



# Numerical simulation of multi-rotor diffuser augmented wind turbine system using the actuator line method in PyFR solver

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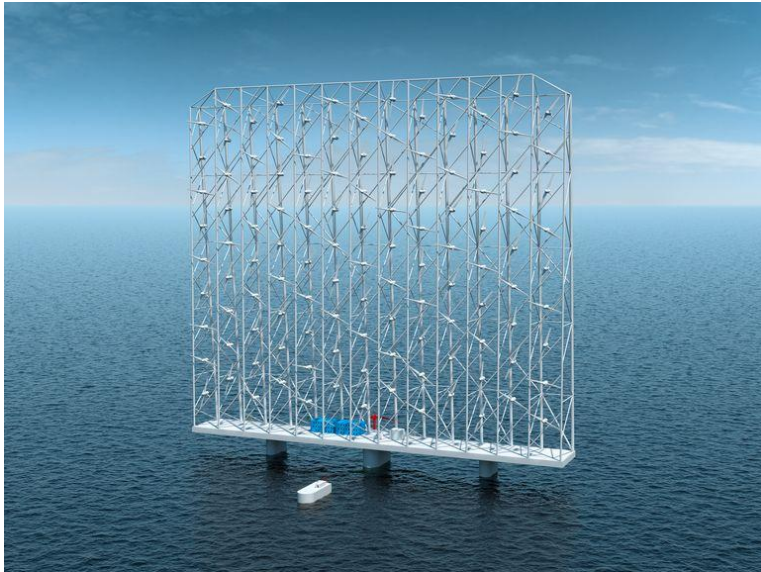
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# Introduction

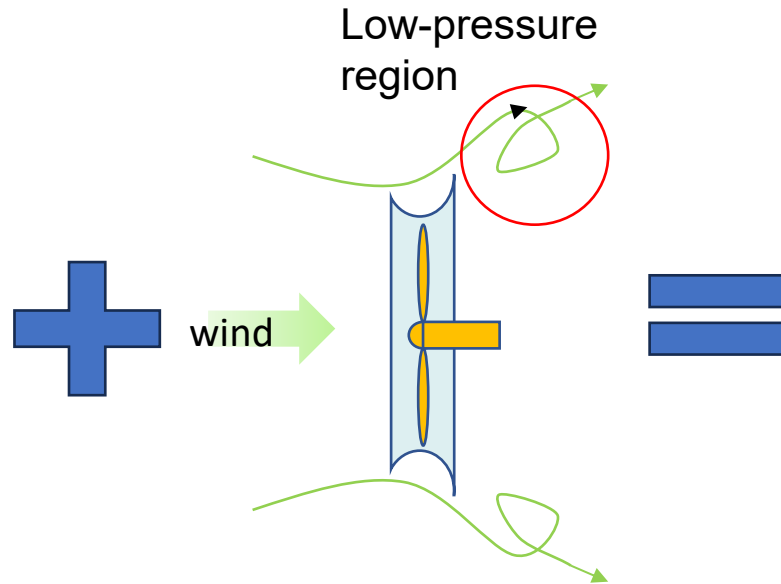
## Multi-rotor diffuser-augmented wind turbines system



Multi-rotor system (MRS)

### Advantages:

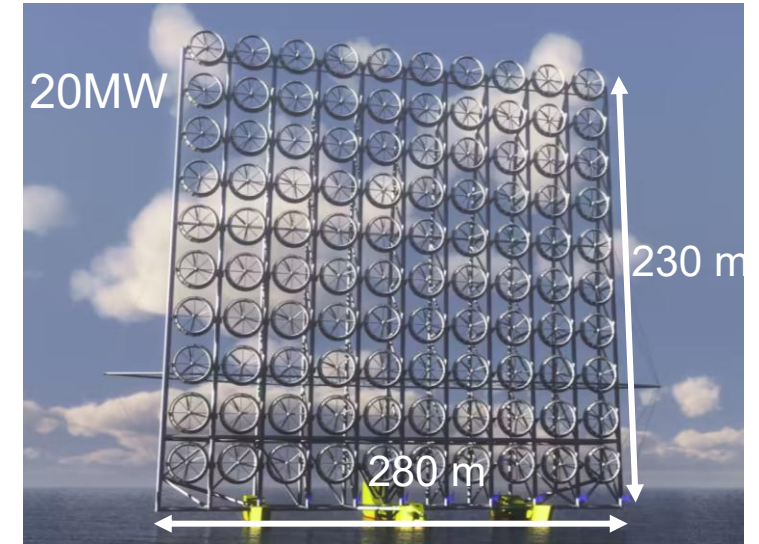
- Cost efficiency
- Maintainability
- Scalability



Diffuser augmented wind turbine (DAWT)

### Advantages:

- Increased efficiency
- Compact design



Novel floating offshore multi-rotor DAWT system

### Challenges:

- Blockage effect
- Wake interactions

# Numerical Method

## Actuator Line Method -- Blade model

### The incompressible Navier–Stokes equation

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\nabla P + \nu \nabla^2 \vec{V} + \vec{f}$$

### The blade element body force

$$\vec{f} = \frac{1}{2} \rho U_{rel}^2 w_b c_b (C_{l,b} \vec{e}_l + C_{d,b} \vec{e}_d) \cdot F_{tip}$$

