

RESEARCH ARTICLE OPEN ACCESS

The Effect of Rotor Size on the Teeter Behavior of Two-Bladed Wind Turbines

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Received: 23 January 2025 | **Revised:** 22 June 2025 | **Accepted:** 30 September 2025

Funding: We want to send special thanks to the Federal Ministry of Education and Research, Siemens Gamesa Renewable Energies, and Energie Baden Württemberg (EnBW) for funding and support of “X-Rotor – two-bladed wind turbines” (13FH1I04IA) and “X-Rotor – two-bladed floating wind turbines” (ref. 13FH1I11IA).

Keywords: large wind turbines | Lock number | pitch-teeter-velocity coupling | scaling effects | square-cube law | teetering | two-bladed

ABSTRACT

The rotor of a two-bladed turbine allows the implementation of an additional degree of freedom around the rotating hub y-axis. This idea is over 60 years old, and various research and commercial turbines have had a teeter hinge. Fatigue and extreme loads can be reduced significantly with a teeter hinge. Teetered turbines have been mainly below or around a rotor diameter of 100 m, and most of the teetered turbines have been built in the 1980s. Today, rotor blades have become much larger and, in relation to their sizes, lighter (indicated by the Lock number, the ratio of aerodynamic, and inertia forces). Both have an impact on teeter dynamics. This paper investigates the effects of teeter behavior over a wide range of turbine sizes using the analytical equations of the teeter movement combined with numerical simulations of the CART2 (600 kW), 2-B Energy's 2B6 (6 MW), and the 2B20HAW-T (20 MW, based on the INNWIND.eu turbine). It extends the existing teeter equations by the effects of pitch-teeter-velocity coupling and concludes that modern turbines with a high Lock number show a significant increase in aerodynamic damping, which leads to a different teeter behavior, making this rather old turbine concept worth considering in more detail.

1 | Introduction

A teeter-hub offers, when designed appropriately, considerable fatigue and extreme load reductions for two-bladed turbines [1–6], leading to a potential reduction in the cost of energy [5]. Nevertheless, it is a turbine concept that is nowadays only rarely discussed and almost impossible to find in the real world as a built turbine. This was different during the beginning of industrial wind energy, when many of the research turbines of the 1980s were two-bladed teetered turbines [7, 8]. However, the weight and thus inertia of today's (large) wind turbine blades have increased less than the aerodynamic forces. The

corresponding ratio, the so-called Lock number, consequently has risen significantly during the past decades, as shown herein.

It makes re-evaluating teetered turbines quite important today, since the Lock number has a large impact on teeter behavior.

Furthermore, today's modern control strategies offer new possibilities for handling teeter dynamics.

Several of the two-bladed research wind turbines in the 1980s (e.g., NASA MOD-0 [9], LS-1 [10], MOD-2 [11], WTS-3 and 4 [12, 13], MOD-5B [14], and Growian [15, 16]) used a teeter

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hinge, and most of the work on the benefits of teetering is from this time. The basic equations of the teeter movement have been introduced by Garrad et al. [1] and Anderson et al. [2], with little additions since then. Their main focus has been on the analytical equation of teeter mechanics and expected teeter angles, along with the potential of teetering for fatigue load reduction.

The effects of pitch-teeter coupling (PTC) for the reduction of teeter excursions have been investigated by Hohenemser et al. [17] and Henderson et al. [3]. Larsen et al. [18] numerically evaluated the influence of PTC and pitch-teeter-velocity coupling (PTVC) on teeter angles and fatigue loads, with only the former being reduced.

Although the reduction of fatigue loads has mostly been seen as the main benefit of a teeter hinge, extreme loads coming from teeter end impacts have been regarded as a major challenge, which may make the effects of reduced extreme loads obsolete [7, 12, 19, 20]. Considering extreme loads, Schorbach et al. [6, 21] have shown that extreme loads coming from large teeter movements may be reduced significantly when the teeter parameters are designed accordingly.

However, common to almost all of these studies is that the effects of the teeter behavior of turbines of different sizes have not been compared with each other. Today's historical trends for blade mass upscaling, which significantly undercut theoretical scaling laws [22] and change aero-elastic dynamics, have never been investigated with respect to teeter motions.

