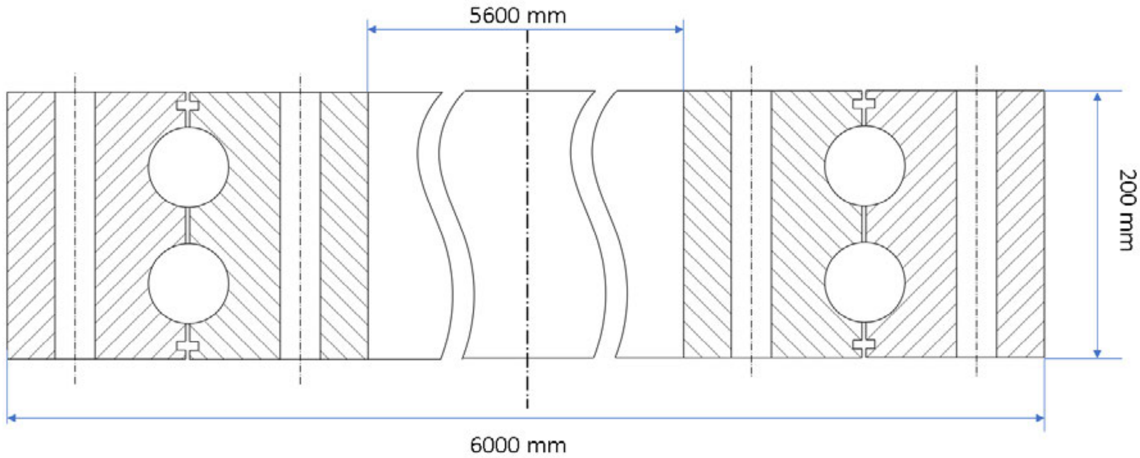

Figure 4.3-2: Geometry of a hypothetical bearing that can be tested using BEAT 6.1

The documentation of this simulation is included in Appendix D. For the sake of simplicity, the load of the simulation is set to be an axial force referenced to the rotation axis of the bearing. This assumption will allow the simulation of only one sector of the outer ring of the bearing instead of simulating a whole outer ring.

The simulation results showed a generality of a biaxial strain field with both of its normal stresses component in the plane tangential to the bearing in the measurement position. Plausibly, the stress normal to the measurement planes is always nonexistent, if no load normal to the measurement plane at the position of measurement is applied. Which is the general case, unless the ambient pressure surrounding the measurement's position is high enough to cause a relevant force normal to the measurement plane.

4.4 Sensitivity analysis

The goal of this analysis is to investigate the correctness of the evaluation of the strain uncertainty according to the approach proposed in this thesis. Furthermore, the evaluation of the strain and its uncertainty will be done with multiple variation of input information based on different assumptions, in order to investigate the contribution of the individual input values.

4.4.1 Validation

In Appendix E a numerical approach to validate the analytical evaluation of the measurement according to this thesis is introduced. It shows that it is safe to linearize all functions used in this approach except for the function of correction the error due to angular misalignment. In Figure 4.4-1 an example of the results of numerical validation is shown, the input values of this investigation are similar to those who are listed in Table B 1, except for the expectation and standard deviation values of the angular misalignment, these were set to be zero.

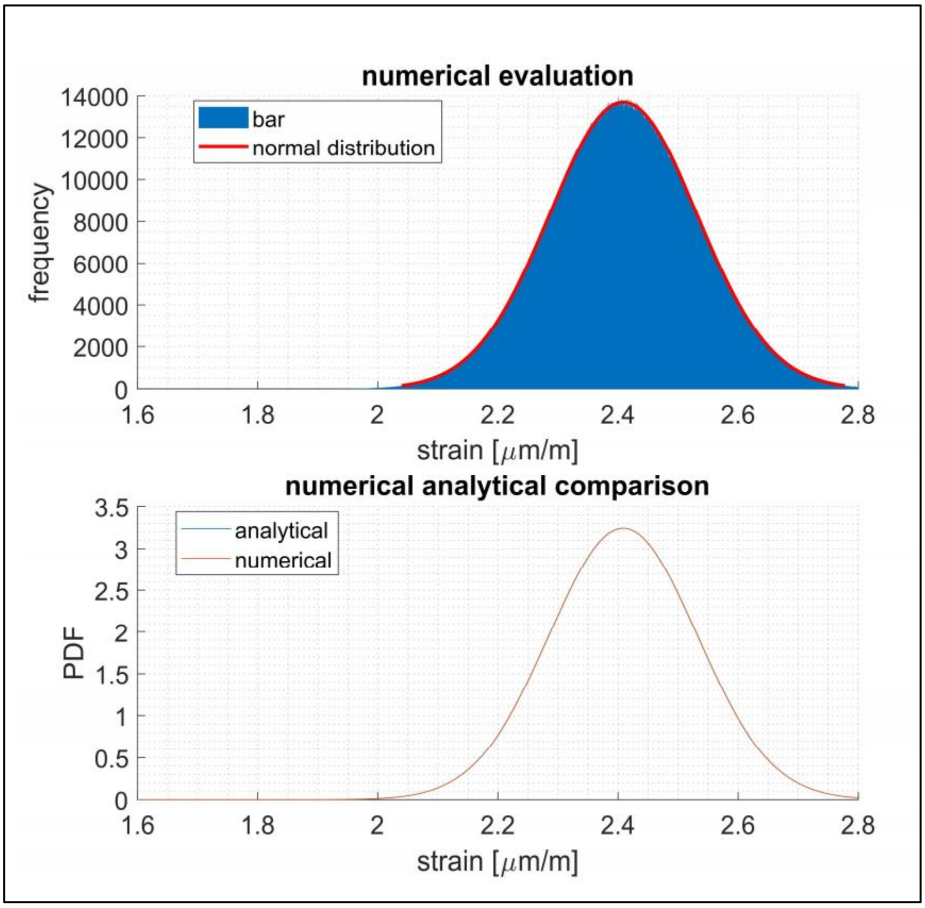

Figure 4.4-1: Numerical validation without considering the angular misalignment, upper graph: blue histogram of the individual numerical evaluations; red curve: the normal distribution of the numerical evaluation of the corrected strain; lower graph: the analytically and numerically evaluated normal distributions (visually almost identical).

This investigation amongst other investigations with the variation of the components of the stress tensor, the change of the resistance and the temperature has shown a numerical-analytical relative deviation of the expectation value of the corrected strain that does not exceed $0.6\,\%$ and a relative deviation of the standard uncertainty that does not exceed $1\,\%$. These results were expected, as the error introduced to the measurement uncertainty due to linearizing the components of uncertainty is mostly nonexistent or negligible, if the angular misalignment is to be excluded.

A general discussion of the validity to linearize the correction function of the error due to angular misalignment is not possible, as it is a multi-dimensional function with several input values, that determine the behavior of the function. The input values of this function and its sensitivity (the slope at the operating point as the best expectation of its value) are:

- The components of the stress tensor and its orientations:

For this study case it's usually a uniaxial stress field perpendicular to direction of the measured strain and parallel to the measurement plane, or a biaxial stress field with the main principal stress perpendicular to the direction of the measured strain but in the measurement plane.

- The expectation of the angle of misalignment:

It is usually set to zero. However, an individual measurement of the angular misalignment and its uncertainty will lower the measurement uncertainty significantly in some cases.

- The standard uncertainty of the angular misalignment:

The usual recommendation is $5°$; however, a sample measurement of an actual orientation of attached strain gauges to a test object (10 strain gauges) showed a value of $1.56° \pm 0.3°$. Based upon; the author recommends $2°$ as a standard uncertainty if the expectation is chosen to be neglected. In the case of a measurement of the actual orientation of the strain gauge, the standard uncertainty of this orientation measurement should be used as the standard uncertainty of the angular misalignment.

Figure 4.4-2 shows an example of a behavior of the corrected strain and the standard uncertainty in relation to the expectation of the angular misalignment. The stress field of this example is considered uniaxial perpendicular to the measured strain. All other input values are usual values that appear in an actual strain measurement of this study case. The standard uncertainty of the angular alignment is set to zero.

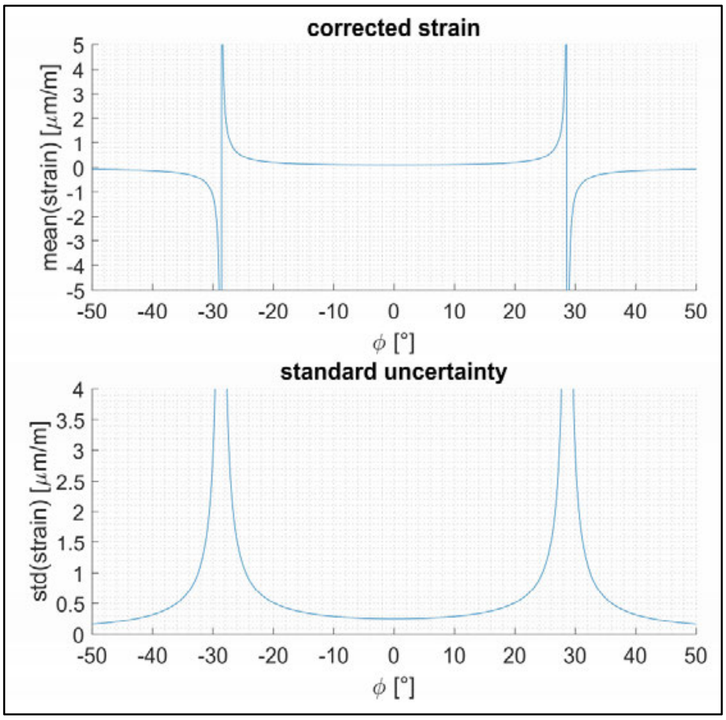

Figure 4.4-2: The relation between corrected strain and angular misalignment

As mentioned above, a general investigation of the validity of the analytical approach is not possible; however, relying on the observed and expected input values it is possible to differentiate between the two possible cases:

- Case a: The angular misalignment is measured for each strain gauge; the mean value and standard deviation of the angular misalignment is evaluated for each strain gauge.

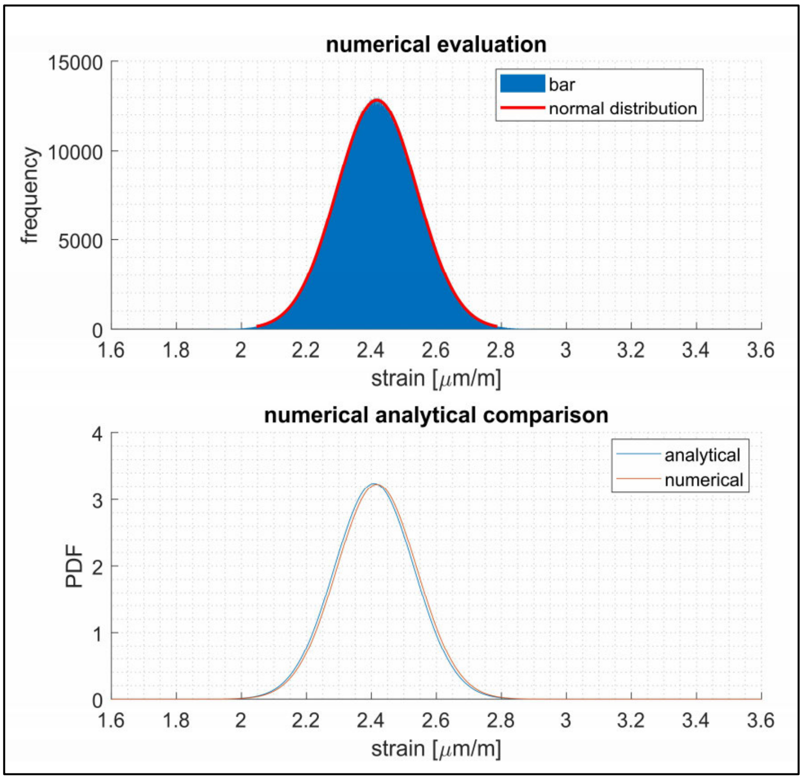

Figure 4.4-3: Numerical validation of case a

The example in Figure 4.4-3 shows that the numerical-analytical relative deviation of the corrected strain is about $0.35\ \%$ and a relative deviation of the standard uncertainty reaches $0.4\ \%$, if the angular misalignment set to $\phi = 1.56°$ and its standard uncertainty is set to $u_\phi = 0.30°$ as an example of individual measurement of the angular misalignment for each strain gauge.

- Case b: The mean value of the angular misalignment is set to zero, its standard uncertainty is approximated with the mean value of the measured angular misalignment of a sample of multiple strain gauges.

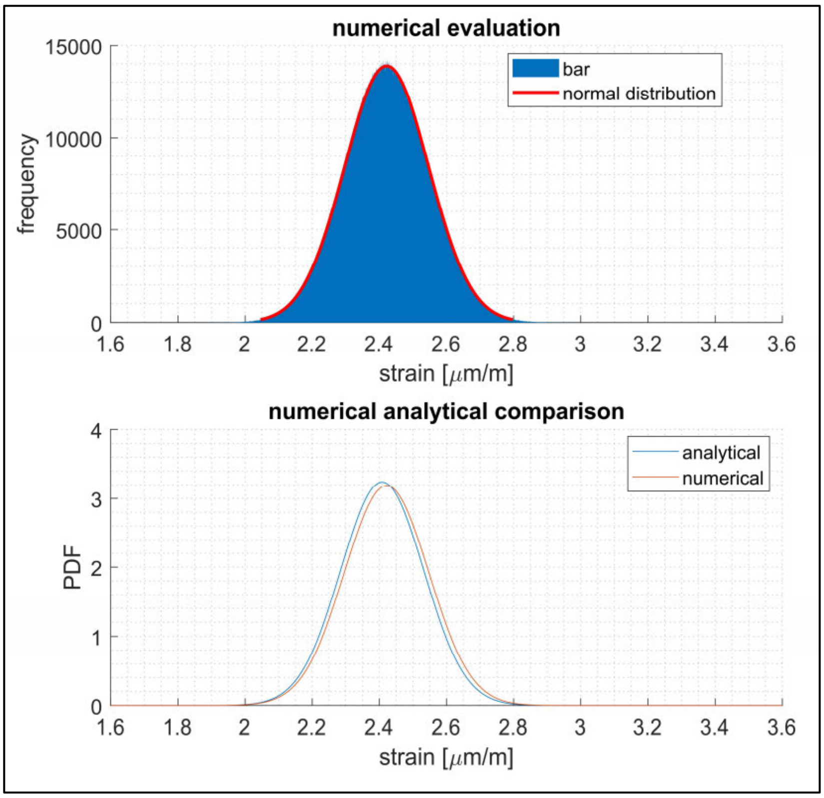

Figure 4.4-4: Numerical validation of case b with a $u_\phi = 2°$

Figure 4.4-4 shows the first example of an evaluation of this case, in which the angular misalignment is set to $\phi = 0°$ and its standard uncertainty is set to $u_\phi = 2°$. The numerical-analytical relative deviation of the corrected strain is about $0.6\ \%$ and a relative deviation of the standard uncertainty reaches $1.8\ \%$.

Figure 4.4-5 shows another example of an evaluation of the case b, where the angular misalignment is set to $\phi = 0°$ and its standard uncertainty is set to $u_{\phi} = 5°$. The numerical-analytical relative deviation of the corrected strain is about $4\ \%$ and a relative deviation of the standard uncertainty reaches $55\ \%$.; however, it should be pointed to the fact that neither the numerically evaluated normal distribution nor the analytically evaluated normal distribution are appropriate to describe the output distribution, as the bar-histogram of the individual output values tend to demonstrate the characteristics of a skewed normal distribution or other asymmetrical similar distributions.