$C_{l,b}$  and  $C_{d,b}$  are the lift and drag coefficients

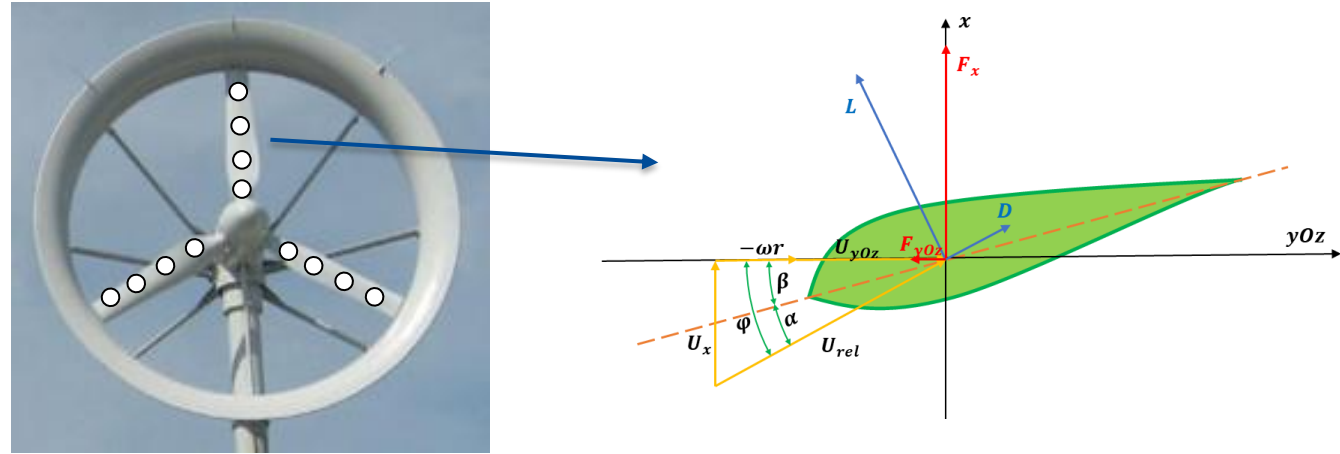
$$C_l, C_d = f(\alpha, Re)$$

$\vec{e}_l$  and  $\vec{e}_d$  are unit vector in the lift and drag directions

$w$  is the segment width,  $c$  is the chord length

$U_{rel}$  is the local relative inflow velocity

$F_{tip}$  is the tip loss correction



The velocity triangle of the cross-sectional blade element

### Projection function for blade element

$$g_{iso}(x, y, z) = \frac{1}{\varepsilon^3 \pi^{3/2}} \exp\left(-\frac{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}{\varepsilon^2}\right)$$

$\varepsilon$  is the Gaussian projection width

# Numerical Method

## Actuator Line Method -- Diffuser model

### The diffuser element body force

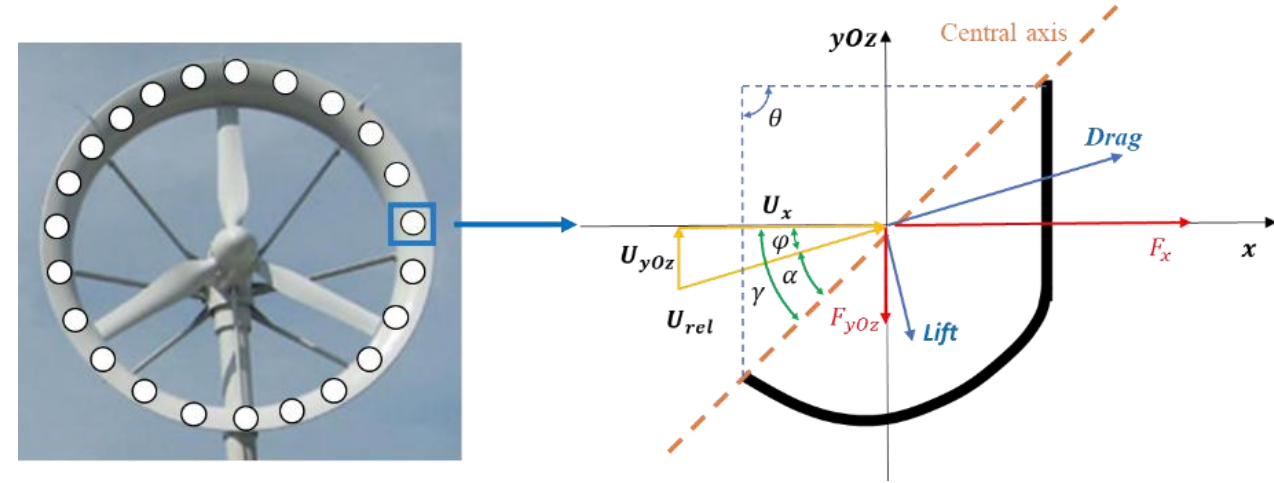
$$\vec{f} = \frac{1}{2} \rho U_{rel}^2 w_d c_d (C_{l,d} \vec{e}_l + C_{d,d} \vec{e}_d)$$

$C_{l,d}$  and  $C_{d,d}$  are the lift and drag coefficients of the diffuser structure

$\vec{e}_l$  and  $\vec{e}_d$  are unit vectors in the lift and drag directions

$w_d$  is the width of the diffuser segment,  $c_d$  is the chord length

$U_{rel}$  is the local relative inflow velocity



The velocity triangle of the cross-sectional diffuser element

### Local attack angle

$$\alpha = \varphi - \gamma \quad \gamma \text{ is the local tilt angle}$$

$$\varphi = \arctan\left(\frac{U_{yOz}}{U_x}\right)$$



# Numerical Method

## Actuator Line Method -- Diffuser model

### Projection function

$$g_{diffuser}(x_t, x_w, \theta) = C \cdot \exp\left(-\frac{(x_t - x_{t,0})^2}{\varepsilon_t^2} - \frac{(x_w - x_{w,0})^2}{\varepsilon_w^2}\right) \cdot g_{diffuser}(\theta)$$

$x_t$ ,  $x_w$  and  $\theta$  are the **thickness**, **width**, and **azimuthal** coordinate directions, respectively

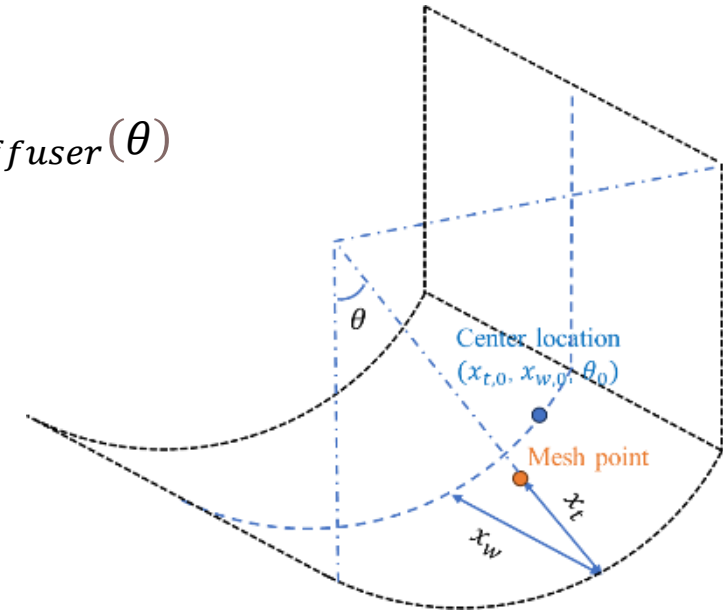
$\varepsilon_t = a_t t$  and  $\varepsilon_w = a_w w$  represent the **Gaussian** widths in the **radial** and **tangential** directions

Normalization factor  $C = \left[2\pi\varepsilon_t\varepsilon_r \cdot \int_{-\frac{\pi}{2}}^0 f(\theta) d\theta\right]^{-1}$

### Force magnitude function

$$g_{diffuser}(\theta) = \frac{2}{\omega} \phi\left(\frac{\theta - \theta_0}{\omega}\right) \Phi\left(\tau\left(\frac{\theta - \theta_0}{\omega}\right)\right)$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \Phi(x) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$$



The schematic diagram of force projection

$\theta_0$ ,  $\omega$  and  $\tau$  are the center location, scale and shape parameter, respectively

# Numerical Method

## PyFR Slover

The Flux Reconstruction approach is a unifying framework for high-order schemes.

