Hochschule für Angewandte Wissenschaften Hamburg

Hamburg University of Applied Sciences

AIRCRAFT DESIGN AND SYSTEMS GROUP (AERO)

Methods for Operating Empty Mass Estimation in Aircraft Design

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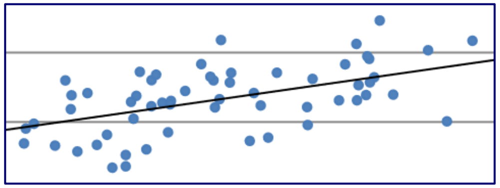
Based on the Project of Jan Lehnert



$$\frac{m_{OE}}{m_{MTO}} = 0{,}247 + 0{,}988 \cdot \left(\frac{T_{TO}}{m_{MTO}\, g}\right)$$

$$\frac{m_{OE}}{m_{MTO}} = 0{,}5967 - 0{,}00000166 \cdot R$$

R in NM

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Abstract

Purpose – Calculation methods from Torenbeek, Raymer, Marckwardt, and Loftin are examined and compared. The question is whether there is a more precise method for passenger aircraft to determine the operating empty mass fraction based on new statistics.

Methodology – Equations for estimating the operating empty mass fraction are determined from parameters including thrust-to-weight ratio, wing loading, design range, payload, and number of engines (on the wing). Only aircraft parameters are used, which are already known in preliminary sizing and for which a physical connection exist to the operating empty mass fraction.

Findings – New methods surpass the accuracy of the classic calculation methods. This is achieved by using more and other design parameters and their optimal mathematical combination.

Practical implications – The risk of incorrect mass estimation in preliminary sizing is reduced.

Acknowledgment

Jan Lehnert prepared a project:

"Methoden zur Ermittlung des Betriebsleermassenanteils im Flugzeugentwurf"

(Methods to Determine the Operating Empty Mass in Aircraft Design)

at

Hamburg University of Applied Sciences,

Aircraft Design and Systems Group (AERO).

http://library.ProfScholz.de

https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2018-05-24.014

His work is referenced here as: **Lehnert 2018**

HAW HAMBURG

Projekt

Methoden zur Ermittlung des Betriebsleermassenanteils im Flugzeugentwurf

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Equations and Numbers from Textbooks

Equations and Numbers from Textbooks

"First Law of Aircraft Design" and Operating Empty Mass Ratio

$$m_{MTO} = m_{OE} + m_F + m_{MPL}$$

Basic mass summation

$$1 = \frac{m_{OE}}{m_{MTO}} + \frac{m_F}{m_{MTO}} + \frac{m_{MPL}}{m_{MTO}}$$

Unity Equation (according to Torenbeek)

$$m_{MTO} = \frac{m_{MPL}}{1 - \frac{m_F}{m_{MTO}} - \boxed{\frac{m_{OE}}{m_{MTO}}}}$$

First Law of Aircraft Design (**Scholz 2015**)

m_F	Fuel Mass
m_{MPL}	Maximum Payload
m_{PL}	Payload
m_{MTO}	Maximum take-off mass
m_{OE}	Operating empty mass