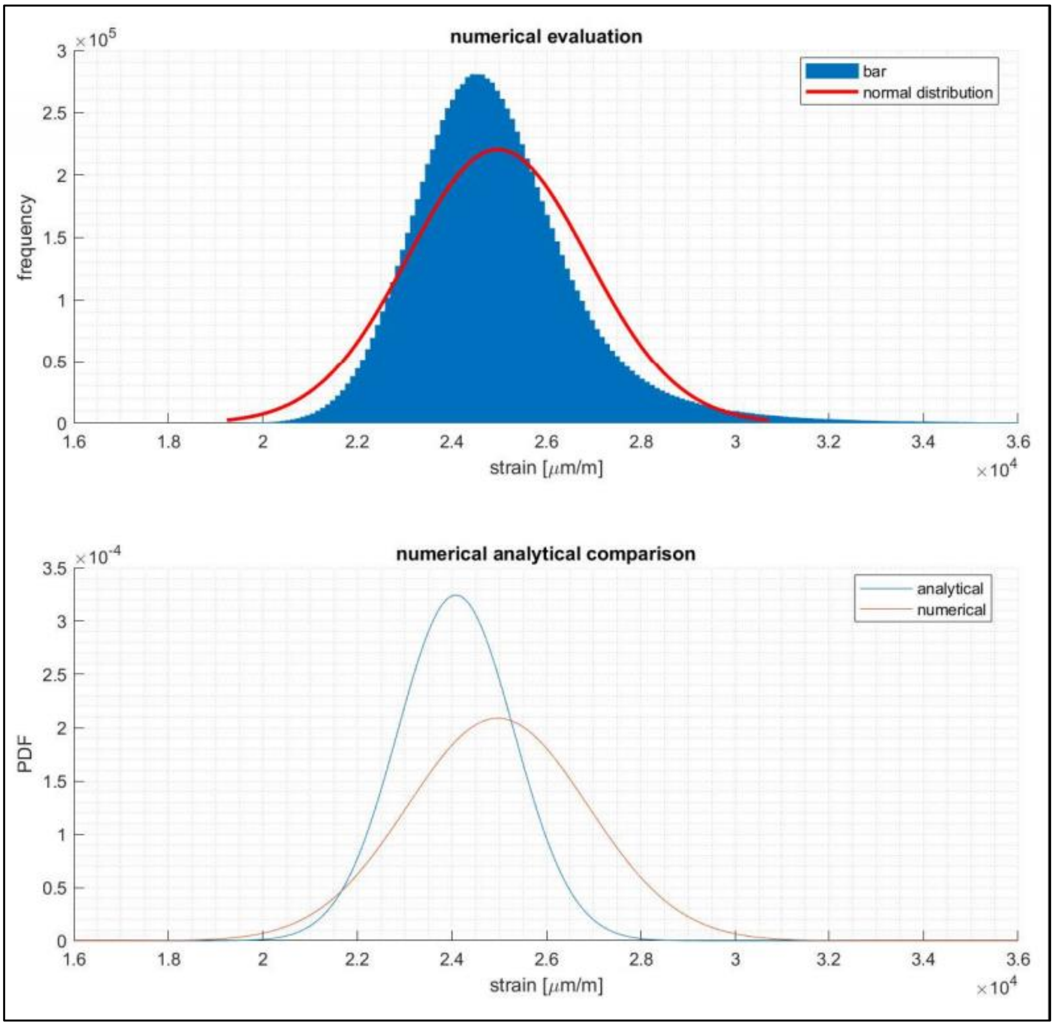

Figure 4.4-5: Numerical validation of case b with a $u_{\phi} = 5°$

4.4.2 Sensitivity analysis

This section was meant to investigate the individual effect of each input quantity on the behavior of the evaluation of the measurement uncertainty, as an attempt to prioritize the input quantities. The purpose of the investigation is to shed light on the practicability of neglecting some of the factors that might affect the measurement and its uncertainty.

The proposed method is to evaluate the corrected strain and its uncertainty with the variation of the input quantity; these variations are characterized as follows:

- Case 1: All input quantities will be considered.
- Case 2: The transverse sensitivity will be neglected.
- Case 3: The thermal correction of the gauge factor will not be carried out.
- Case 4: The apparent strain will be neglected.
- Case 5: The correction of the angular misalignment will not be carried out.
- Case 6: A combination of the cases 2 till 5.

After the evaluation according to the assumptions described by these cases, the deviation referenced to Case 1 will be calculated and graphically presented.

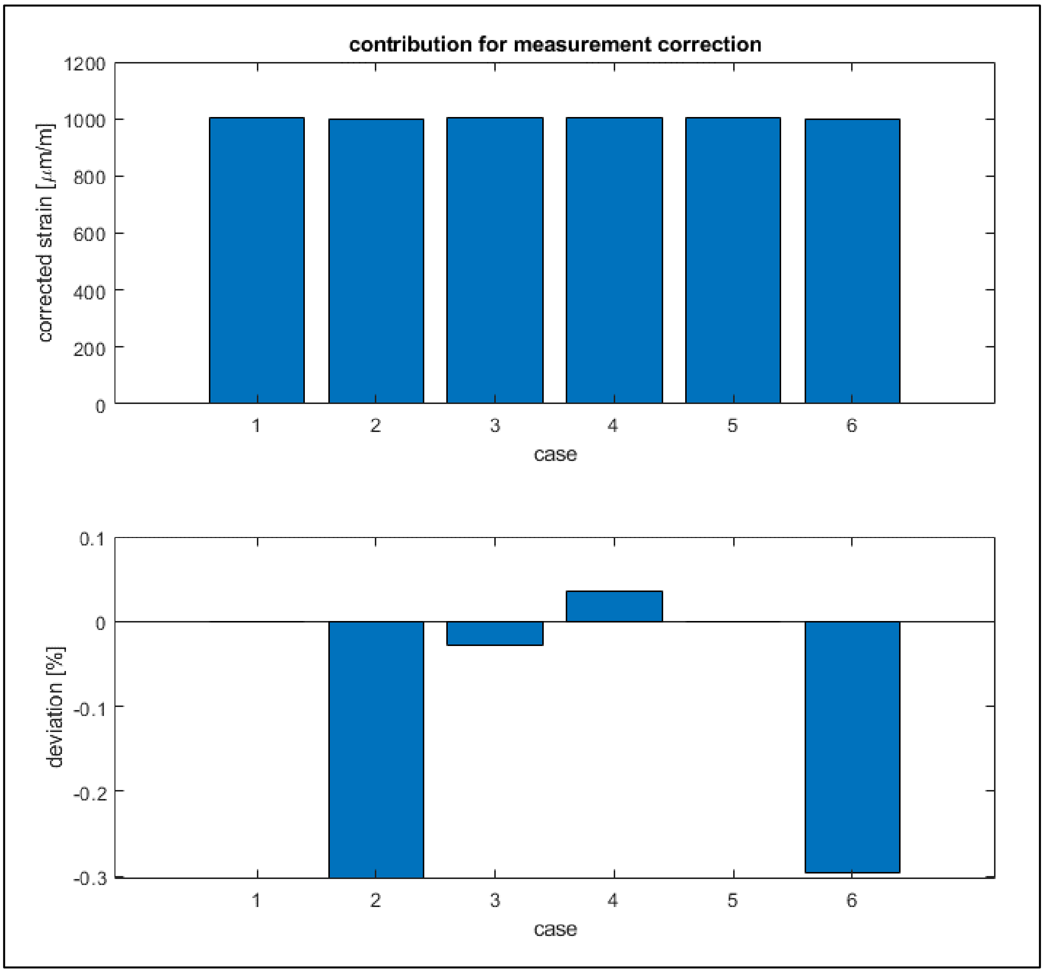

Figure 4.4-6: Results of sensitivity analysis for the corrected strain

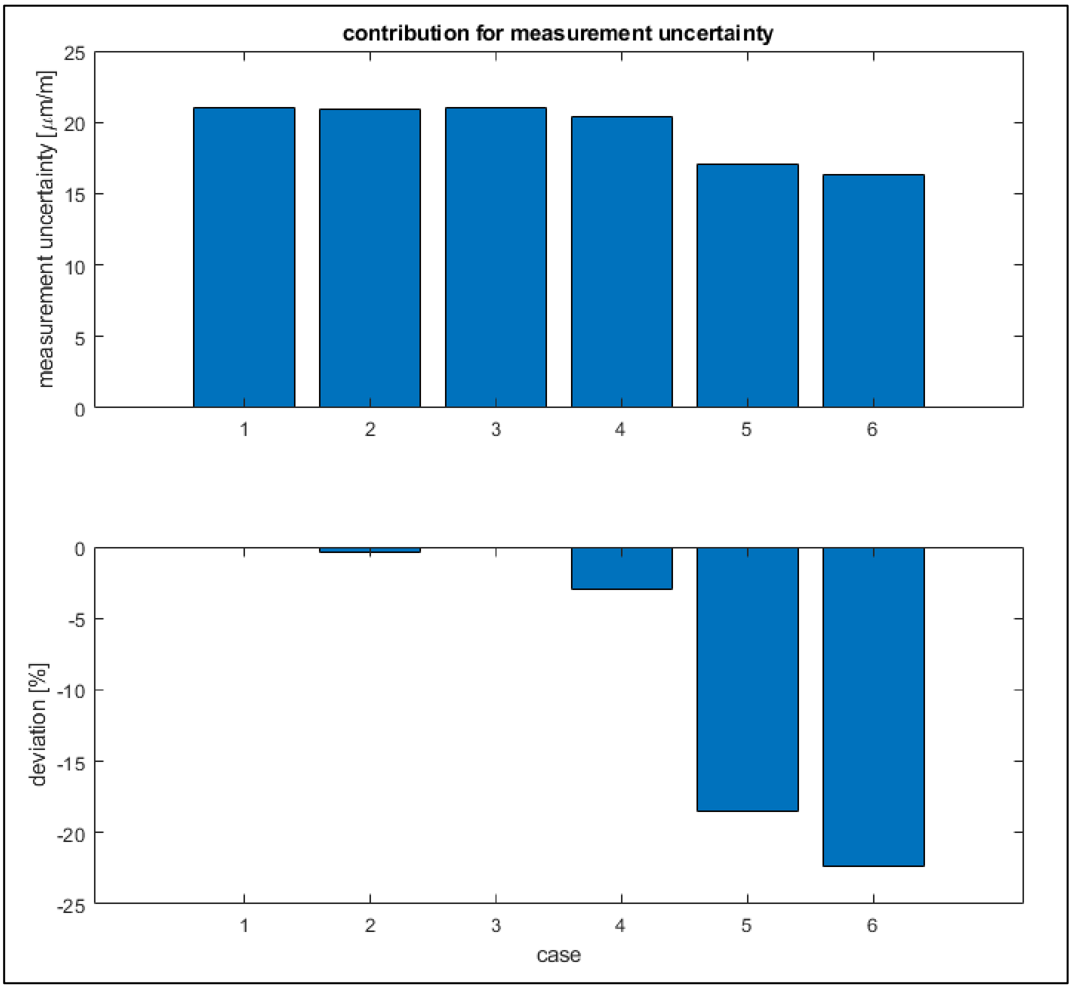

Figure 4.4-7: Results of sensitivity analysis for the measurement uncertainty

Figure 4.4-7 and Figure 4.4-6 show the result of such an investigation. Unfortunately, a general tendency could not be observed, as the relation between each input quantity and the output quantity is not exclusively linear, but transgressive to other input quantities.

5 Conclusion

In this thesis, an approach to evaluate the corrected strain using a mathematical model that considers multiple factors that might affect a strain measurement using a single one-directional grid strain gauge with a quarter Wheatstone bridge configuration were proposed, the considered factors are:

- The thermal dependency of the gauge factor
- The thermal behavior of the strain gauge
- The transverse sensitivity of the strain gauge
- The angular misalignment of the strain gauge

In addition, the author has derived a general approach to correct a strain measurement error due to angular misalignment. The validity of this approach was investigated using simulation of various measurement settings.

A general approach to evaluate the measurement uncertainty based on the mathematical model was also proposed, relying on the statistical methods introduced by the guide to express the measurement uncertainty. And a Matlab-tool based on this approach were presented, in addition to a practical calculating example.

A study case of a strain measurement on blade bearing of large wind turbines in the scope of a stress analysis were carried out. Including a numerical validation of the evaluation of the corrected strain and its measurement uncertainty according to the approaches proposed in this thesis. The result of this validation proved the correctness of the mathematical model to correct the strain and to evaluate its uncertainty with restrictions regarding the correction of the angular misalignment.

A sensitivity analysis showed no general tendencies of the contribution of individual input quantities to the corrected strain and its measurement uncertainty.

Appendix A Matlab tool

The tool is integrated in a Matlab-script that reads the databank and evaluate the strain measurement and its uncertainty; and it functions in three separated steps:

- Reading and filtering the databank and searching for the relevant data sets that fulfills given criteria.
- Evaluating the measurement and its uncertainty
- Reporting the result as a PDF-file; alternatively, exploring the results can be carried out using a figure-based browser.

The tool evaluates the corrected strain and its uncertainty according to 3.9.1 and 3.10. In addition, the error introduced using the sensitivity functions will be plotted. This estimation of the error is derived from Eq 2.2-7, and will be held according to:

$$\delta_x = f_M(x + \Delta x) - \left[f_M(x) + \frac{\partial f'(x)}{\partial x}(\Delta x) \right]$$

Where

f_M is the function of the quantity to be evaluated.
$\bar{x}$ is the mean value of the input quantity.
Δx is a finite change of the input quantity.
δ_x is the error introduced due to approximating the evaluation using the first-order Tylor expansion.

Tool input:
A vector of the input values, an example follows:

```
disp(Result.T_input)
                   Value         Unit
                 ________    _____________

    sig11              40    {'MPa'      }
    sig22             200    {'MPa'      }
    sig33               0    {'MPa'      }
    tau12              25    {'MPa'      }
    k_RK             2.04    {'-'        }
    u_k_RK         0.0204    {'-'        }
    T_ref              23    {'°C'       }
    alpha_k       9.3e-05    {'1/K'      }
    u_alpha_k       1e-05    {'1/K'      }
    T                  20    {'°C'       }
    u_T                 2    {'°C'       }
    R                 350    {'ohm'      }
    u_R              1.05    {'ohm'      }
    DeltaR          0.714    {'ohm'      }
    u_DeltaR       0.0238    {'ohm'      }
    c_s0           -10.23    {'mum/m'    }
    c_s1             1.56    {'mum/m/K'  }
    c_s2           -0.058    {'mum/m/K^2'}
    c_s3         0.000234    {'2.34e-4'  }
    u_eps_s_m           5    {'mum/m'    }
    u_acc          0.0002    {'-'        }
    u_rep          0.0001    {'-'        }
    u_lin          0.0002    {'-'        }
    phi           0.17453    {'radian'   }
    u_phi        0.087266    {'radian'   }
    nu                0.3    {'-'        }
    nu_0              0.3    {'-'        }
    q               0.001    {'-'        }
```

Tool output:

- A structure field that contains:
  - A vector of the input values.
  - A table of the corrected strain and its components
  - A table of the corrected strain uncertainty and its components
  - A table of the apparent strain uncertainty and its components
  - A table of the measured strain uncertainty and its components
  - A table of the corrected gauge factor and its components
- Graphs:
  - 8 approximation error graphs of every use of a sensitivity function
  - Visual presentation of the corrected strain uncertainty

An example of the output:

```
Result = struct with fields:
             T_input: [28×2 table]
    T_eps_evalutaion: [1×6 table]
            Tu_eps_c: [1×4 table]
          T_u_eps_sg: [1×4 table]
           T_u_eps_r: [1×6 table]
           T_u_eps_s: [1×4 table]
               T_u_k: [1×5 table]
                 T_c: [1×9 table]


disp(Result.T_eps_evalutaion)
      k       feps_c1    eps_r    eps_s     eps_sg    eps_c
    ______    _______    _____    ______    ______    ______

    2.0394    0.81109    10003    -0.358    10002     8112.9


disp(Result.Tu_eps_c)
    u_eps_c    u_eps_c^2     u_eps_c_sg^2    u_eps_c_phi^2
    _______    __________    ____________    _____________

    404.54     1.6365e+05       80337            83314


disp(Result.T_u_eps_r)
    u_eps_r    u_eps_r^2     u_m^2    u_eps_r_k^2    u_eps_r_R^2    u_eps_r_DeltaR^2
    _______    __________    _____    ___________    ___________    ________________

    349.42     1.2209e+05    9.005       10009          900.5          1.1117e+05


disp(Result.T_u_eps_s)
    u_eps_s    u_eps_s^2    u_eps_s_T^2    u_eps_s_m^2
    _______    _________    ___________    ___________

     5.091      25.919        0.91853          25


disp( Result.T_u_k)
      u_k          u_k^2         u_k_T^2      u_k_k_RK^2    u_k_alpha_k^2
    ________    __________    __________    __________    _____________

    0.020398    0.00041608    1.4397e-07    0.00041593     3.7454e-09
```