### Advantage

1. High accuracy--Achieving more precise results with a smaller number of grids
2. High efficiency--Only requires neighboring cells to update physical quantities within a cell



PyFR: An open source framework for solving advection–diffusion type problems on streaming architectures using the flux reconstruction approach<sup>☆</sup>

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<https://doi.org/10.1016/j.cpc.2014.07.011>

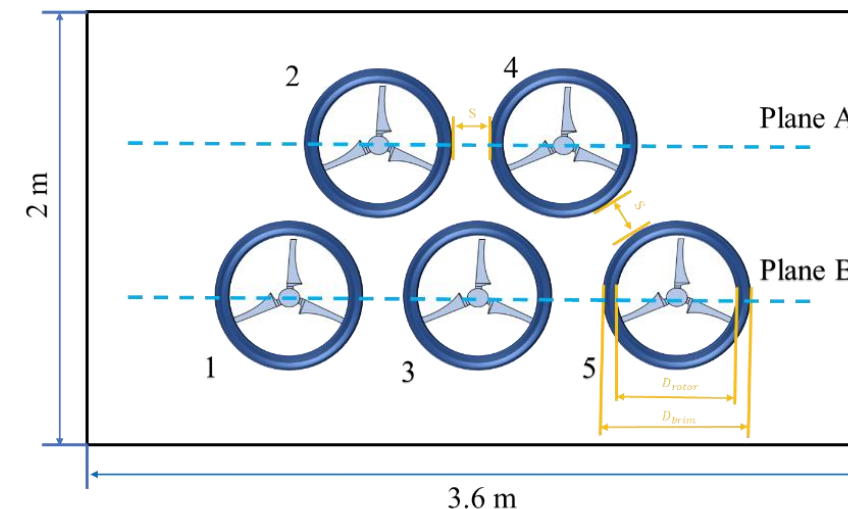


# Results and discussion

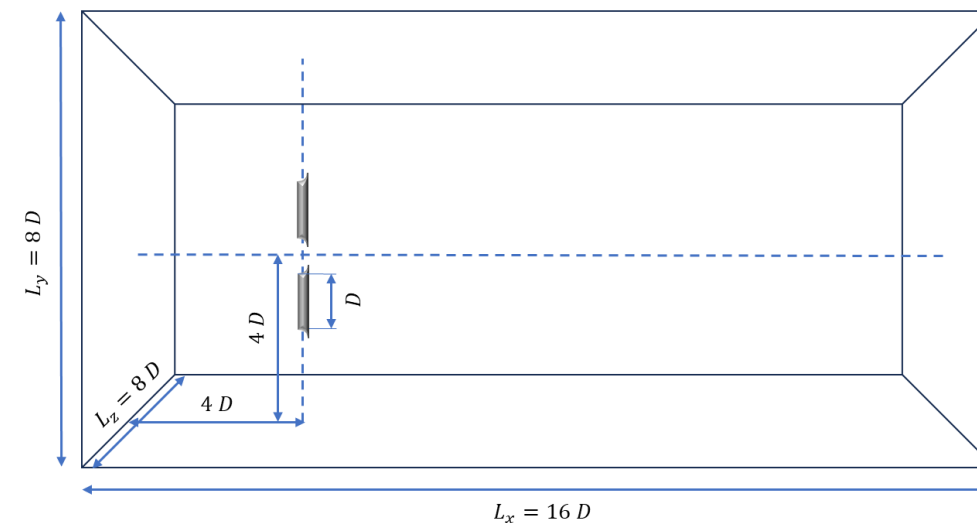
## 5-rotor DAWT system

### Parameter of the multi-rotor system experiment

| Parameter                      | Value             |
|--------------------------------|-------------------|
| Rotor diameter ( $D_{rotor}$ ) | 0.5 m             |
| Brim diameter ( $D_{brim}$ )   | 0.669 m           |
| Tip speed ratio (TSR)          | 4                 |
| Inflow speed                   | 6 m/s             |
| Lens shape                     | Cii type brim 10% |
| Gap ratio ( $S$ )              | $0.2D_{rotor}$    |