$$\frac{m_{OE}}{m_{MTO}}$$

Operating Empty Mass ratio

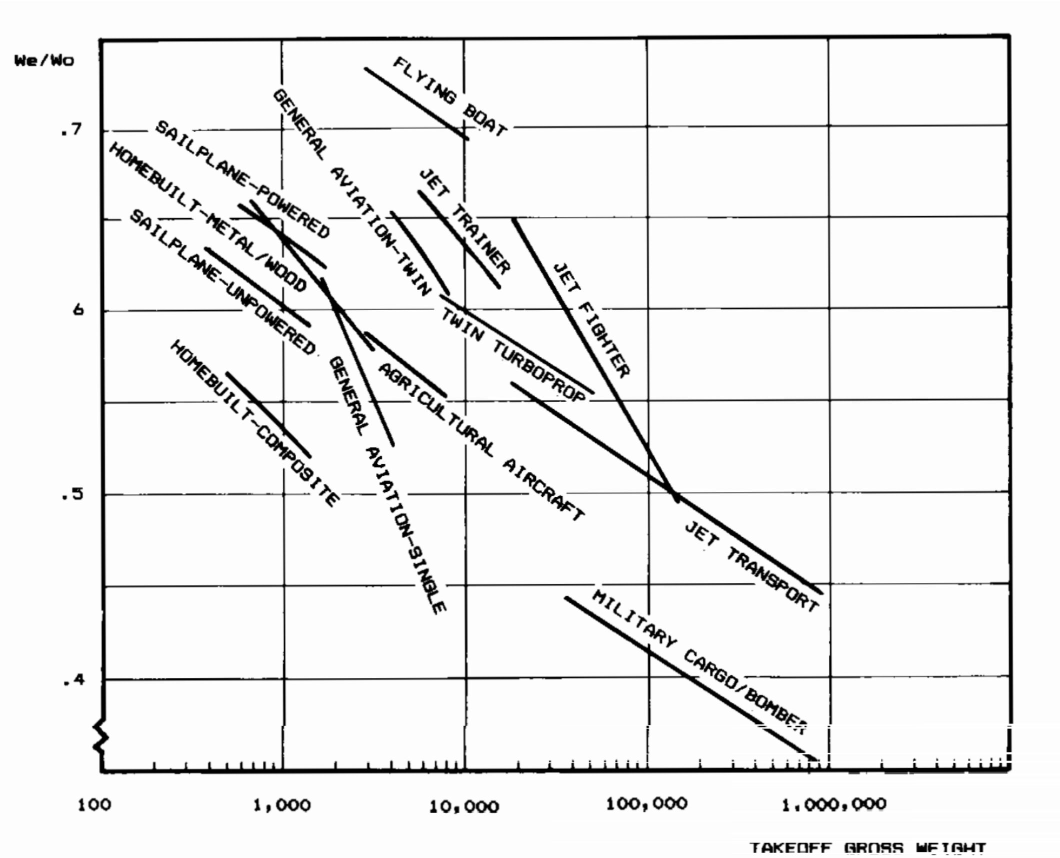
Some Numbers from Torenbeek

Operating Empty Mass ratio

Airplane Category		Percentage of MTOW			
		Airframe structure	propulsion Group	fixed eq. and serv.	empty weight
Passenger Transport					
short-haul	jets	31,5	8,0	13,5	53,0
	turboprops	32,0	12,5	13,5	58,0
	pistons	29,5	20,5	15,5	65,5
long-haul	jets	24,5	8,5	9,0	42,0
	turboprops	27,0	12,0	12,0	51,0
	pistons	25,5	17,5	11,0	54,0
Freighters					
short-haul	turboprops	35,0	13,0	8,0	56,0
long-haul	turboprops	26,5	10,0	7,0	43,5
Executive Jets		27,5	8,0	15,5	51,0

Torenbeek 1982
fromTU Delft.

Some Numbers from Raymer



Empty weight fraction trends.

Operating Empty Mass ratio

Apparent reduction with aircraft size. Physically, the reduction is due to range.

There is a correlation between aircraft size and range. This explains the reduction of Operating Empty Mass ratio with size.

Raymer 1992. His book published with AIAA.

Equation from Marckwardt

$$\frac{m_{OE}}{m_{MTO}} = 0{,}591 \cdot \left(\frac{R[km]}{1000}\right)^{-0{,}113} \cdot \left(\frac{m_{MTO}[kg]}{1000}\right)^{0{,}0572} \cdot n_E{}^{-0{,}206}$$

$$m_{MTO} = \frac{m_{MPL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$$

Operating Empty Mass ratio

According to Marckwardt (HAW Hamburg), OEM ratio reduces with range, *R* and number of engines, n_E on the wing. MTOM rather increases OEM ratio, but very little. This can all be seen from the exponents in the first equation. MTOM is initially unknown. This leads to an iteration with OEM ratio set to 0.5 initially. The iteration converges after only three steps, because MTOM has very little influence.

Please see my Aircraft Design lecture notes (**Scholz 2015**) for details.

Equation from Loftin

$$\frac{m_{OE}}{m_{MTO}} = 0{,}23 + 1{,}04 \cdot \left(\frac{T_{TO}}{m_{MTO} \cdot g}\right)$$

Operating Empty Mass ratio

Loftin 1980 (NASA) calculates OEM ratio from what is know after the matching chart is drawn in aircraft preliminary sizing. The simple equation is based on aircraft data from 1980. It still works well.

Please see my Aircraft Design lecture notes (**Scholz 2015**) for details.

New Equations

Database from Jenkinson Used for the New Equations

Data of 67 Passenger Jet Aircraft

Produced by Lloyd Jenkinson, Loughborough University with students.

His book: **Jenkinson 1999**.