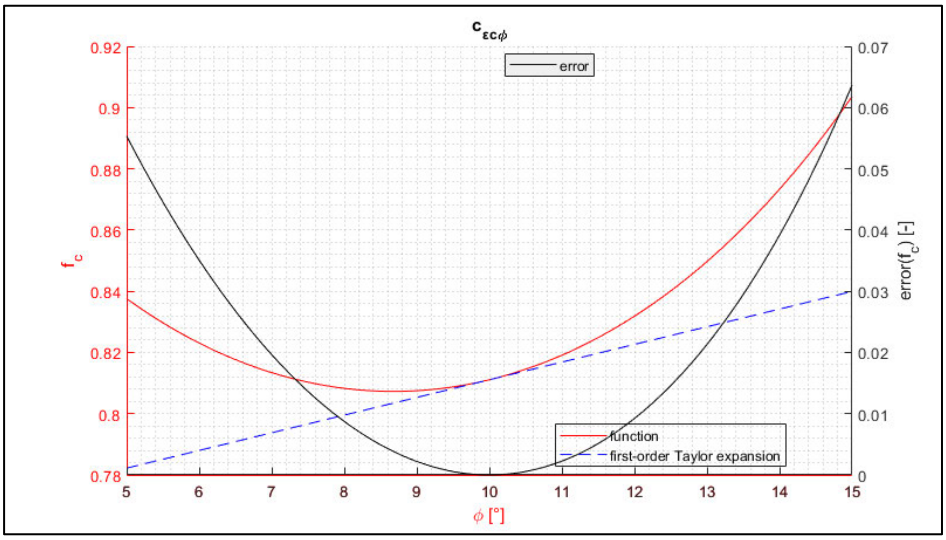

Figure A 1: Error of approximating the corrected strain in relation to the angular misalignment

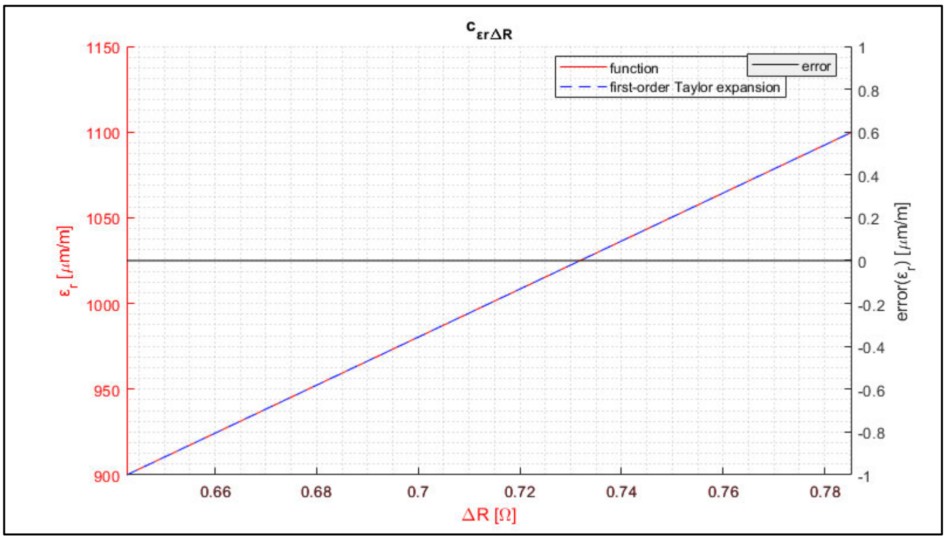

Figure A 2: Error of approximating the measured strain in relation to the change of resistance

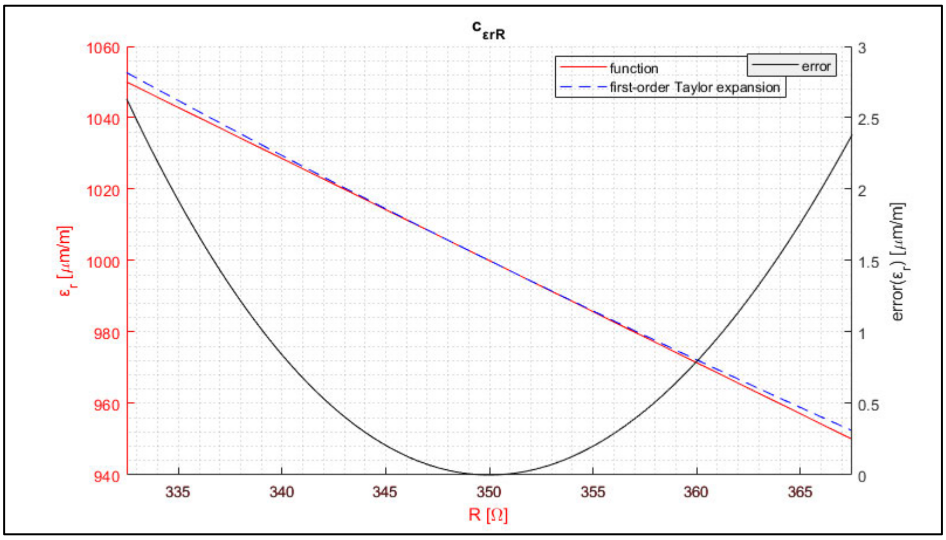

Figure A 3: Error of approximating the measured strain in relation to the resistance of the unstressed strain gauge

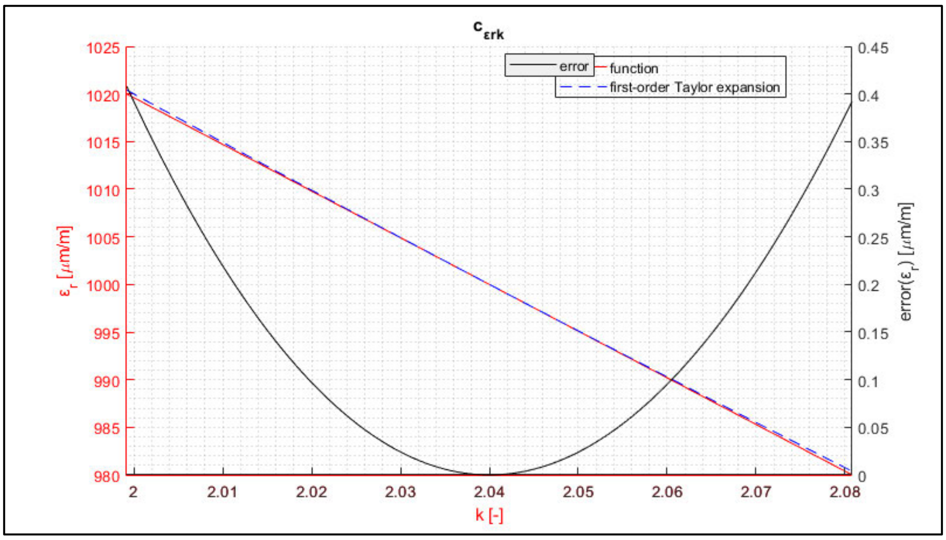

Figure A 4: Error of approximating the measured strain in relation to the gauge factor

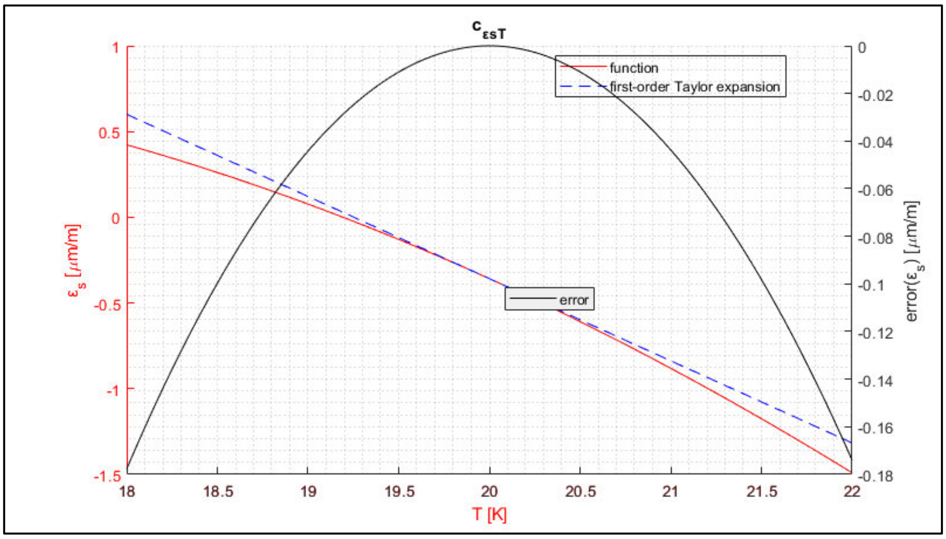

Figure A 5: Error of approximating the apparent strain in relation to temperature

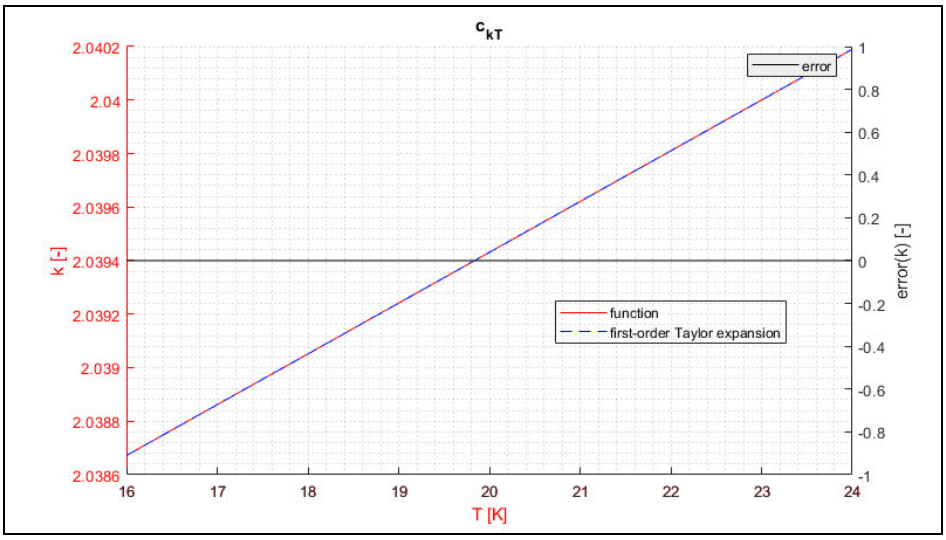

Figure A 6: Error of approximating the gauge factor in relation to the temperature

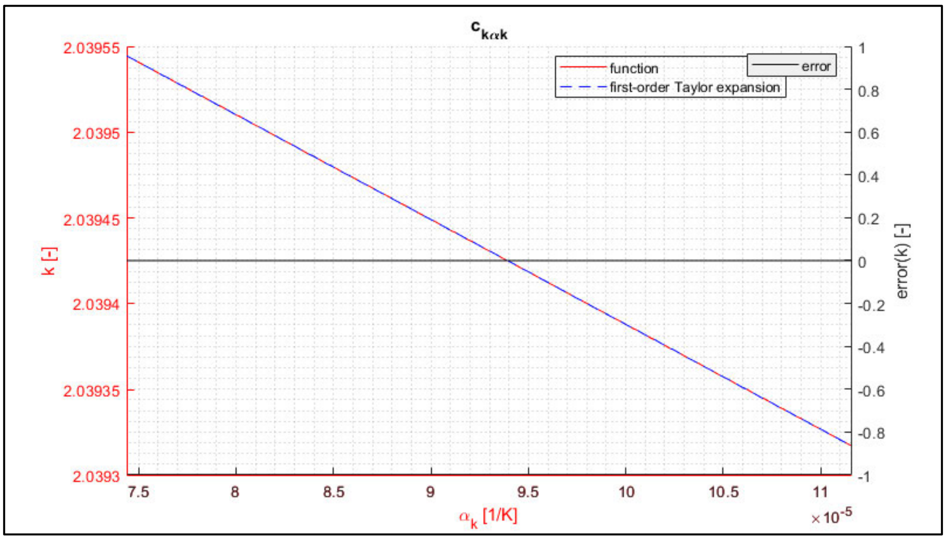

Figure A 7: Error of approximating the gauge factor in relation to the thermal coefficient of the gauge factor

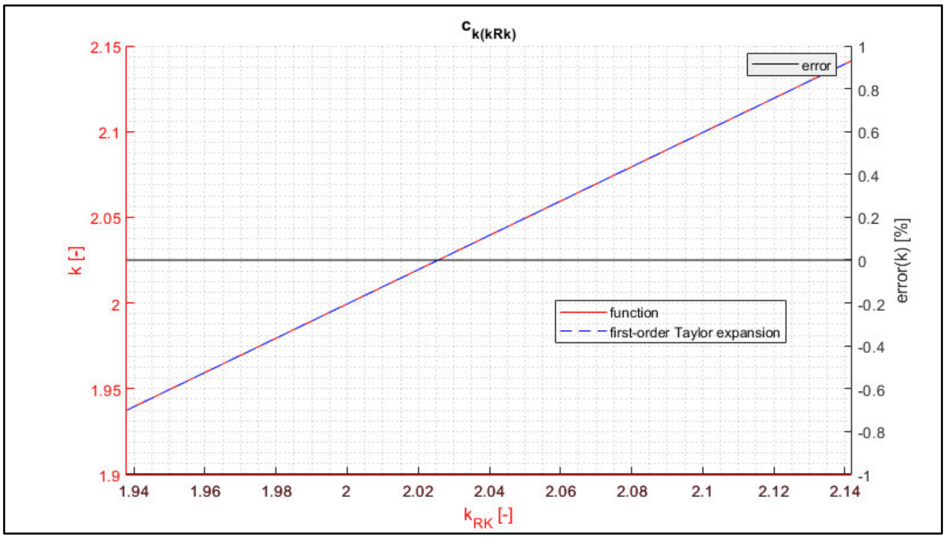

Figure A 8: Error of approximating the gauge factor in relation to the reference gauge factor