Schematic diagram of 5-rotor system experiment



Side view of the computational domain

# Results and Analysis

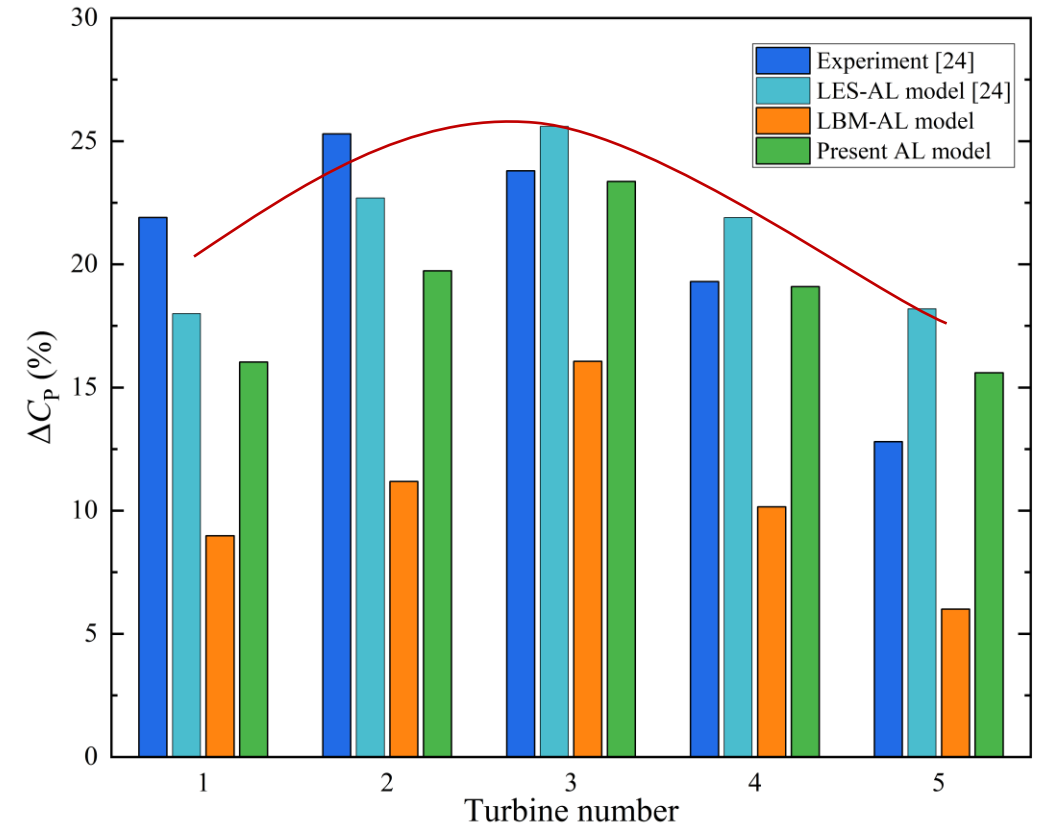
## 5-rotor DAWT system

$$\Delta C_{P,i} = \frac{C_{P,i}}{C_{P,standalone}} - 1$$

$$C_P = \frac{P}{\frac{1}{2} \rho A_{D_{brim}} U_{\infty}^3}$$

The results demonstrate that the AL-PyFR(DAWT) model accurately predicts  $\Delta C_P$  for each rotor in the 5-rotor system.

This asymmetry arises because all rotors rotated in the same direction.



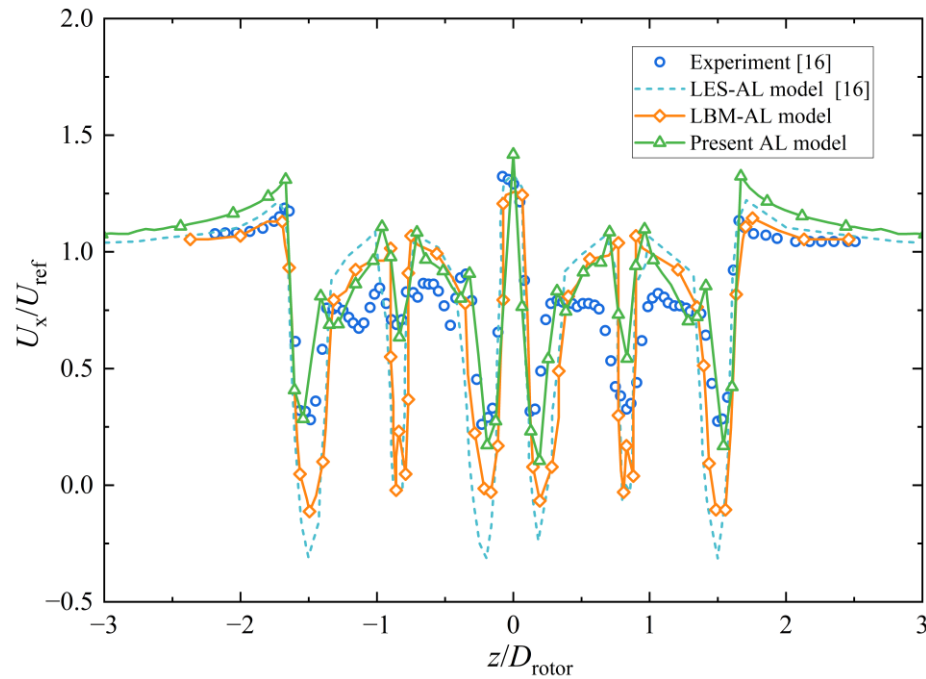
$\Delta C_P$  comparison of each rotor in the 5-rotor system



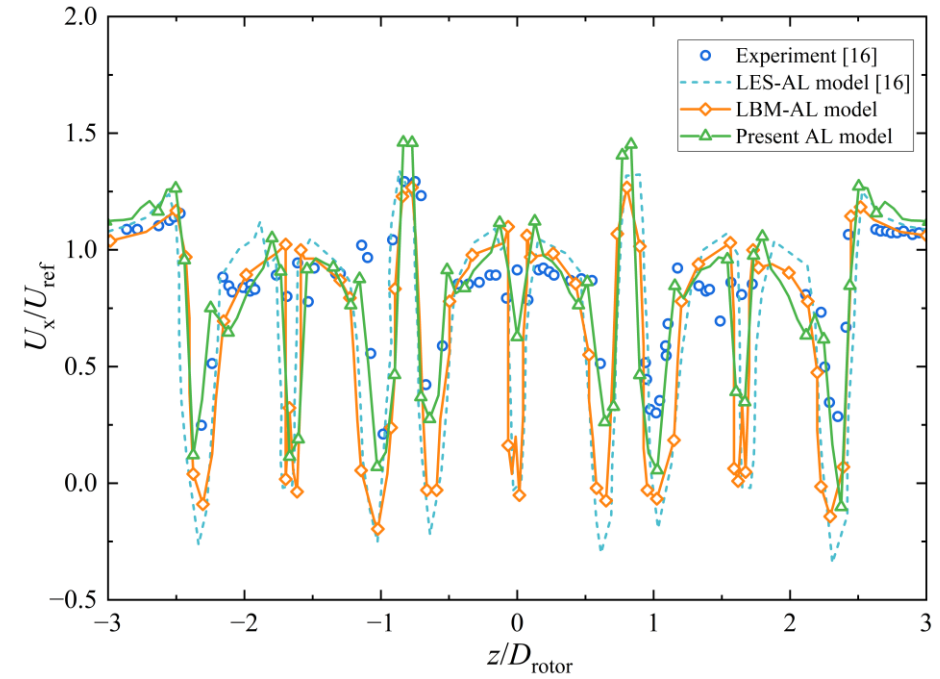
# 3. Results and Analysis

## 5-rotor DAWT system

The AL-PyFR(DAWT) model wake velocity profiles align well with both experiment result and the predictions from other numerical models.