	Manufacturer	Type	Model		Manufacturer	Type	Model
1	AIRBUS	A300-	600R	35	DOUG.		DC8-63
2	AIRBUS	A310-	300	36	DOUG.		DC8-73
3	AIRBUS	A319-	100	37	DOUG.		DC 9-10
4	AIRBUS	A320-	200	38	DOUG.		DC 9-30
5	AIRBUS	A321-	200	39	DOUG.		DC 9-40
6	AIRBUS	A330-	200	40	DOUG.		DC 9-50
7	AIRBUS	A330-	300	41	McDON.	DOUG	MD-81
8	AIRBUS	A340-	200	42	McDON.	DOUG	MD-82
9	AIRBUS	A340-	300	43	McDON.	DOUG	MD-83
10	AIRBUS	A340-	500	44	McDON.	DOUG	MD-87
11	AIRBUS	A340-	600	45	McDON.	DOUG	MD-90
12	AIRBUS	A380-	100	46	DOUG.	DC10-	10
13	BOEING	707-	320C	47	DOUG.	DC10-	30
14	BOEING	717-	200	48	McDON.	DOUG	MD-11
15	BOEING	727-	200Adv	49	McDON.	DOUG	MD12LR
16	BOEING	737-	200	50	McDON.	DOUG	MD12HC
17	BOEING	737-	300	51	LOCKHD	L1011	100
18	BOEING	737-	400	52	ILYUSHIN	Il-2M-	MK
19	BOEING	737-	500	53	ILYUSHIN	Il-96-	300
20	BOEING	737-	600	54	ILYUSHIN	Il-96	M
21	BOEING	737-	700	55	TUPOLEV	Tu-	134
22	BOEING	737-	800	56	TUPOLEV	Tu-	154M
23	BOEING	747-	100	57	TUPOLEV	Tu-204	-200
24	BOEING	747-	200	58	TUPOLEV		Tu-334
25	BOEING	747-	400	59	BAe		RJ70
26	BOEING	757-	200	60	BAe		RJ85
27	BOEING	757-	300	61	BAe		RJ100
28	BOEING	767-	200	62	BAe		RJ115
29	BOEING	767-	200ER	63	CADAIR	Reg. Jet	100
30	BOEING	767-	300	64	CADAIR	Reg. Jet	100ER
31	BOEING	767-	300ER	65	EMBRAER	EMB-	145
32	BOEING	777-	200	66	FOKKER		F70
33	BOEING	777-	200IGW	67	FOKKER		F100
34	BOEING	777-	300				

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Reevaluating the Equation from Loftin

old (1980): $$\frac{m_{OE}}{m_{MTO}} = 0{,}23 + 1{,}04 \cdot \left(\frac{T_{TO}}{m_{MTO} \cdot g}\right)$$

new (2018): $$\frac{m_{OE}}{m_{MTO}} = 0{,}25 + 0{,}99 \cdot \left(\frac{T_{TO}}{m_{MTO} \cdot g}\right)$$

After 38 years the equation results pretty much unchanged from regression.

Lehnert 2018

Possible Structure of New Equations

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Linear Regression: $f(X) = a + b \cdot X$