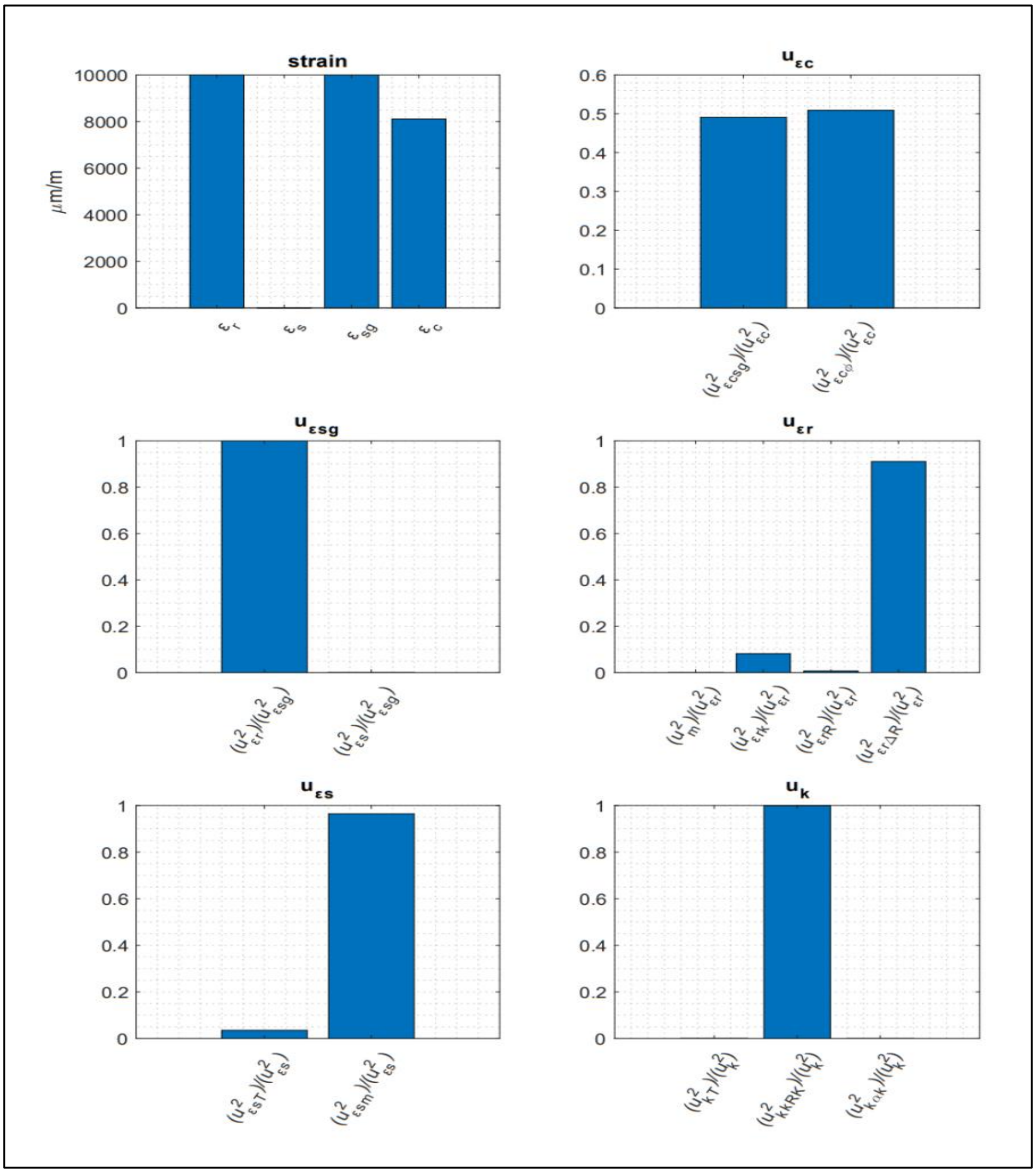

Figure A 9: Visual presentation of the component of the corrected strain uncertainty

The script that is used:

Script A 1

```
function Result = usg(INPUT,FIGURE_CONTROL)
%%  usg function evaluates strain measurement and its uncertainty
%   usg(INPUT,FIGURE_CONTROL) gives back the Result as structure (line 135)
%   figures will be ploted and saved if (FIGURE_CONTROL==1)
%   example: usg(Inputarry,1)
%
%   the input is a vertical verctor preferably with double values
%   the 28 variables and its appropriate units are given by lines from 16
%   till 43
%   the approach of evaluation is described in a bachelor thesis
```

```
%
%   Nabil Safieh, Fraunhofer IWES, Hamburg, Augusr 2020
```

```
%% preparation
```

```
sig11       = INPUT(1);% [MPa] normal stress in direction of measurement
sig22       = INPUT(2);% [MPa] normal stress perpendicular to direction of measurement; contained in measurement plane
sig33       = INPUT(3); % [MPa] normal stress perpendicular to direction of measurement; perpendicular to measurement plane
tau12       = INPUT(4); % [MPa] shear stress in measurement plane
k_RK        = INPUT(5); % [-] gauge factor @ T_ref.
u_k_RK      = INPUT(6); % [-] std of gauge factor @ T_ref.
T_ref       = INPUT(7); % [°C] the reference temperature.
alpha_k     = INPUT(8); % [1/K] coefficient of temperature-dependence of the gauge factor.
u_alpha_k   = INPUT(9); % [1/K] std of coefficient of temperature-dependence of the gauge factor.
T           = INPUT(10);% [°C] the temperature at the measurement.
u_T         = INPUT(11);% [°C] std of the temperature at the measurement.
R           = INPUT(12);% [ohm] the resistance of the unstressed strain gauge.
u_R         = INPUT(13);% [ohm] std of the resistance of the unstressed strain gauge.
DeltaR      = INPUT(14);% [ohm] the measured change in resistance under deformation.
u_DeltaR    = INPUT(15);% [ohm] std of the measured change in resistance under deformation.
c_s0        = INPUT(16);% [mum/m] coefficient 0 of apparent strain
c_s1        = INPUT(17);% [mum/m/K] coefficient 1 of apparent strain
c_s2        = INPUT(18);% [mum/m/K^2] coefficient 2 of apparent strain
c_s3        = INPUT(19);% [mum/m/K^3] coefficient 3 of apparent strain
u_eps_s_m   = INPUT(20);% [mum/m] std of apparent strain
u_acc       = INPUT(21);% [-] accuracy
u_rep       = INPUT(22);% [-] repeatability
u_lin       = INPUT(23);% [-] linearity
phi         = INPUT(24);% [°] angular misalignment
u_phi       = INPUT(25);% [°] std of angular misalignment
nu          = INPUT(26);% [-] poisson's ratio of test object
nu_0        = INPUT(27);% [-] poisson's ratio of strain gauge
q           = INPUT(28);% [-] transverse sensitivity of strain gauge

% stress tensor; the principal normal stresses and the shear stress in the measurement plane are needed, other shear stresses are irrelevant
sigma_1st = [sig11 tau12 0; tau12 sig22 0;0 0 sig33];

% ease of handling
C_spoly     = [c_s3 c_s2 c_s1 c_s0];
```

```
% constructing an input table to be exported with the result structure
```

```
T_input = array2table ([sig11,sig22,sig33,tau12,k_RK,u_k_RK,T_ref,...
    alpha_k,u_alpha_k,T,u_T,R,u_R,DeltaR,u_DeltaR,c_s0,c_s1,c_s2,...
    c_s3,u_eps_s_m,u_acc,u_rep,u_lin,phi,u_phi,nu,nu_0,q]' ...
    ,"RowNames",{'sig11','sig22','sig33','tau12','k_RK','u_k_RK',...
    'T_ref','alpha_k','u_alpha_k','T','u_T','R','u_R','DeltaR',...
    'u_DeltaR','c_s0','c_s1','c_s2','c_s3','u_eps_s_m','u_acc',...
    'u_rep','u_lin','phi','u_phi','nu','nu_0','q'},"VariableNames",{'Value'});
T_input.Unit(:) = ({'MPa','MPa','MPa','MPa','-','-','°C','1/K',...
    '1/K','°C','°C','ohm','ohm','ohm','ohm','mum/m','mum/m/K',...
    'mum/m/K^2','2.34e-4','mum/m','-','-','-','degree','degree','-','-','-'}');
```

```
%% execution commands
```

```
% evaluate the corrected strain
[k,eps_r,~,eps_sg,feps_c1,~,~,~,T_eps_evalutaion] = ...
    eps_evaluation (k_RK,T_ref,alpha_k,T,R,DeltaR,C_spoly,sigma_1st,phi,nu,nu_0,q);

% evaluate the sensitivity function of correcting the angular misalignment
[c_eps_c_phi,~,~,~] = DIFF_eps (sigma_1st,phi,nu,nu_0,q);

% evaluate the measurement uncertainty
[~,T_u_k,T_u_eps_s,T_u_eps_r,T_u_eps_sg,Tu_eps_c] = ...
    u_eps_evaluation(c_eps_c_phi,feps_c1,eps_r,k,k_RK,T_ref,alpha_k,T,R,...
    DeltaR,C_spoly,u_alpha_k,u_k_RK,u_T,u_eps_s_m,u_DeltaR,u_R,u_acc,u_lin,u_rep,u_phi,eps_sg);
```

```
%% figure generation

 if FIGURE_CONTROL == 1

     figure('Renderer', 'painters', 'Position', [10 10 700 1000])
     subplot(3,2,1)
     bar (T_eps_evalutaion{1,3:end})
     TICKS = {[char(949) '_r'],[char(949) '_s'],[char(949) '_s_g'],[char(949) '_c']};
     xticklabels(TICKS)
     title('strain')
     ylabel('\mum/m')
     xtickangle (60); grid minor

     subplot(3,2,2)
     bar(Tu_eps_c{1,3:end}/Tu_eps_c{1,2})
     TICKS = { [ '(u_' char(949) '_c_s_g^2)/(u_' char(949) '_c^2)' ] , ['(u_' char(949)
'_c_\phi^2)/(u_' char(949) '_c^2)' ] };
     xticklabels(TICKS)
     xtickangle (60)
     title(['u_' char(949) '_c']); grid minor
     subplot(3,2,3)
     bar(T_u_eps_sg{1,3:end}/T_u_eps_sg{1,2})
     TICKS ={ [ '(u_' char(949) '_r^2)/(u_' char(949) '_s_g^2)' ] , ['(u_' char(949) '_s^2)/(u_'
char(949) '_s_g^2)' ] };
     xticklabels(TICKS)
     title(['u_' char(949) '_s_g'])
     xtickangle (60); grid minor

   subplot(3,2,4)
     bar(T_u_eps_r{1,3:end}/T_u_eps_r{1,2})
     TICKS = { [ '(u_m^2)/(u_' char(949) '_r^2)' ] , ['(u_' char(949) '_r_k^2)/(u_' char(949) '_r^2)'
],['(u_' char(949) '_r_R^2)/(u_' char(949) '_r^2)' ] , ['(u_' char(949) '_r_\Delta_R^2)/(u_' char(949)
'_r^2)' ] };
     xticklabels(TICKS)
     title(['u_' char(949) '_r'])
     xtickangle (60); grid minor

     subplot(3,2,5)
     bar(T_u_eps_s{1,3:end}/T_u_eps_s{1,2})
     TICKS = { [ '(u_' char(949) '_s_T^2)/(u_' char(949) '_s^2)' ] , ['(u_' char(949) '_s_m^2)/(u_'
char(949) '_s^2)' ] };
     xticklabels(TICKS)
     title(['u_' char(949) '_s'])
     xtickangle (60)
     grid minor

     subplot(3,2,6)
     bar(T_u_k{1,3:end}/T_u_k{1,2})
     TICKS = {'(u_k_T^2)/(u_k^2)','(u_k_k_R_K^2)/(u_k^2)','(u_k_\alpha_k^2)/(u_k^2)'};
     xticklabels(TICKS)
     title('u_k')
     xtickangle (60)
     grid minor
     saveas(gcf,'RESULT','pdf')

 end
```

```
%% constructing the result structure

 Result.T_input = T_input;                      % input vector
 Result.T_eps_evalutaion = T_eps_evalutaion; % evaluation vector [mum/m]
 Result.Tu_eps_c = Tu_eps_c;                    % uncertainty vector [mum/m]
 Result.T_u_eps_sg = T_u_eps_sg;     % components of uncertainty of the measured strain after
compensating for the apparent strain
 Result.T_u_eps_r = T_u_eps_r;                  % components of uncertainty of the measured strain
 Result.T_u_eps_s = T_u_eps_s;                  % components of uncertainty of apperant strain
 Result.T_u_k = T_u_k;                          % components of uncertainty of gauge factor
```

```
%% uncertainty evaluation function
```

```
function [u_eps_c,T_u_k,T_u_eps_s,T_u_eps_r,T_u_eps_sg,Tu_eps_c] =...
        u_eps_evaluation (c_eps_c_phi,feps_c1,eps_r,k,k_RK,T_ref,alpha_k,...
        T,R,DeltaR,C_spoly,u_alpha_k,u_k_RK,u_T,u_eps_s_m,u_DeltaR,u_R,...
        u_acc,u_lin,u_rep,u_phi,eps_sg)

    c_k_alpha_k     = T*k_RK;                        % [K]sen. function (alpha_k)    EQ 3.10-4
    c_k_k_RK        = 1 + alpha_k*(T-T_ref);         % [-]sen. function (k_RK)       EQ 3.10-3
    c_k_T           = k_RK *alpha_k;                 % [K]sen. function (T)          EQ 3.10-2
    u_k_alpha_k     = c_k_alpha_k*u_alpha_k;
    u_k_k_RK        = c_k_k_RK*u_k_RK;
    u_k_T           = c_k_T*u_T;
    u_k             = sqrt (u_k_alpha_k^2 + u_k_k_RK^2 +  u_k_T^2); % [-]std k          EQ 3.10-1
    c_eps_s_t       = polyval(polyder(C_spoly),T);     % [mum/m]                     EQ 3.10-6
    u_eps_s_T       = c_eps_s_t*u_T;
    u_eps_s         = sqrt (u_eps_s_T^2 + u_eps_s_m^2);      % [mum/m]               EQ 3.10-5
    c_eps_r_DeltaR  = 1/R/k;
    c_eps_r_R       = DeltaR/(R^2)/k;
    c_eps_r_k       = DeltaR/R/(k^2);
    u_eps_r_DeltaR  = 10e6*c_eps_r_DeltaR * u_DeltaR;         % [mum/m]              EQ 3.10-8
    u_eps_r_R       = 10e6*c_eps_r_R * u_R;                   % [mum/m]              EQ 3.10-10
    u_eps_r_k       = 10e6*c_eps_r_k * u_k;                   % [mum/m]              EQ 3.10-9
    u_m             = eps_r*sqrt(u_acc^2 + u_lin^2 + u_rep^2);% [mum/m]              EQ 3.10-11
    u_eps_r         = sqrt (u_eps_r_DeltaR^2 + u_eps_r_R^2 + u_eps_r_k^2 + u_m^2); %  EQ 3.10-7
    u_eps_sg        = sqrt (u_eps_r^2 + u_eps_s^2);       % [mum/m]                  EQ 3.10-12
    u_eps_c_sg      = u_eps_sg * feps_c1;
    u_eps_c_phi     = u_phi * c_eps_c_phi  * eps_sg;      % [mum/m]                  EQ 3.10-16
    u_eps_c         = sqrt(u_eps_c_sg^2 + u_eps_c_phi^2); % [mum/m]                  EQ 3.10-14

    % constructing the result tables
    T_u_k = array2table([u_k,u_k^2,u_k_T^2,u_k_k_RK^2,u_k_alpha_k^2],...
        "VariableNames",{'u_k','u_k^2','u_k_T^2','u_k_k_RK^2','u_k_alpha_k^2'});
    T_u_eps_s = array2table([u_eps_s,u_eps_s^2,u_eps_s_T^2,u_eps_s_m^2],...
        "VariableNames",{'u_eps_s','u_eps_s^2','u_eps_s_T^2','u_eps_s_m^2'});
    T_u_eps_r       = array2table([u_eps_r,u_eps_r^2,u_m^2,u_eps_r_k^2,...
        u_eps_r_R^2,u_eps_r_DeltaR^2],"VariableNames",{'u_eps_r','u_eps_r^2',...
        'u_m^2','u_eps_r_k^2','u_eps_r_R^2','u_eps_r_DeltaR^2'});
    T_u_eps_sg      = array2table([u_eps_sg,u_eps_sg^2,u_eps_r^2,u_eps_s^2],...
        "VariableNames",{'u_eps_sg','u_eps_sg^2','u_eps_r^2','u_eps_s^2'});
    Tu_eps_c        = array2table ([u_eps_c,u_eps_c^2,u_eps_c_sg^2,u_eps_c_phi^2],...
        "VariableNames",{'u_eps_c','u_eps_c^2','u_eps_c_sg^2','u_eps_c_phi^2'});
end
```

```
%%  strain evaluation function
```

```
function [k,eps_r,eps_s,eps_sg,feps_c1,eps_c,epsR1E,epsR2E,T_eps_evalutaion] =...
        eps_evaluation (k_RK,T_ref,alpha_k,T,R,DeltaR,C_spoly,sigma_1st,phi,nu,nu_0,q)

    k       = k_RK * (1 + alpha_k*(T-T_ref) );          % [-] gauge factor             EQ 3.9-1
    eps_r   = 10e6*DeltaR/R/k;                          % [mum/m] measured strain      EQ 3.9-2
    eps_s   = polyval(C_spoly,T);                       % [mum/m] apparent strain      EQ 3.9-3
    eps_sg  = eps_r + eps_s;                            % [mum/m] mechanical strain    EQ 3.9-4
    [feps_c1,~,epsR1E,epsR2E] = eps_correction (sigma_1st,phi,nu,nu_0,q); % [-] correction factor
EQ 3.9-6
    eps_c   = eps_sg*feps_c1;                            % [mum/m] final strain         EQ 3.9-5

    T_eps_evalutaion = array2table([k,feps_c1,eps_r,eps_s,eps_sg,eps_c],...
        "VariableNames",{'k','feps_c1','eps_r','eps_s','eps_sg','eps_c'});
end
```

```
%% angular misalignment correction function
```

```
function [fc1,fc2,epsR1E,epsR2E] = eps_correction (sigma_1st,phi,nu,nu_0,q)

    ROT = [cos(phi) -sin(phi) 0 ; sin(phi) cos(phi) 0 ;0 0 1];          % rotation matrix about
    sigma_2nd = ROT*sigma_1st*ROT'; % second transformed stress tensor
EQ 3.6-8
    [eps1objE,eps2objE] = eps_sig (sigma_1st,nu); % strain tesnor components 11 & 22 of first
transformed tensor      EQ 3.6-14 & EQ 3.6-15
    [epsR1E,epsR2E] = eps_sig (sigma_2nd,nu);  % strain tesnor components 11 & 22 of second
transformed tensorEQ 3.6-17 & EQ 3.6-18
    epsSG1 = (epsR1E +q*epsR2E)/(1-nu_0*q);%strain gauge equivelant in direction 1  EQ 3.6-19
```

```
        epsSG2 = (epsR2E +q*epsR1E)/(1-nu_0*q);%strain gauge equivelant in direction 2  EQ 3.6-20
        fc1 = eps1objE/epsSG1; % correction ratio of strain in direction 1              EQ 3.6-29
        fc2 = eps2objE/epsSG2; % correction ratio of strain in direction 2              EQ 3.6-30
    end
```

```
%%  angular misalignment sensitivity function
```

```
    function [c_eps_c_phi,C31,C21,C1] = DIFF_eps (sigma_1st,phi,nu,nu_0,q)

        ROT = [cos(phi) -sin(phi) 0 ; sin(phi) cos(phi) 0 ;0 0 1];          % rotation matrix about
        sigma_2nd               = ROT*sigma_1st*ROT';                        % second transformed
stress tensor                                       EQ 3.6-8
        [eps1objE,~]            = eps_sig (sigma_1st,nu);                    % strain tesnor
components 11 & 22 of first transformed tensor      EQ 3.6-14 & EQ 3.6-15
        [epsR1E,epsR2E]         = eps_sig (sigma_2nd,nu);                    % strain tesnor
components 11 & 22 of second transformed tensor     EQ 3.6-17 & EQ 3.6-18
        C1 = 1-nu_0*q;                                                       % partial EQ 3.10-18
        C21 = (epsR1E + q*epsR2E)^-2;                                        % partial EQ 3.10-19
        % C22 = (epsR2E + q*epsR1E)^-2;                                      % in case of corrrecitng
two strains

        dif_eps1 = 2*sigma_1st(1,2)*sin(phi)^2 -2*sigma_1st(1,2)*cos(phi)^2 -nu*(2*sigma_1st(1,2) ...
*cos(phi)^2 - 2*sigma_1st(1,2)*sin(phi)^2 + 2*sigma_1st(1,1)*cos(phi)*sin(phi) -2*sigma_1st(2,2) ...
*cos(phi)*sin(phi)) - 2*sigma_1st(1,1)*cos(phi)*sin(phi) + 2*sigma_1st(2,2)*cos(phi)*sin(phi); % EQ
3.10-22

        dif_eps2 = 2*sigma_1st(1,2)*cos(phi)^2 -2*sigma_1st(1,2)*sin(phi)^2 +nu*(2*sigma_1st(1,2) ...
 *cos(phi)^2 - 2*sigma_1st(1,2)*sin(phi)^2 + 2*sigma_1st(1,1)*cos(phi)*sin(phi) - 2*sigma_1st(2,2) ...
 *cos(phi)*sin(phi))+ 2*sigma_1st(1,1)*cos(phi)*sin(phi) - 2*sigma_1st(2,2)*cos(phi)*sin(phi); % EQ
3.10-23
        C31 = dif_eps1 + q*dif_eps2;                                         % partial EQ 3.10-22 & 23
        % C32 = dif_eps2 + q*dif_eps1;                                       % in case of corrrecitng
two strains
        c_eps_c_phi = -eps1objE*C1*C21*C31;                                  % EQ 3.10-16
        % DIF_fc2_phi = -eps2objE*C1*C22*C32;                                % in case of corrrecitng
two strains
    end
```

```
%% strains of a stress tensor
```

```
    function [eps1,eps2] = eps_sig (SIGMA,nu)
        % strain tensor comp 11 & 22 of stress tensor SIGMA; nu is Poisson's ratio
        eps1 = SIGMA(1,1) -nu*(SIGMA(2,2)+SIGMA(3,3));  %strain in direction 1
        eps2 = SIGMA(2,2) -nu*(SIGMA(1,1)+SIGMA(3,3));  %strain in direction 2
    end
end
```