Mean wake velocity profile at plane A

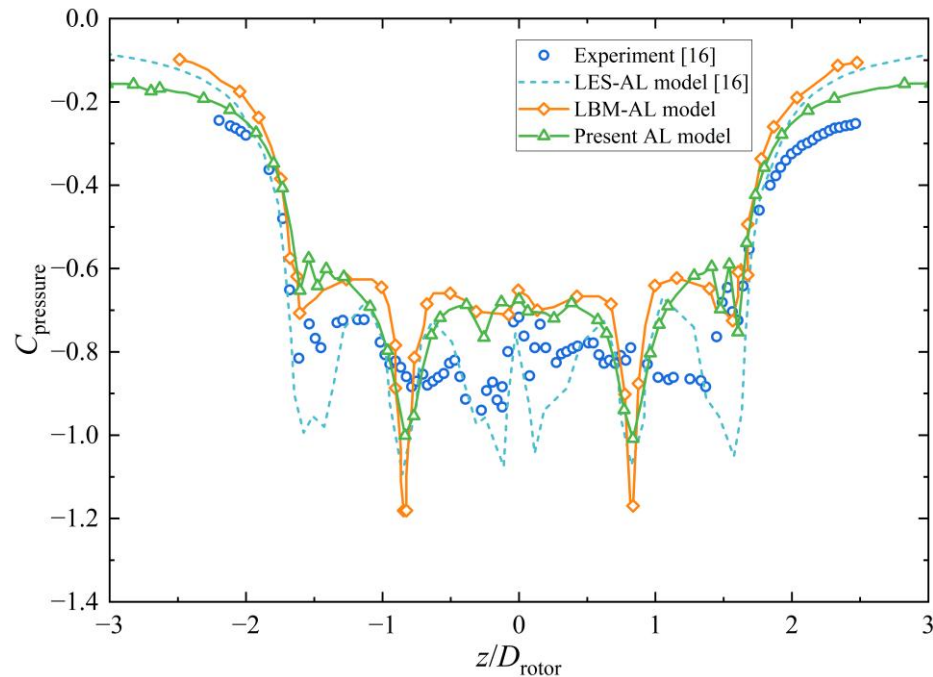


Mean wake velocity profile at plane B

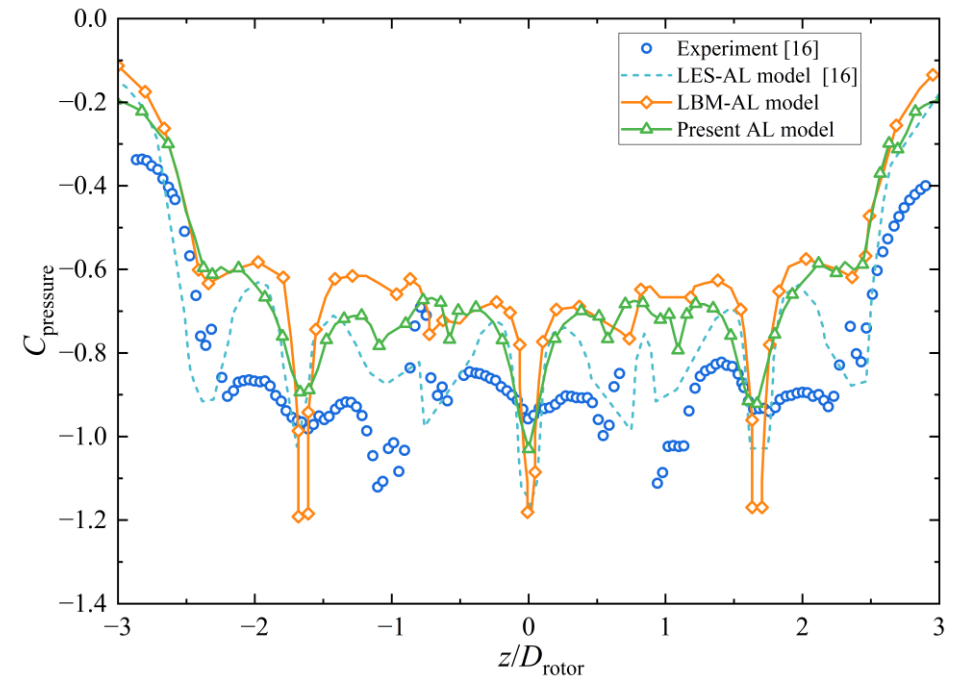
Comparison of the mean wake velocity  $x = 0.4D_{rotor}$