Multiple Regression: $f(X,Y,Z) = m \cdot X + n \cdot Y + p \cdot Z$

Nonlinear Regression: $f(X,Y,Z) = X^m + Y^n + Z^p$

$f(X,Y,Z) = X^m \cdot Y^n \cdot Z^p$

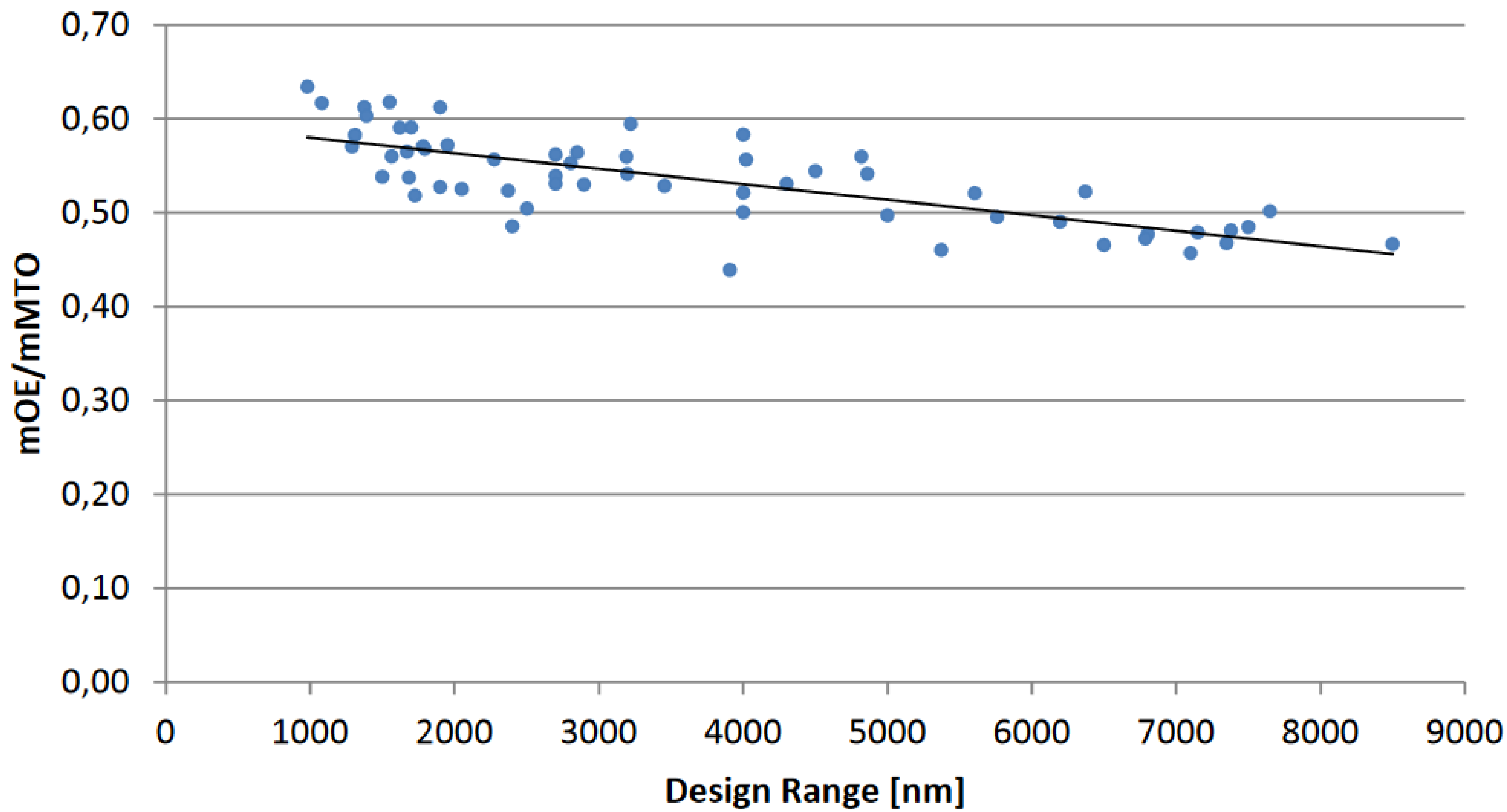
$f(X,Y,Z) = X^m \cdot Y^n + Z^p$

$f(X,Y,Z) = m \cdot X^n + p \cdot Y^q + r \cdot Z^s$

The best: $f(X,Y,Z) = k \cdot X^m \cdot Y^n \cdot Z^p$

New Equations

Equation from Design Range, Plotting the Data



Long-range aircraft need to be efficient to make money. This is why OEM ratio is decreasing with range. **Lehnert 2018**

Equation from Design Range (*R* in NM)

$$\frac{m_{OE}}{m_{MTO}} = 0{,}5967 - 0{,}00000166 \cdot R$$

R^2	F [%]
0,57	4,48

Regression coefficient, R^2 and average relative error, F are used to determine the accuracy of the equation.
$R^2 > 0.5$ is useful
$R^2 > 0.7$ is a good accuracy

Lehnert 2018

Equation from Thrust-to-Weight Ratio and Wing Loading

$$\frac{m_{OE}}{m_{MTO}} = 4{,}456 \cdot \left(\frac{T_{TO}}{m_{MTO}g}\right)^{0{,}363} \cdot \left(\frac{m_{MTO}}{S_W}\right)^{-0{,}262}$$

R^2	$F\,[\%]$
0,68	4,29

The matching chart in jet aircraft preliminary sizing plots thrust-to-weight ratio versus wing loading. This makes these two parameters natural candidates for an equation. The equation is an extension of Loftin's equation.

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New Equations

Equation from Thrust-to-Weight Ratio, Wing Loading, Range, Payload, and Number of Engines <u>on</u> the Wing

$$\frac{m_{OE}}{m_{MTO}} = 3{,}533 \cdot \left(\frac{T_{TO}}{m_{MTO}\,g}\right)^{0{,}2497} \cdot \left(\frac{m_{MTO}}{S_W}\right)^{-0{,}2044} \cdot R^{-0{,}0659} \cdot {m_{PL}}^{0{,}0247} \cdot {n_e}^{0{,}0086}$$

R^2	$F\,[\%]$
0,77	3,36

Now we add more parameters readily available in preliminary sizing. The more, the better. Range, *R* has proven its usefulness already, engines <u>on</u> the wing reduce wing root bending moment, but number of engines, n_E has little influence. The same is true for payload (exponent close to zero).

Lehnert 2018

New Equations

Equation from Thrust-to-Weight Ratio, Wing Loading, and Range

$$\frac{m_{OE}}{m_{MTO}} = 3{,}298 \cdot \left(\frac{T_{TO}}{m_{MTO}\,g}\right)^{0{,}2412} \cdot \left(\frac{m_{MTO}}{S_W}\right)^{-0{,}1863} \cdot R^{-0{,}04105}$$

R^2	$F\,[\%]$
0,77	3,40

The Winner !

Omitting unnecessary parameters keeps the accuracy almost the same as in the previous equation but simplifies its application.

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