Script A 1

Appendix B Calculation (numerical) example

This is an example of evaluating the corrected strain and its combined uncertainty according to 3.9.1 and 3.10 . The used input values describe a realistic possible test setting; but are not data of an actual test. The input values are listed in the following table:

Value	*Distribution*	*Unit*	*Expectation value*	*Standard uncertainty*
ΔR	Normal (cat. A)	Ω	0.714	0.0238
R	Normal (cat. B)	Ω	350	1.05
k_{RK}	Normal (cat. B)	$-$	2.04	0.0204
α_k	Normal (cat. B)	$1/K$	$93 \cdot 10^{-6}$	$10 \cdot 10^{-6}$
T_R	Constant	$°C$	23	0
T	Normal (cat. A)	$°C$	20	$0{,}1°C$
ϕ	Normal (cat. B)	$-$	$0.0349 = 2°$	$0.0175 = 1°$
q	Constant	$-$	$0.1 \cdot 10^{-2}$	0
$\varepsilon_l/\varepsilon_q$	Constant	$-$	0.3	0
c_{s0}	Constant	$\mu m/m$	-10.23	0
c_{s1}	Constant	$\mu m/mK$	1.56	0
c_{s2}	Constant	$\mu m/mK$	$-5.8 \cdot 10^{-2}$	0
c_{s3}	Constant	$\mu m/mK^3$	$2.34 \cdot 10^{-4}$	0
$u_{\varepsilon s,m}$	Normal (cat. B)	$\mu m/m$	0	5
u_{acc}	Normal (cat. B)	$-$	0	$0.02/100$
u_{rep}	Normal (cat. B)	$-$	0	$0.01/100$
u_{lin}	Normal (cat. B)	$\mu m/m$	0	$0.02/100$

Table B 1: input values of the evaluation

The stress tensor at the measurement position:

$$\boldsymbol{\sigma} = \begin{bmatrix} 125 & 25 & 25 \\ 25 & 300 & 25 \\ 25 & 25 & 250 \end{bmatrix} MPa$$

Evaluation of corrected strain

The starting point is calculating the corrected gauge factor according to Eq 3.9-1:

$$k = k_{RK}(1 + \alpha_k(T - T_R)) = 2.04(1 + 93 \cdot 10^{-6}(20 - 23)) = 2{,}039$$

The strain measured by the strain gauge is then according to Eq 3.9-2:

$$\varepsilon_r = \frac{\Delta R}{Rk} = \frac{0.714}{350 \cdot 2.039} = 1 \cdot 10^3 \ \mu m/m$$

The apparent strain according to Eq 3.9-3 is:

$$\varepsilon_s = c_0 + c_1 T + c_2 T^2 + c_3 T^3$$

$$\varepsilon_s = -10.23 \ + 1.56 \cdot 20 + -5.8 \cdot 10^{-2} \cdot 20^2 \ + 2.34 \cdot 10^{-4} \cdot 20^3 = \ -358 \cdot 10^{-3} \ \mu m/m$$

The measured strain after compensating for the apparent strain is then according to Eq 3.9-4:

$$\varepsilon_{sg} = \varepsilon_s + \varepsilon_r = 1 \cdot 10^3 - 358 \cdot 10^{-3} \approx 1 \cdot 10^3 \, \mu m/m$$

The second transformed stress tensor is according to Eq 3.6-8:

$$\boldsymbol{\sigma}' = \boldsymbol{R\sigma R^T}$$

With the rotation matrix:

$$\boldsymbol{R}(\phi = 2°) = \begin{bmatrix} \cos 2° & -\sin 2° & 0 \\ \sin 2° & \cos 2° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.9994 & -0.0349 & 0 \\ -0.0349 & 0.9994 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\sigma}' = \begin{bmatrix} 123.5 & 18.84 & 24.11 \\ 18.84 & 301.5 & 25.86 \\ 24.11 & 25.86 & 250 \end{bmatrix} MPa$$

The product of the strain and the Young's modulus is according to Eq 3.9-7:

$$\varepsilon_{object} E = \sigma_1 - \nu(\sigma_2 + \sigma_3) = 125 - 0.3\,(300 + 250) = -40\, MPa$$

The strain component above is the strain intended to be measured and derived from the first transformation stress tensor. The relevant strain components of the second transformation are given by Eq 3.9-9 and Eq 3.9-10 or alternatively as follows:

$$\varepsilon_1{}'E = \sigma'_1 - \nu(\sigma'_2 + \sigma'_3) = 123.5 - 0.3(301.5 + 250) = -41.99\, MPa$$

$$\varepsilon_2' E = \sigma'_2 - \nu(\sigma'_1 + \sigma'_3) = 301.5 - 0.3(123.5 + 250) = 189.5\, MPa$$

The measured strain with considering the angular misalignment is given by Eq 3.9-8:

$$\varepsilon_{sg,app} E = \frac{\varepsilon'_1 E + q\, \varepsilon'_2 E}{1 - \nu_0 q} = \frac{-41.99\, MPa + 10^{-3} \cdot 189.5\, MPa}{1 - 0.3 \cdot 10^{-3}} = -41.81\, MPa$$

According to Eq 3.9-6, the correction function has the value:

$$f_{\varepsilon,corrected} = \frac{\varepsilon_{object}}{\varepsilon_{sg,app}} = \frac{\varepsilon_{object} E}{\varepsilon_{sg,app} E} = \frac{-40}{-41.81} = 0.9566$$

The corrected strain is then according to Eq 3.9-5:

$$\varepsilon_c = \varepsilon_{sg}\, f_{\varepsilon,corrected} = 1 \cdot 10^3 \cdot 0.9566 = 956.6\, \mu m/m$$

Evaluation of measurement uncertainty:

The measurement uncertainty of the reference gauge factor is according to Eq 3.10-3:

$$u_{k_{RK}} c_{k,k_{RK}} = u_{k_{RK}} \frac{\partial k}{\partial k_{RK}} = u_{k_{RK}} \big(1 + \alpha_k (T - T_R)\big) = \frac{2.4}{100} \big(1 + 93 \cdot 10^{-6} (20 - 23)\big)$$

$$u_{k_{RK}} c_{k,k_{RK}} = 204 \cdot 10^{-4}$$

The measurement uncertainty component of the coefficient of temperature-dependence of the gauge factor is according to Eq 3.10-4:

$$u_{\alpha_k} c_{k,\alpha_k} = u_{\alpha_k} \frac{\partial k}{\partial \alpha_k} = u_{\alpha_k} k_{RK} T = 10^{-6} \cdot 2.04 \cdot 20 = 4.08 \cdot 10^{-4}$$

The measurement uncertainty component of the temperature measurement according to Eq 3.10-2:

$$u_T c_{k,T} = u_T \frac{\partial k}{\partial T} = u_T k_{RK} \alpha_k = 1 \cdot 2.04 \cdot 93 \cdot 10^{-6} = 1.897 \cdot 10^{-4}$$

The standard uncertainty of the gauge factor is according to Eq 3.10-1:

$$u_k = \sqrt{u_T^2 c_{k,T}^2 + u_{k_{RK}}^2 c_{k,k_{RK}}^2 + u_{\alpha_k}^2 c_{k,\alpha_k}^2}$$

$$u_k = \sqrt{(204 \cdot 10^{-4})^2 + (4.08 \cdot 10^{-4})^2 + (1.897 \cdot 10^{-4})^2} \approx 204 \cdot 10^{-4}$$

The uncertainty component of the apparent strain with respect to the temperature is according to Eq 3.10-6:

$$u_T c_{\varepsilon_s,T} = u_T \frac{\partial \varepsilon_s}{\partial T} = 1 \left(1.56 + \frac{-5.8 \cdot 10^{-2}}{2} \cdot 20 + \frac{2.34 \cdot 10^{-4}}{3} 20^2\right) = -0.0479\ \mu m/m$$

The standard uncertainty of the apparent strain is according to Eq 3.10-5:

$$u_{\varepsilon_s} = \sqrt{u_T^2 c_{\varepsilon_s,T}^2 + u_{\varepsilon_{s;m}}^2} = \sqrt{(-0.0479)^2 + (5)^2} \approx 5\ \mu m/m$$

The uncertainty component of the measured strain relevant to resistance change is according to Eq 3.10-8:

$$u_{\Delta R} c_{\varepsilon_r,\Delta R} = u_{\Delta R} \frac{\partial \varepsilon_r}{\partial \Delta R} = \frac{u_{\Delta R}}{Rk} = \frac{0.0238}{350 \cdot 2.039} = 33.34\ \mu m/m$$

The uncertainty component of the measured strain relevant to resistance of the unstressed strain gauge is according to Eq 3.10-10:

$$u_R c_{\varepsilon_r,R} = u_R \frac{\partial \varepsilon_r}{\partial R} = -\frac{u_R\, \Delta R}{R^2 k} = -\frac{1.05 \cdot 0.714}{350^2 \cdot 2.039} = 3.001\ \mu m/m$$

The uncertainty component of the measured strain relevant to the gauge factor is according to Eq 3.10-9:

$$u_k c_{\varepsilon_r,k} = u_k \frac{\partial \varepsilon_r}{\partial k} = -\frac{u_k\, \Delta R}{k^2 R} = -\frac{204 \cdot 10^{-4} \cdot 0.714}{2.039^2 \cdot 350} = 10.01\ \mu m/m$$

the uncertainty of the measured strain related to operating modules is according to Eq 3.10-11:

$$u_m = \varepsilon_r \sqrt{u_{lin}^2 + u_{rep}^2 + u_{acc}^2} = 1 \cdot 10^3 \sqrt{\left(\frac{0.02}{100}\right)^2 + \left(\frac{0.01}{100}\right)^2 + \left(\frac{0.02}{100}\right)^2} = 0.3\ \mu m/m$$

The uncertainty of the measured strain is according to Eq 3.10-7:

$$u_{\varepsilon_r} = \sqrt{u_{\Delta R}^2 c_{\varepsilon_r,\Delta R}^2 + u_k^2 c_{\varepsilon_r,k}^2 + u_R^2 c_{\varepsilon_r,R}^2 + u_m^2} = \sqrt{33.34^2 + 10.01^2 + 3.001^2 + 0.3^2}$$

$$u_{\varepsilon_r} = 34.94\ \mu m/m$$

The uncertainty of the measured strain after compensating for the apparent strain is according to Eq 3.10-12:

$$u_{\varepsilon_{sg}} = \sqrt{u_{\varepsilon_r}^2 c_{\varepsilon_{sg},\varepsilon_r}^2 + u_{\varepsilon_s}^2 c_{\varepsilon_{sg},\varepsilon_s}^2} = \sqrt{349.4^2 \cdot 1^2 + 5^2 \cdot 1^2} = 34.95\ \mu m/m$$