### 3. Results and Analysis

#### 5-rotor DAWT system



Mean pressure profile at plane A



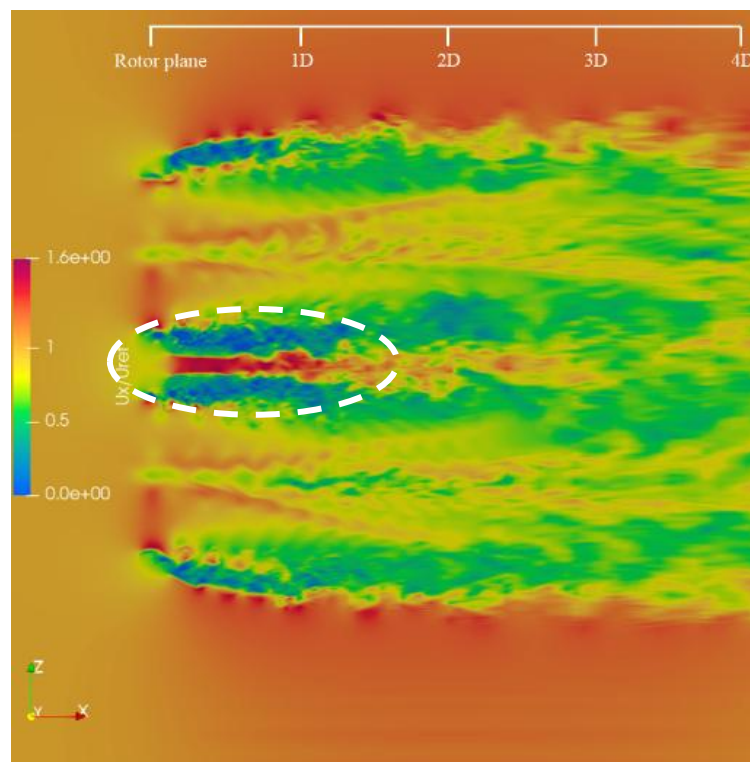
Mean pressure profile at plane B

Comparison of the mean wake pressure  $x = 0.4D_{\text{rotor}}$

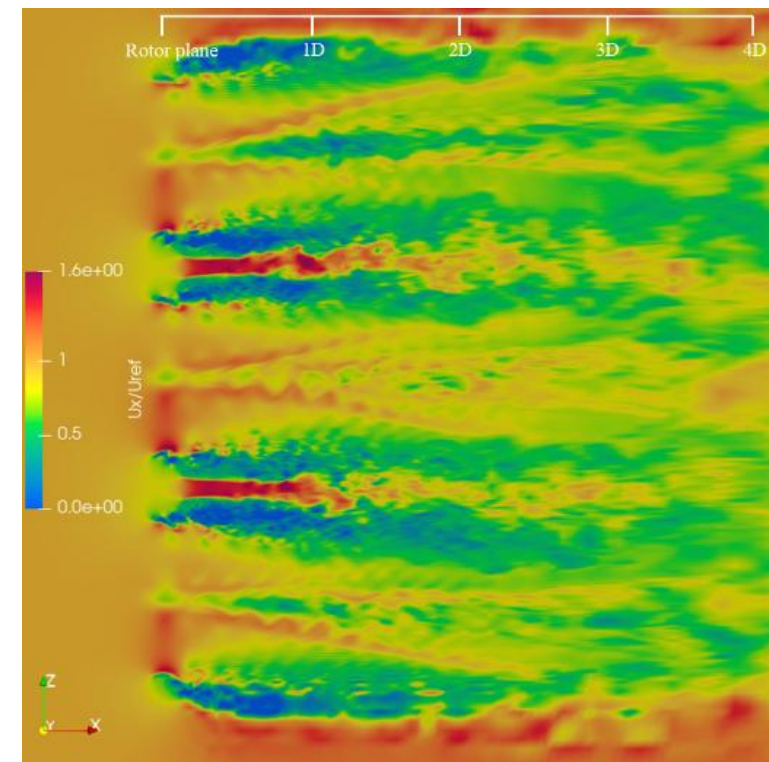
### 3. Results and Analysis

#### 5-rotor DAWT system

This high-speed region arises from the blockage effect among the rotors, resulting in overall higher wake speeds compared to a single rotor case.



Plane A



Plane B

Contours of the instantaneous streamwise velocity in the  $x - y$  plane

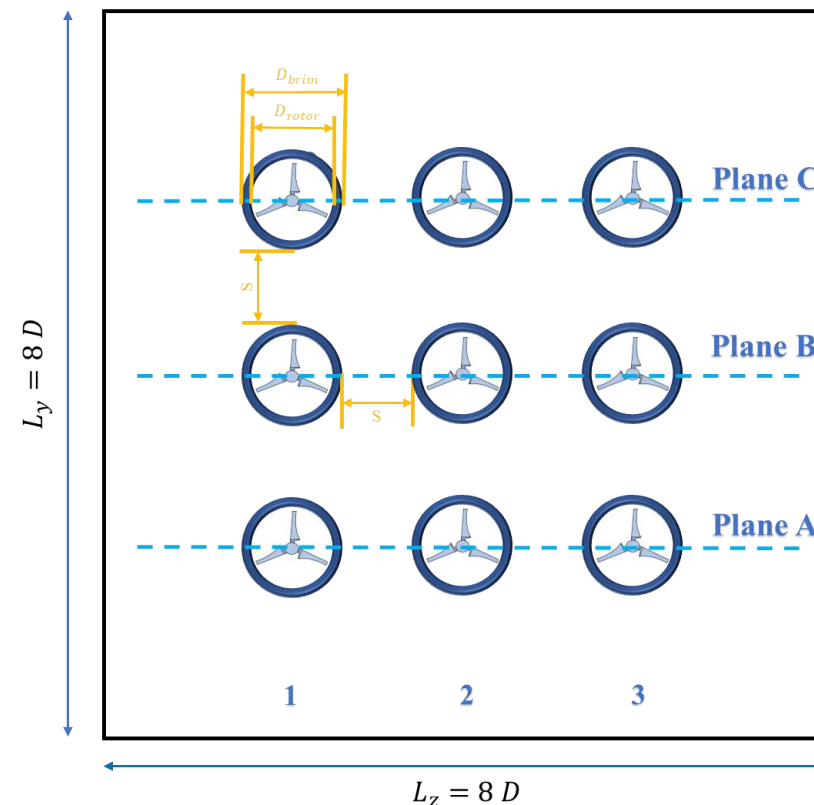
### 3. Results and Analysis

#### 9-rotor DAWT system

The focus is to investigate the influence of rotor spacing and TSR on both aerodynamic efficiency and structural stability.

#### Parameter of the 9-rotor DAWTs system

| Parameter                      | Value             |
|--------------------------------|-------------------|
| Rotor diameter ( $D_{rotor}$ ) | 1 m               |
| Brim diameter ( $D_{brim}$ )   | 1.35 m            |
| Tip speed ratio (TSR)          | 3~5               |
| Inflow speed                   | 10 m/s            |
| Lens shape                     | Cii type brim 10% |
| Gap ratio ( $S$ )              | 0.05~0.4          |