To evaluate the uncertainty of the final corrected strain, the sensitivity function according to Eq 3.10-22 and Eq 3.10-23 must be evaluated:

$$E\frac{\partial \varepsilon'_1}{\partial \phi} = 2\cos\phi\ \sin\phi\,(\sigma_2 - \sigma_1) + 2\,\tau(\sin^2\phi - \cos^2\phi) - \nu\left(2\,\tau(\cos^2\phi - \sin^2\phi) + 2\cos\phi\ \sin\phi\,(\sigma_1 - \sigma_2)\right)$$

$$E\frac{\partial \varepsilon'_1}{\partial \phi} = 2\cos 2°\ \sin 2°\,(300 - 125) + 2\cdot 25\,(\sin^2 2° - \cos^2 2°) - 0.3\left(2\cdot 25(\cos^2 2° - \sin^2 2°) + 2\cos 2°\ \sin 2°\,(125 - 300)\right)$$

$$E\frac{\partial \varepsilon'_1}{\partial \phi} = -48.97\ MPa$$

$$E\frac{\partial \varepsilon'_2}{\partial \phi} = 2\cos\phi\ \sin\phi\,(\sigma_1 - \sigma_2) + 2\,\tau(\cos^2\phi - \sin^2\phi) + \nu\left(2\,\tau(\cos^2\phi - \sin^2\phi) + 2\cos\phi\ \sin\phi\,(\sigma_1 - \sigma_2)\right)$$

$$E\frac{\partial \varepsilon'_2}{\partial \phi} = 2\cos 2°\ \sin 2°\,(125 - 300) + 2\cdot 25(\cos^2 2° - \sin^2 2°) + 0.3\left(2\cdot 25(\cos^2 2° - \sin^2 2°) + 2\cos 2°\ \sin 2°\,(125 - 300)\right)$$

$$E\frac{\partial \varepsilon'_2}{\partial \phi} = 48.97\ MPa$$

The derivative of the combined sensitivity is then according to Eq 3.10-19:

$$E\frac{\partial(\varepsilon'_1 + q\,\varepsilon'_2)}{\partial\phi} = \frac{\partial\varepsilon'_1}{\partial\phi} + q\frac{\partial\,\varepsilon'_2}{\partial\phi}$$

$$E\frac{\partial(\varepsilon'_1 + q\,\varepsilon'_2)}{\partial\phi} = -48.97 + \frac{0.1}{100}48.97$$

$$E\frac{\partial(\varepsilon'_1 + q\,\varepsilon'_2)}{\partial\phi} = -48.92\;MPa$$

$$1/E\frac{\partial((\varepsilon'_1 + q\,\varepsilon'_2)^{-1})}{\partial\phi} = -\;\frac{\partial(\varepsilon'_1 + q\,\varepsilon'_2)}{\partial\phi}(\varepsilon'_1 + q\,\varepsilon'_2)^{-2}$$

$$1/E\frac{\partial((\varepsilon'_1 + q\,\varepsilon'_2)^{-1})}{\partial\phi} = -\frac{-48.92}{\left(-41.99 + \frac{0.1}{100}189.5\right)^2}$$

$$1/E\frac{\partial((\varepsilon'_1 + q\,\varepsilon'_2)^{-1})}{\partial\phi} = 0.0280\;(MPa)^{-1}$$

Then according to Eq 3.10-18

$$1/E\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial\phi} = (1 - \nu_0 q)\frac{\partial((\varepsilon'_1 + q\,\varepsilon'_2)^{-1})}{\partial\phi}$$

$$1/E\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial\phi} = \left(1 - 0.3\frac{0.1}{100}\right)0.0280$$

$$1/E\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial\phi} = 0.0280(MPa)^{-1}$$

And the derivative of the correction function is given by Eq 3.10-17:

$$\frac{\partial\,f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial\phi} = \varepsilon_{object}\frac{\partial\left(\varepsilon_{sg,app}^{-1}\right)}{\partial\phi}$$

$$\frac{\partial\,f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial\phi} = -40\cdot 0.0280$$

$$\frac{\partial\,f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial\phi} = -1.1196$$

And the sensitivity related to the correction function is given by Eq 3.10-16:

$$c_{\varepsilon_c,\phi} = \frac{\partial\varepsilon_c}{\partial\phi} = \varepsilon_{sg}\frac{\partial\,f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)}{\partial\phi}$$

$$c_{\varepsilon_c,\phi} = 1.0002 \cdot 10^3(-1.1196\ ) = -11199\ \mu m/m$$

The uncertainty component of the angular misalignment is according to Eq 3.10-16

$$u_\phi c_{\varepsilon_c,\phi} = \frac{1°\ \pi}{180°}(-1119.9) = \ -19.54\ \ \mu m/m$$

The uncertainty component of the corrected that is related to the strain gauge measurement is according to Eq 3.10-15:

$$u_{\varepsilon_{sg}}\ c_{\varepsilon_c,\varepsilon_{sg}} = \ u_{\varepsilon_{sg}}\ f_{\varepsilon,corrected}\left(c_{\sigma_2}, c_{\sigma_3}, c_\tau, \nu, \phi, q, \nu_0\right)$$

$$u_{\varepsilon_{sg}}\ c_{\varepsilon_c,\varepsilon_{sg}} = \ 34.95 \cdot\ 0.9566 = 33.77\ \ \mu m/m$$

And the final measurement uncertainty of the corrected strain is according to Eq 3.10-14

$$u_{\varepsilon_c} = \sqrt{u_{\varepsilon_{sg}}^2 c_{\varepsilon_c,\varepsilon_{sg}}^2 + u_\phi^2 c_{\varepsilon_c,\phi}^2}$$

$$u_{\varepsilon_c} = \sqrt{(33.77\ \ )^2 + (-19.54)^2} = 39.01\ \mu m/m$$

The coefficient of variance of the measurement is then:

$$COV_{\varepsilon_c} = \frac{u_{\varepsilon_c}}{\varepsilon_c} = \frac{39.01}{956.6} = 0.0409 = 4.09\%$$

The final representation of the evaluation is:

$$\varepsilon = (956.6 \pm\ 39.01)\frac{\mu m}{m}\ ; k_p = \ 1$$

Where k_p represents the coverage factor. With a value of $k_p = \ 1$, the coverage factor corresponds to a level of confidence of $p = 68.27\ \%$

Appendix C Validation of the correction related to the angular misalignment

In this appendix, a way to validate the mathematical approach of correcting the measurement error induced by the angular misalignment and the implementation of this correction in a tool will be introduced.

This validation is based on comparing the analytically transformed strain (using the Matlab-tool) with a strain transformation using an FE-model.

Information about the simulation (Figure C 1):

Simulation type:	static structural analysis
Geometry:	cube
Material model:	steel with ideal elastic behavior
Mesh:	automatic (program controlled with a pre-defined max-face size)
Element type:	3D solid
Boundary condition:	total displacement restriction applied to one face
Load:	force applied to one face (one-step ramped, arbitrary orientation)

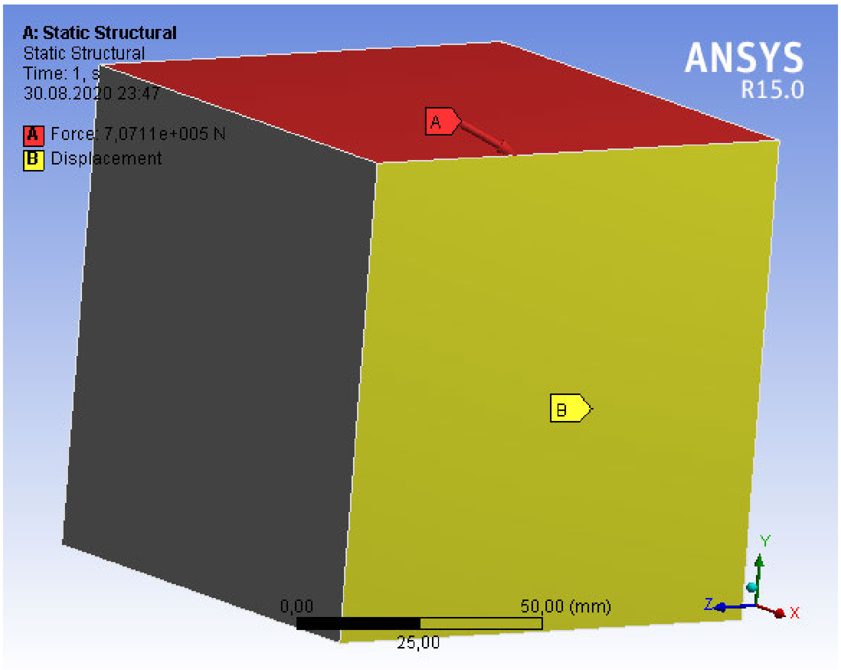

Figure C 1: The FE-Model of the validation of the strain transformation function

After performing the simulations, the strain is evaluated at the same positions in two different orientation that are shown in Figure C 2:

- The first orientation in the direction of the strain that is intended to be measured.
- The second orientation considers the angular misalignment.

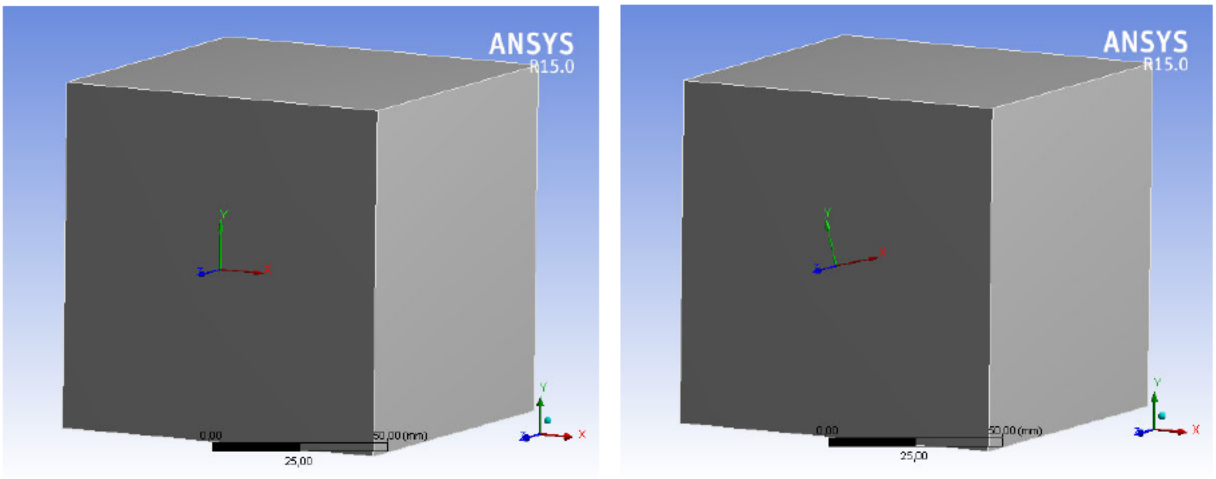

Figure C 2: Tow different orientations of the evaluated strain at the same position.

For the evaluation of the strains, two coordinate systems were defined in the same position with two different orientations. The second coordinate system is obtained by rotating the first coordinate systems around the axis perpendicular to the cube-face with a rotation angle equal to the angular misalignment.

The results of this validation show a mean relative deviation defined as:

$$dev = 100\frac{\varepsilon_{an:rot} - \varepsilon_{sim:rot}}{\varepsilon_{an;rot}}$$

of $0.5\ \%$ and a maximum of $3.5\ \%$. The author assumes that this deviation is a result of the accuracy of the numerical solution of the simulation due to discretization and averaging errors, amongst other factors.

Normed_stress = 10x6 table

	sigma 11	sigma 22	sigma 33	tau 12	tau23	tau13
	[MPa/MPa]	[MPa/MPa]	[MPa/MPa]	[MPa/MPa]	[MPa/MPa]	[MPa/MPa]
1	1	0.4000	0	0	0	0
2	1	7.8379	0.0110	-0.6372	0	-0.0764
3	1	0.7524	-0.0010	0.0382	0	0.3614
4	1	16.3092	0.2288	40.4238	0.3321	0
5	1	16.3092	0.2288	40.4238	0.3321	0
6	1	16.3092	0.2288	40.4238	0.3321	0
7	1	16.3092	0.2288	40.4238	0.3321	0
8	1	16.3092	0.2288	40.4238	0.3321	0
9	1	1.0154	2.2981	0	1.1403	-1.9399
10	1	1.0154	2.2981	0	1.1403	-1.9399

Table C 1: Normed stress tensor of ten deferent simulations

Phi = 10x2 table

	Phi_degree	Phi_radian
	[°]	[-]
1	5	0.0873
2	-10	-0.1745
3	5	0.0873
4	-5	-0.0873
5	-10	-0.1745
6	-15	-0.2618
7	-20	-0.3491
8	-25	-0.4363
9	5	0.0873
10	10	0.1745

Table C 2: The rotation of the second stress tensor for each simulation

Normed_strain = 10x4 table

	epsilon 11 cs1	epsilon 11 cs2	epsilon 22 cs1	epsilon 22 cs2
	[mum/m / mum/m]	[mum/m / mum/m]	[mum/m / mum/m]	[mum/m / mum/m]
1	1	1.0275	0.1442	0.1168
2	1	0.9896	-5.4156	-5.4046
3	1	0.9921	0.5719	0.5798
4	1	-0.7528	1.2627	3.0205
5	1	-0.2731	-0.1810	1.0957
6	1	-0.1686	-0.4953	0.6766
7	1	-0.1240	-0.6295	0.4976
8	1	-0.1000	-0.7017	0.4013
9	1	0.9581	4.1186	4.1604
10	1	0.8796	3.6993	3.8195

Table C 3: The evaluated strain components for each simulation

Correction_evaluation = 10x4 table

	analytical_1	simulation_1	deviation_1	analytical_2	simulation_2	deviation_2
	[-]	[-]	[%]	[-]	[-]	[%]
1	1.0275	1.0275	2.0543	08096	08096	-0.0001
2	0.9888	0.9896	0.0799	0.9980	0.9980	-0.0010
3	1.0145	0.9921	-2.2104	0.9761	1.0138	3.8618
4	-0.7453	-0.7528	1.0074	2.3920	2.3920	-0.0004
5	-0.2711	-0.2731	0.7274	-6.0539	-6.0536	-0.0062
6	-0.1675	-0.1686	0.6593	-1.3662	-1.3661	-0.0048
7	-0.1232	-0.1240	0.6329	-0.7904	-0.7904	-0.0040
8	-0.0994	-0.1000	0.6126	-0.5720	-0.5719	-0.0036
9	0.9752	0.9581	-1.7455	1.0059	1.0101	0.4228
10	0.9082	0.8796	-3.1400	1.0238	1.0325	0.8522

Table C 4: The analytical and simulated results of the strain transformation

Appendix D Simulation of bearing sector under load

The simulation considers a signal axial force (referenced to the bearing's rotation axis) that is applied to the bearing. As a rotational symmetry is given in this case, only a one hundredth of a complete revolution sector of the outer ring of the bearing is simulated. The used geometry is given in Figure 4.3-2.

Information about the simulation (**Error! Reference source not found.**):

Simulation type:	static structural analysis
Geometry:	3.6° sector of the bearing's outer ring
Material model:	steel with ideal elastic behavior
Mesh:	automatic (program controlled with a pre-defined max-face size), shown in **Error! Reference source not found.**.
Element type:	3D solid
Boundary condition:	fixed support applied to the railway **Error! Reference source not found.**
	displacement restriction to the sectorial cross-sections **Error! Reference source not found.**.
Load:	bolt-connection's force applied to two surfaces **Error! Reference source not found.**
	axial force (referenced to the bearing's rotation axis) applied to one surface.

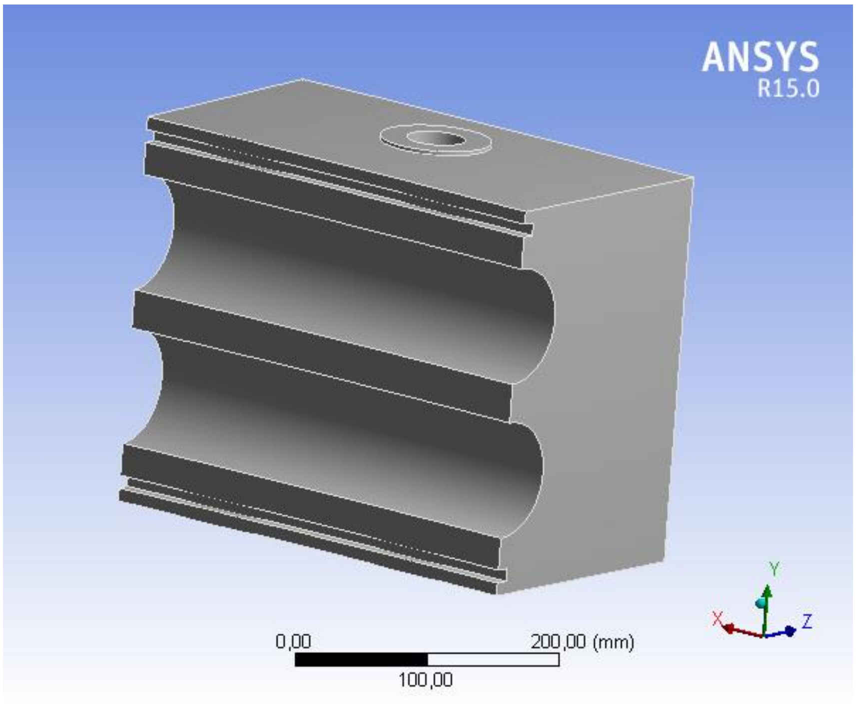

Figure D 1: The simulated sector of the outer ring of the bearing.

The geometry was generated through revolving a cross-sectional sketch of the outer ring with an angle of 3.6°, the bolt-hole were then extruded and subtracted using a Boolean operation, and a hollow cylinder were then generated to represent the washer at one end of the bolt-hole.

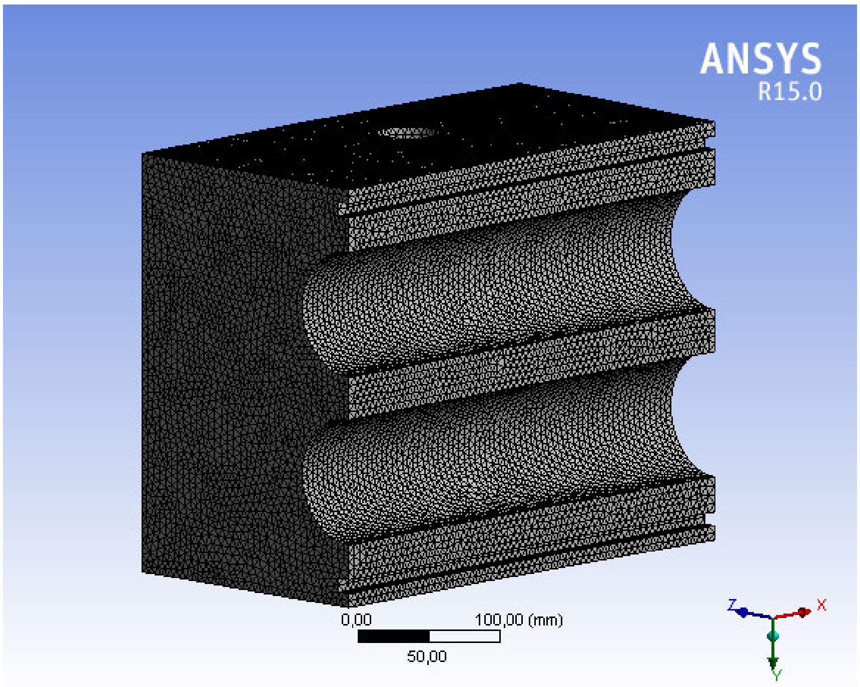

Figure D 2: The mesh of the soled object.

The mesh was generated automatically by the program. A max-face size was defined in order to minimize the averaging error of the probed stress tensor.

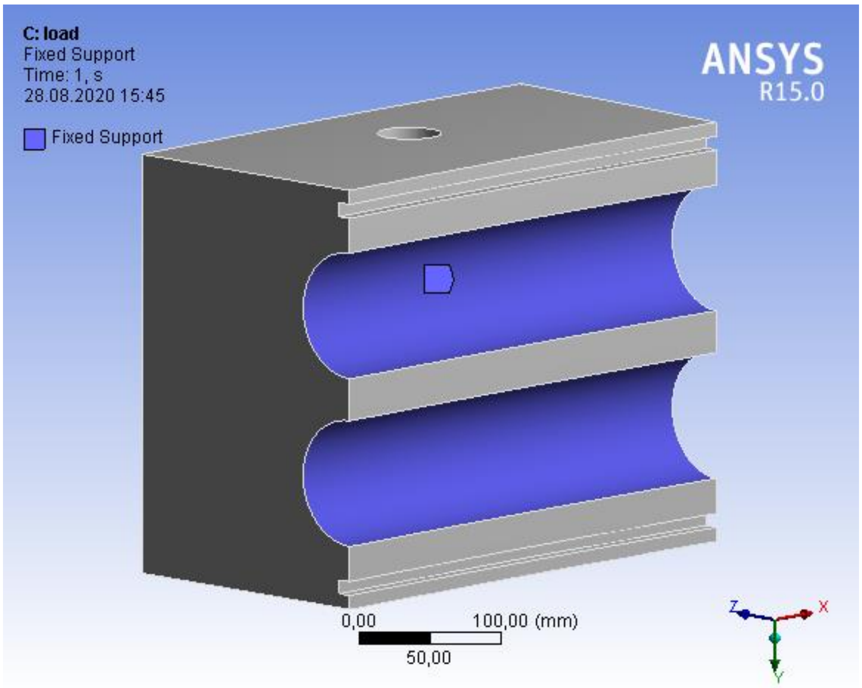

Figure D 3: Boundary condition 1, sixed support at the railway.

To contact between the rolling-balls and the railway was not simulated; instead, a fixed support boundary condition was set to the railway, for the sake of simplicity, assuming that this simplification will not affect the object of this simulation, which is to probe the stress tensor allocated on the outer surface of the outer ring (the flat side opposite to the railway).

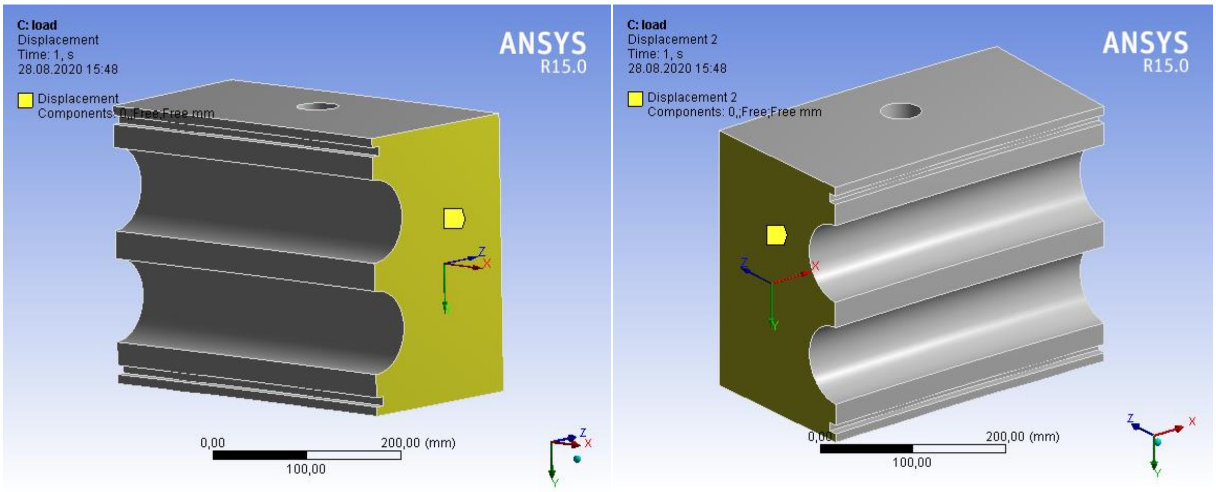

Figure D 4: Boundary condition: the sectorial cross-sections are restricted to displace tangentially referenced to the bearing's rotation-axis.

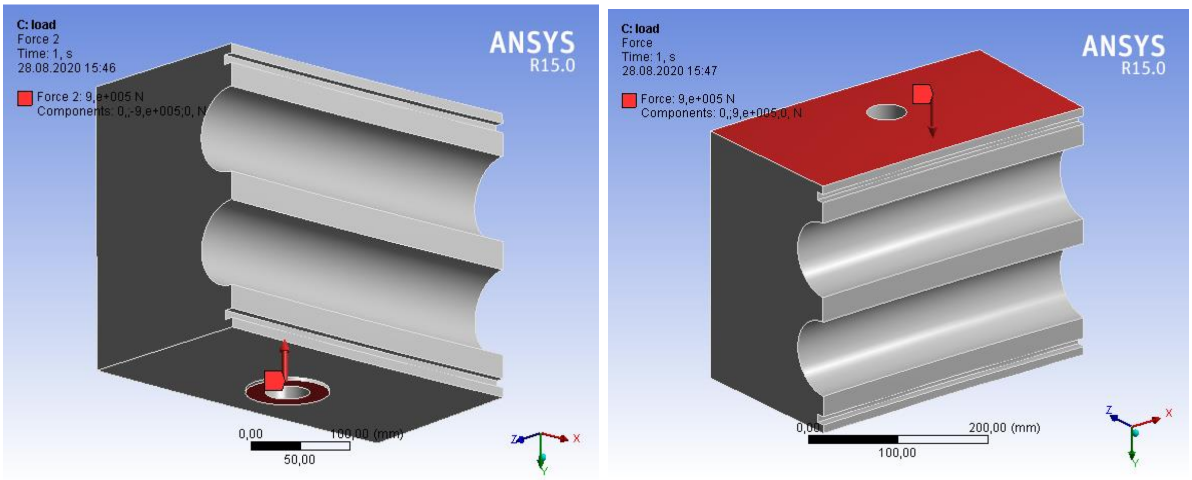

Figure D 5: Load application of the blot-connection's force. Left: the lower face is in contact with the washer face. Right: The upper side is in contact with the force transition element.

The load application was carried out in one single ramp. As a result, the nonlinear stress-load or stiffness relation will not be detectable. However, the purpose of this simulation is to show the general behavior of the stress tensor and not to evaluate exact stress tensor components that correspond to certain load-cases.

The simulation results showed a generality of a biaxial strain field with both of its normal stress components in the plane tangential to the bearing in the measurement position. Plausibly, the stress normal to the measurement planes is always nonexistent, if no load normal to the measurement plane at the position of measurement is applied. Which is the general case, unless the ambient pressure surrounding the measurement's position is high enough to cause a relevant force normal to the measurement plane. The shear stress in the strain measurement plane was always near 0, that was also expected, as the load consists of only one axial force perpendicular to the measured strain.

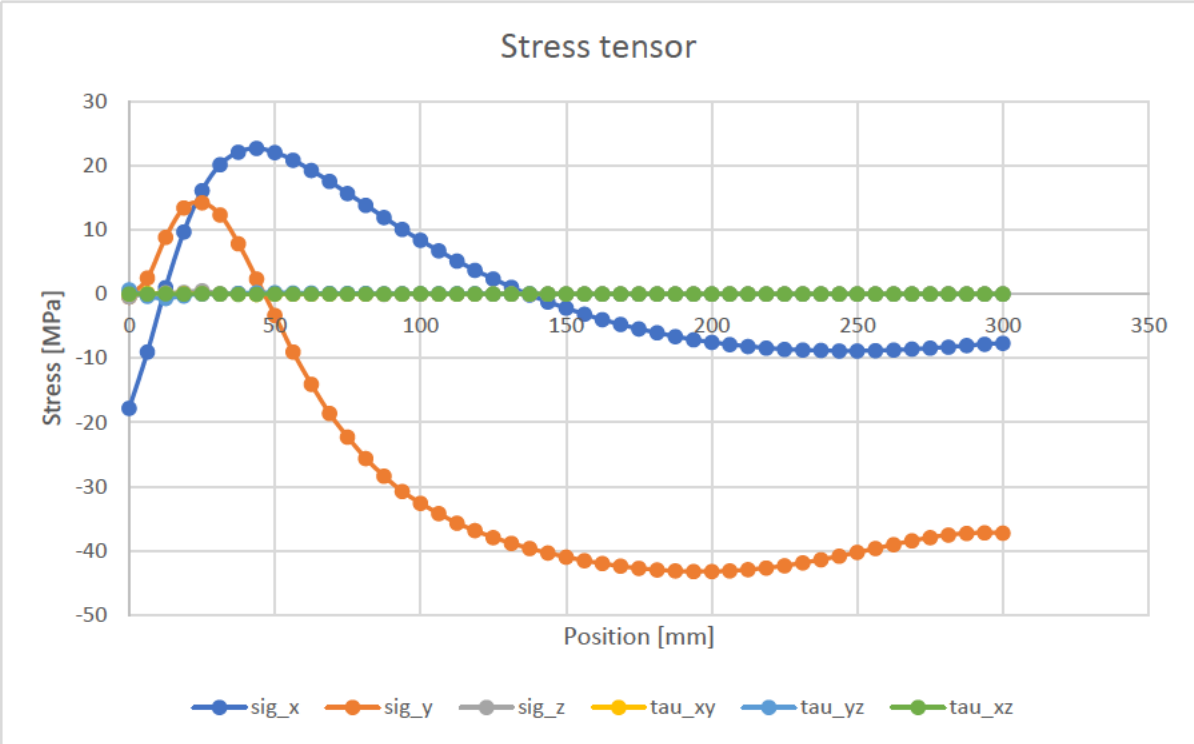

Figure D 6: Simulation results: stress components on the outer surface of the outer ring

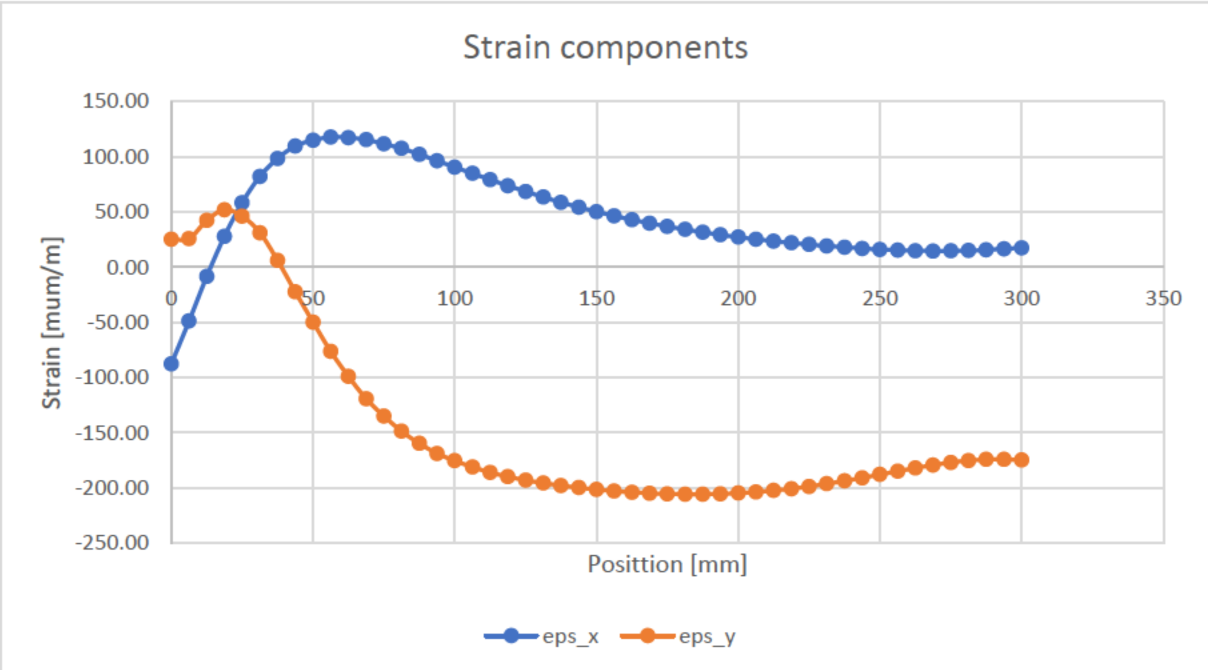

Figure D 7: Simulation results: strain components on the outer surface of the outer ring

Appendix E Numerical validation of the evaluation of measurement uncertainty

This appendix will introduce a numerical validation of the evaluation of the measurement uncertainty based on the mathematical approach given in 3.9. To do so, the analytical evaluation of the measurement uncertainty introduced in 3.10 will be compared with a numerical evaluation of the measurement uncertainty.