Cross-section of the computational domain at rotor plane

# 3. Results and Analysis

## 9-rotor DAWT system

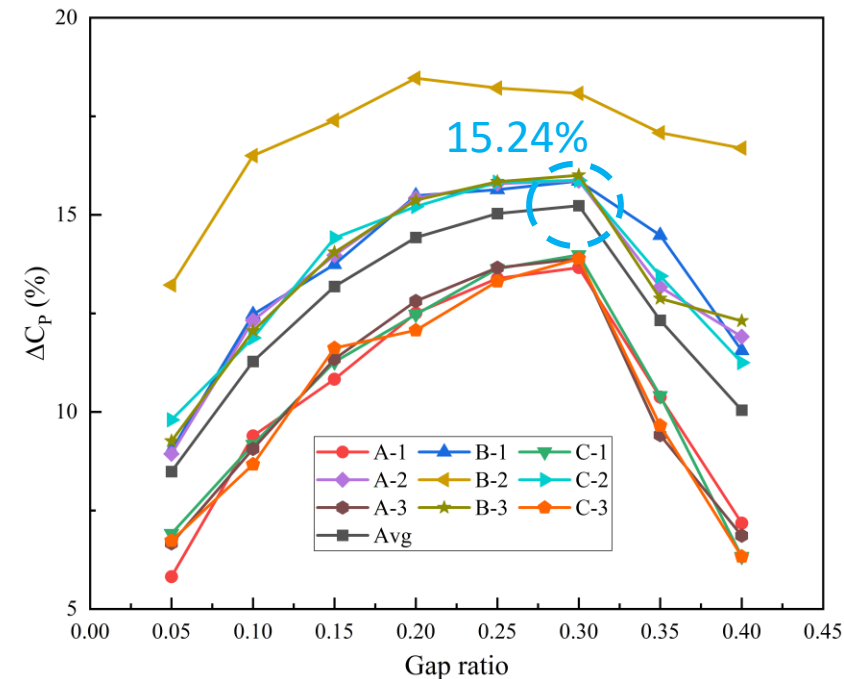
The central rotor demonstrating the highest efficiency and lowest sensitivity to spacing.

$$\Delta C_{T,rotor,i} = \frac{C_{T,rotor,i}}{C_{T,rotor,standalone}} - 1$$

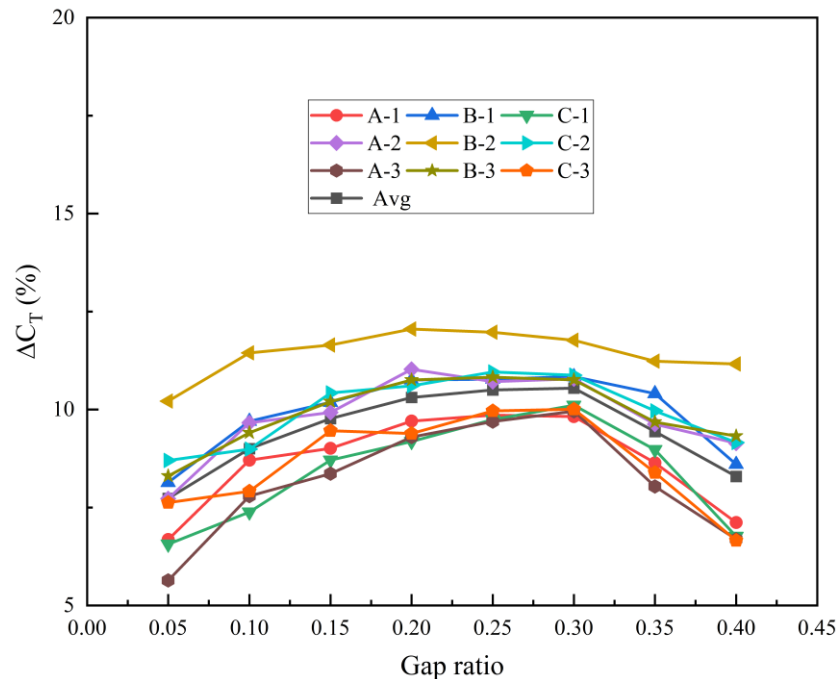
$$C_{T,rotor} = \frac{T_{blade}}{\frac{1}{2}\rho A_{D_{rotor}} U_{\infty}^2}$$

$$\Delta C_{T,brim,i} = \frac{C_{T,brim,i}}{C_{T,brim,standalone}} - 1$$

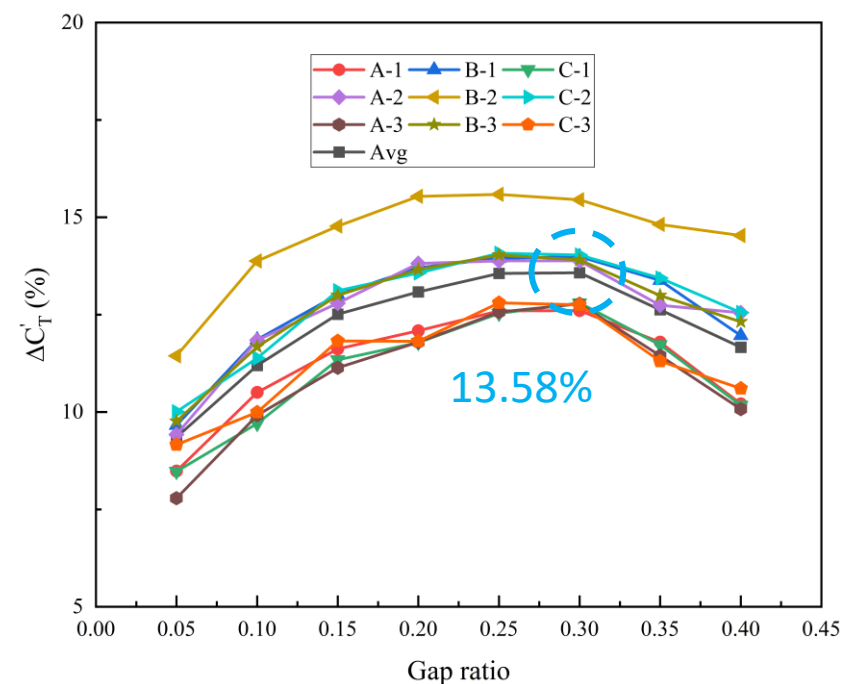
$$C_{T,brim} = \frac{T_{blade} + T_{diffuser}}{\frac{1}{2}\rho A_{D_{brim}} U_{\infty}^2}$$



Variation of  $\Delta C_p$  under different gap ratio



Variation of  $\Delta C_T$  under different gap ratio



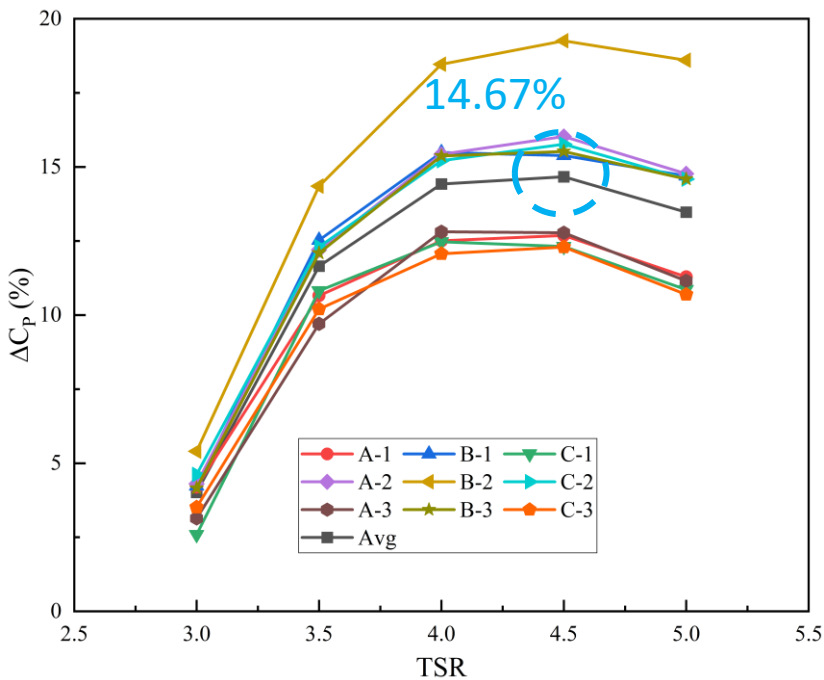
Variation of  $\Delta C'_T$  under different gap ratio



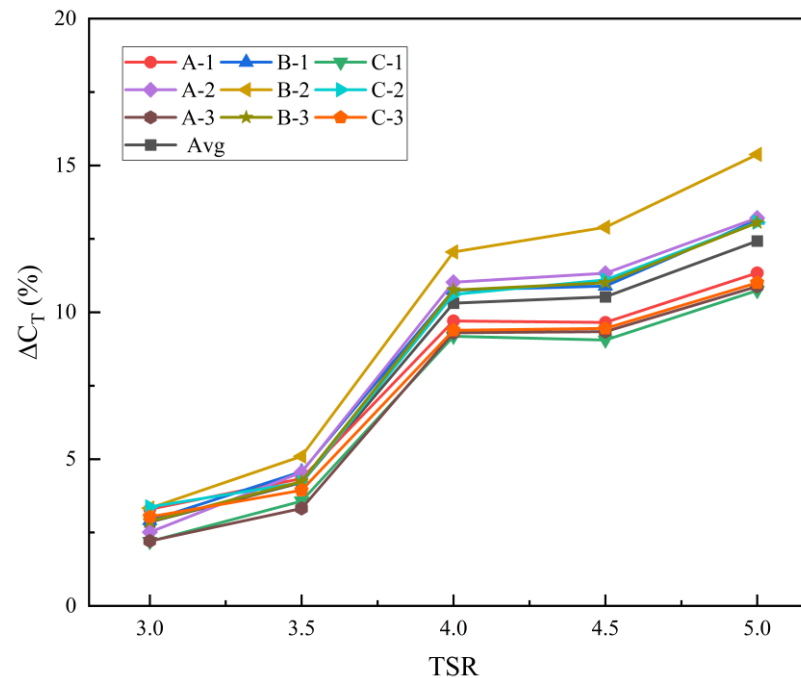
# 3. Results and Analysis

## 9-rotor DAWT system

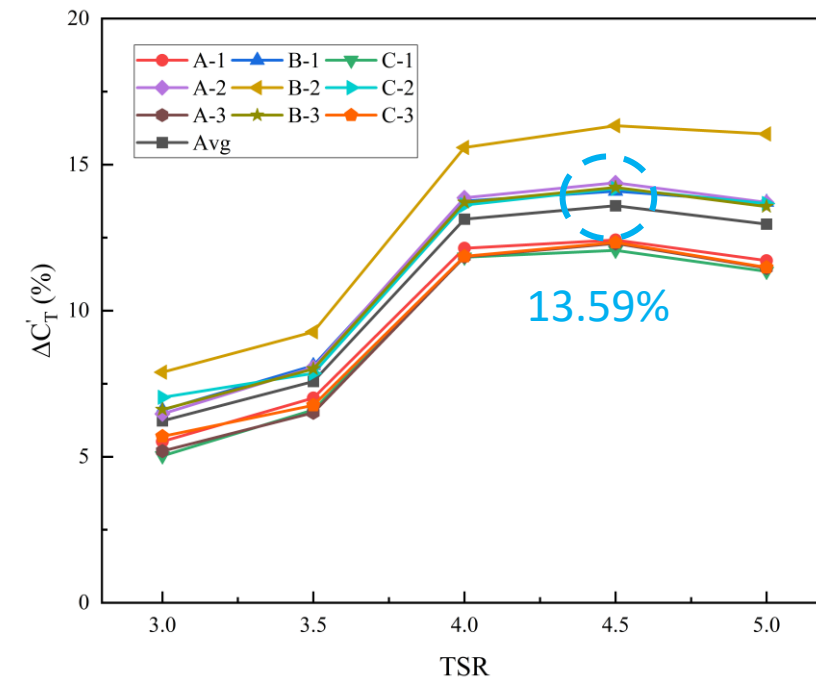
Running at the optimal TSR for single rotor case (TSR=4) might not be optimal for the MRDAWT system.



Variation of  $\Delta C_p$  under different TSR



Variation of  $\Delta C_T$  under different TSR



Variation of  $\Delta C'_T$  under different TSR



# Conclusion

- The performance of AL-PyFR(DAWT) model in MRDAWTs system prediction is validated by comparing with LES-AL model, LBM-AL model and experiment results of a 5-rotor system .
- The AL-PyFR(DAWT) model successfully captured  $\Delta C_p$  due to blockage effect and the asymmetric  $\Delta C_p$  distribution linked to wake interaction and unidirectional rotation.
- In the 9-rotor system, the gap spacing sensitivity analysis revealed that the maximum average system efficiency occurred at  $S = 0.3$ .
- In the TSR sensitivity analysis, optimal system performance was achieved at  $TSR = 4.5$  for both  $\Delta C_p$  and  $\Delta C_T$ .
- These results demonstrate that MRDAWTs system performance is highly dependent on rotor layout and operating conditions, providing valuable insights for system optimization.

# Future Work

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- Develop a generalized diffuser force projection framework using machine learning or adaptive algorithms, eliminating the need for manual adjustment.
- Propose a tip-loss correction model specifically for DAWTs, improving accuracy in blade-tip flow predictions.
- Extend the model to include floating platform motions by coupling with structural solvers, enabling realistic offshore simulations.
- Integrate with optimization algorithms to co-optimize rotor layout and operating parameters, balancing efficiency and structural loading.

# Thank you.

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