The idea behind this validation is to simulate a normal distribution that describes the measurement uncertainty based on the same input values that are used in the analytical evaluation. The numerical evaluation starts with generating sets of the input quantities using the mean values and the standard deviations of the input quantities. The obtained sets will then contain normal distributed random values as possible quantifications of the input quantities:

$$X_i = normrnd(\tilde{\mu}, \tilde{\sigma}, n)$$

$$X_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,n}\}$$

Where

X_i	the set of numbers that describes the quantity i.
$\tilde{\mu}$	the initial mean value of the quantity i.
$\tilde{\sigma}$	the initial standard deviation of the quantity i.
$x_{i,1}, x_{i,2}, \dots, x_{i,n}$	the numbers contained in the set X_i.
n	the number of elements contained in the set X_i.
$normrnd$	the generator of normal distributed random numbers.

Because the mean values and the standard deviations of the generated sets might differ minimally from the initial given values due to the limited accuracy of the process used while generating these sets; it is necessary to re-evaluate the actual mean values and standard deviations after generation:

$$\mu_{x_i} = \frac{1}{n}\sum_{j=1}^{n} x_{i,j}$$

Were μ_{x_i} represents the actual mean value of the generated set.

$$\sigma_{x_i} = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_{i,j} - \mu_{x_i}\right)^2}$$

Were σ_{x_i} represents the actual standard deviation of the generated set.

For the validation of the strain measurement uncertainty the following sets are generated:

$$X_{\Delta R} = normrnd(\widetilde{\Delta R}, \widetilde{u_{\Delta R}}, n)$$

$$X_R = normrnd(\tilde{R}, \widetilde{u_R}, n)$$

$$X_{k_{RK}} = normrnd(\widetilde{k_{RK}}, \widetilde{u_{k_{RK}}}, n)$$

$$X_{\alpha_k} = normrnd(\widetilde{\alpha_k}, \widetilde{u_{\alpha_k}}, n)$$

$$X_T = normrnd(\tilde{T}, \widetilde{u_T}, n)$$

$$X_\phi = normrnd(\tilde{\phi}, \widetilde{u_\phi}, n)$$

Based on the numerically generated sets, input sets to evaluate the corrected strain according to the mathematical approach will be constructed as follows:

$$\varepsilon_{numrical,m} = f_\varepsilon\left(x_{\Delta R,m}, x_{R,m}, x_{k_{RK},m}, x_{\alpha_k,m}, x_{T,m} x_{\phi,m}\right)$$

Where

$$m \in (1, n) \cap m \in N$$

A new set of evaluated strains should then be constructed:

$$X_{\varepsilon_{numrical}} = \{\varepsilon_{numrical,1}, \varepsilon_{numrical,2}, \dots, \varepsilon_{numrical,n}\}$$

The final step is then to evaluate the mean value and the standard deviation of the constructed sets of the corrected strains:

$$\varepsilon_{numrical} = \frac{1}{n}\sum_{j=1}^{n} \varepsilon_{numrical,j}$$

$$u_{\varepsilon_{numrical}} = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(\varepsilon_{numrical,j} - \varepsilon_{numrical}\right)^2}$$

$$COV_{numrical} = \frac{u_{\varepsilon_{numrical}}}{\varepsilon_{numrical}}$$

The mean values and the standard deviations of the numerically generated sets that describes the input quantities are then used to evaluation the strain and its uncertainty analytically according to 3.9 and 3.10:

$$\varepsilon_{analytical} = f_{\varepsilon_{analytical}}\left(\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_i}\right)$$

$$u_{\varepsilon_{analytical}} = f_{u_{\varepsilon_{analytical}}}\left(\mu_{x_1}, \sigma_{x_1}, \mu_{x_2}, \sigma_{x_2}, \dots, \mu_{x_i}, \sigma_{x_i}\right)$$

$$COV_{analytical} = \frac{u_{\varepsilon_{analytical}}}{\varepsilon_{analytical}}$$

To quantify the deviation of the numerical evaluation from the analytical evaluation, the following quantities are calculated:

$$DIV_{\varepsilon} = 100 \frac{\varepsilon_{analytical} - \varepsilon_{numrical}}{\varepsilon_{analytical}} \%$$

$$DIV_{u} = 100 \frac{u_{\varepsilon_{analytical}} - u_{\varepsilon_{numrical}}}{u_{\varepsilon_{analytical}}} \%$$

$$DIV_{COV} = 100 \frac{COV_{analytical} - COV_{numrical}}{COV_{analytical}} \%$$

Notes:

The number of the elements of the numerical evaluation of the strain is naturally the same of the number of elements of all sets of input quantities and should be adequately high to assure a realistic distribution of the output quantity. The Author chose 10^7 elements after iteratively validating the approach using several numbers of elements.

Script E 1: Example of numerical validation

```
% defining a prencipale stress tensor; this will ease the prosses of generating the second transformation stress tensor

  SIGMA11      = 20;
  SIGMA22      = 10;
  SIGMA3       = 0;
  Gamma        = 20;
  gamma        = gamma*pi/180;
  SIGMA        = [SIGMA11 0 0; 0 SIGMA22 0; 0 0 SIGMA3]
```

```
SIGMA = 3×3
    20     0     0
     0    10     0
     0     0     0
```

```
% defining a rotation matrix

  ROTATION = [cos(gamma) -sin(gamma) 0; sin(gamma) cos(gamma) 0; 0 0 1]
```

```
ROTATION = 3×3
    0.9397   -0.3420         0
    0.3420    0.9397         0
         0         0    1.0000
```

```
% performing the first transformation
  SR = ROTATION*SIGMA*ROTATION'
```

```
SR = 3×3
   18.8302    3.2139         0
    3.2139   11.1698         0
         0         0         0
```

```
% defining the variabls of the first transformation tensor
```

```
sig11 = SR(1,1);
sig22 = SR(2,2);
sig33 = SR(3,3);
tau12 = SR(1,2);
```

```
% defining the input values of the evaluation
```

```
k_RK        = 2.04;          % [-] gauge factor @ T_ref.
u_k_RK      = k_RK/100;      % [-] std of gauge factor @ T_ref.
T_ref       = 23;            % [°C] the reference temperature.
alpha_k     = 93e-6;         % [1/K] coefficient of temperature-dependence of K-factor.
u_alpha_k   = 10e-6;         % [1/K] std of alpha_k.
T           = 20;            % [°C] the temperature at the measurement.
u_T         = 1;             % [°C] std of the temperature at the measurement.
R           = 350;           % [ohm] the resistance of the unstressed strain gauge.
u_R         = R*.003;        % [ohm] std of R.
DeltaR      = .714;          % [ohm] the measured change in resistance under deformation.
u_DeltaR    = 5*DeltaR/100;  % [ohm] std of DelataR.
c_s0        = -10.23;        % [mum/m] coefficient 0 of apparent strain
c_s1        = 1.56;          % [mum/m/K] coefficient 1 of apparent strain
c_s2        = -5.8e-2;       % [mum/m/K^2] coefficient 2 of apparent strain
c_s3        = 2.34e-4;       % [mum/m/K^3] coefficient 3 of apparent strain
u_eps_s_m   =  0;%5;         % [mum/m] std of apparent strain; set to zero

C_spoly     = [c_s3 c_s2 c_s1 c_s0];

u_acc       = 0;%.02/100;    % [-] accuracy
u_rep       = 0;%.01/100;    % [-] repeatability
u_lin       = 0;%02/100;     % [-] linearity

phi = 0;
u_phi = 5;

nu = .3;
nu_0 = .3;
q =.1/100;
```

```
% defining the function of generating the normal distributions
```

```
function V = N_d (mu,sigma)
  res = 1000000;
  rng('shuffle')
  V = normrnd(mu,sigma,[1,res]);
end
```

```
% generating the normal distributions
```

```
    N_k_RK      = N_d (k_RK,u_k_RK);
    N_phi       = N_d (phi ,u_phi);
    N_alpha_k   = N_d (alpha_k,u_alpha_k);
    N_T         = N_d (T,u_T);
    N_DeltaR    = N_d (DeltaR,u_DeltaR);
    N_R         = N_d (R,u_R);

    sigma_1st = [sig11 tau12 0; tau12 sig22 0;0 0 sig33]; % stress tensor
    eps_num = ones(1,length(N_k_RK));
```

```
% evaluating the components of the strain vector numerically
```

```
    for i = 1:length(N_k_RK)
```

```
    eps_num(i) = eps_evaluation1 (N_k_RK(i),T_ref,N_alpha_k(i),N_T(i),N_R(i),...
N_DeltaR(i),C_spoly,sigma_1st,N_phi(i)*pi/180,nu,nu_0,q);
    end

% evaluating the mean value and the standard deviation of the input values

    k_RK        = mean(N_k_RK);
    u_k_RK      = std(N_k_RK);
    alpha_k     = mean(N_alpha_k);
    u_alpha_k   = std(N_alpha_k);
    T           = mean(N_T);
    u_T         = std(N_T);
    R           = mean(N_R);
    u_R         = std(N_R);
    DeltaR      = mean(N_DeltaR);
    u_DeltaR    = std(N_DeltaR);
    phi         = mean(N_phi);
    u_phi       = std(N_phi);

% generating the vector of input values

    INPUT =
([sig11,sig22,sig33,tau12,k_RK,u_k_RK,T_ref,alpha_k,u_alpha_k,T,u_T,R,u_R,DeltaR,u_DeltaR,c
_s0,c_s1,c_s2,c_s3,u_eps_s_m,u_acc,u_rep,u_lin,phi,u_phi,nu,nu_0,q]);

% evaluating the corrected strain and its standard uncertainty of the analytical approach

    Resulat = usg(INPUT);
    MEAN_AN = Resulat.T_eps_evalutaion.eps_c;
    STD_AN  = Resulat.Tu_eps_c.u_eps_c;
    COV_AN  = STD_AN/MEAN_AN;

% evaluating the corrected strain and its standard uncertainty of the numerical approach

    MEAN_NUM = mean (eps_num);
    STD_NUM = std (eps_num);
    COV_NUM = STD_NUM/MEAN_NUM;

% Result of comparison

  RMEAN= 100*(MEAN_AN-MEAN_NUM)/MEAN_AN
```

RMEAN = -0.7322

```
  RSTD = 100*(STD_AN-STD_NUM)/STD_AN
```

RSTD = -2.4770

```
  RCOV = 100*(COV_AN-COV_NUM)/COV_AN
```

RCOV = -1.7320

Script E 1: Example of numerical validation

Appendix F References

[1] Christine Storm (2015), "Invention of the Strain Gauge, Arthur Ruge, 1938", MIT museum, Retrieved: 4/08/2020 12:00. http://museum.mit.edu/150/82

[2] Guide to the Expression of Uncertainty in Measurement (GUM), JCGM 100:2008 *(GUM 1995 with minor corrections*, International Organization for Standardization (ISO), Geneva, *2008*.

[3] KAARLS, R. (1981), BIPM Proc.-Verb. Com. Int. Poids et Mesures 49, A1-A12 (in French); Giacomo, P. (1981), Metrologia 17, 73 -74 (in English)

[4] CIPM (1980), BIPM Proc.-Verb. Com. Int. Poids et Mesures 48, C1-C30 (in French); BIPM (1980), Rapport BIPM-80/3, Report on the BIPM enquiry on error statements, Bur. Intl. Poids et Mesures (Sèvres, France) (in English)

[5] Sommer K, Siebert B R L, "Praxisgerechtes Bestimmen der Messunsicherheit nach GUM", in Technisches Messen 71 (Oldenbourg Verlag 2004), ISSN 0171-8096.

[6] Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" —Propagation of distributions using a Monte Carlo method, International Organization for Standardization (ISO), Geneva, 2008.

[7] Evaluation of measurement data — Supplement 2 to the "Guide to the expression of uncertainty in measurement" —Extension to any number of output quantities, International Organization for Standardization (ISO), Geneva, 2011.

[8] Evaluation of measurement data — Supplement 3 to the "Guide to the expression of uncertainty in measurement" —Modeling, International Organization for Standardization (ISO), Geneva, 2008.

[9] Evaluation of measurement data — An introduction to the "Guide to the expression of uncertainty in measurement" —and related documents, International Organization for Standardization (ISO), Geneva, 2009.

[10] Evaluation of measurement data – Concepts and basic principles (ISO/IEC Guide 98-2), International Organization for Standardization (ISO), Geneva, 2009.

[11] Evaluation of measurement data – The role of measurement uncertainty in conformity assessment (ISO/IEC Guide 98-4), International Organization for Standardization (ISO), Geneva, 2012.

[12] Evaluation of measurement data – Applications of the least-squares method (ISO/IEC Guide 98-5), International Organization for Standardization (ISO), Geneva, current status: deleted; according to ISO, Retrieved: 14/08/2020 12:00 https://www.iso.org/standard/68374.html .

[13] DIN 1319-1 Grundlagen der Messtechnik; Verein Deutscher Ingenieure; 1995

[14] DIN 1319-2 Begriffe für Messmittel; Verein Deutscher Ingenieure; 2005.

[15] DIN 1319-3 Auswertung von Messungen einer einzelnen Messgröße; Verein Deutscher Ingenieure; 1996.

[16] DIN 1319-4 Auswertung von Messunsicherheit; Verein Deutscher Ingenieure 1999.

[17] Sommer K, Weckenmann A, “Systematic approach to the modeling of measurement for uncertainty evaluation”, Article *in* Journal of Physics Conference Series. September 2005.

[18] Joyce J (2003), "Bayes' Theorem", The Stanford Encyclopedia of Philosophy Archive(Spring 2019 ed.), Retrieved: 14/08/2020 12:00 https://plato.stanford.edu/archives/spr2019/entries/bayes-theorem/

[19] ISO 13373-1:2002, “Condition monitoring and diagnostics of machines – Vibration condition monitoring – Part 1: General procedures”.

[20] Keil S, “Dehnungsmessstreifen” ;(Springer Vieweg); 2016; ISBN 978-3-658-13611-6

[21] Hoffmann K, “An Introduction to Stress Analysis and Transducer Design using Strain Gauges”; (HBM Test and Measurement) www.hbn.com.

[22] Ekelof S, “The Genesis of the Wheatstone Bridge”; Article in Published in "Engineering Science and Education Journal", volume 10, no 1, February 2001, pages 37–40.

[23] Parthier R, “Messtechnik”; (Springer Vieweg) 2020; ISBN 978-3-658-27130-5

[24] VDI/VDE 2635 Part 1; “Bonded electric resistance strain gauges, characteristics and testing conditions”, 2015; Verein Deutscher Ingenieure.

[25] Kessel R, Sommer K, “Sensortechnik”; (Springer Vieweg) 2018; ISBN 978-3-642-29941-4

[26] HAPT; Fraunhofer institute for wind energy systems, Retrieved: 24/08/2020 12:00 https://www.iwes.fraunhofer.de/en/research-projects/current-projects/hapt.html/

[27] Rolling Bearings; Methodology; Fraunhofer institute for wind energy systems , Retrieved: 24/08/2020 12:00 https://www.iwes.fraunhofer.de/en/test-centers-and-measurements/drive-train-components.html#tabpanel-1