The primary focus of this work is thus to apply historical trends to the analytical equation of teeter mechanics, validate the analytics numerically, and evaluate the effects of rotor size on teeter angles in extreme load cases—the weak point of teeter-hub systems. In this process, a new extension of the analytical teeter equation for PTVC is developed, and its considerable impact on teeter angles is showcased. The comparisons include three wind turbine models ranging from 600 kW to 20 MW to assess the behavior of the teeter motion between small, medium, and large rotors.

First, the analytical background of the teeter motion is described and extended, as this will play an important role in later chapters. Subsequently, the teeter behavior for different sizes of two-bladed wind turbines will be described analytically, followed by an evaluation of simulations from turbines in three different sizes to support the theory numerically. This includes examining the simulated teeter angles in steady and extreme load conditions to analyze the effect of increased rotor sizes on the teeter motion. Finally, the impact and rotor size dependency of two versions of active PTCs will be discussed.

1.1 | Analytical Background of the Teeter Motion

During normal power production, the rotor teeters back and forth once per revolution. A person standing at the side of the turbine would observe the teeter angle (ζ , in the rotating rotor x - z plane in Figure 1) as a constant angle that is not changing in the frame of reference outside the turbine. Suppose an observer

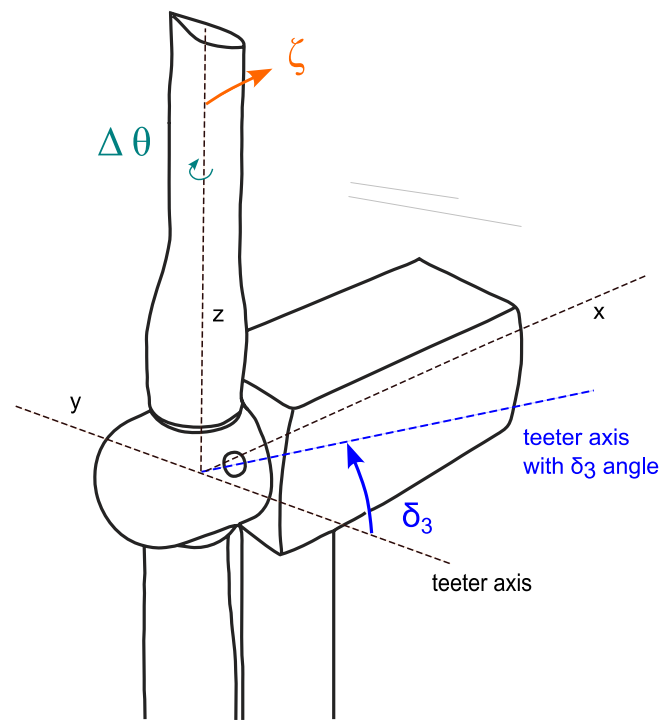


FIGURE 1 | Sketch of a two-bladed teetering-hub with teeter angle ζ , pitch angle difference, $\Delta\theta$, by PTC or PTVC and the optional δ_3 -angle, based on [23].

would be sitting on the blade, he or she would notice the teeter motion once per revolution.

The equation of teeter motion has been written by Henderson et al. [3] as follows:

$$J\ddot{\zeta} + C_a\dot{\zeta} + (K_{cf} + K_\theta)\zeta = M(t) \quad (1)$$

$M(t)$ is the teeter disturbing moment leading to the teeter excursions. Under normal operation, this is wind shear, and any kind of unequal wind conditions on the rotor blades. J is the mass moment of inertia around the teeter axis. C_a is the aerodynamic damping, defined in Equation (7). K_{cf} is the spring rate coming from centrifugal stiffening during the rotation of the rotor, shown in Equation (2). K_θ is an aerodynamic stiffening, presented in Equation (8), which can be achieved by PTC [2].

The moment of the centrifugal force is an important teeter restoring moment. This is also the reason why a teetered turbine needs a teeter brake, as centrifugal stiffening, aerodynamic damping, and aerodynamic stiffening drop with loss of rotor speed [1, 3, 16, 24]. Centrifugal stiffening is dependent on rotor mass moment of inertia and rotor speed:

$$K_{cf} = J\Omega^2 \quad (2)$$

Deriving the undamped natural teeter frequency ω_n from the teeter equation of motion (1) and substituting K_{cf} by Equation (2) leads to

$$\omega_n = \sqrt{\Omega^2 + K_\theta/J} \quad (3)$$

TABLE 1 | Lock numbers of different turbines sorted by year. The status of the turbine indicates whether it is a research turbine (*r*) or a commercial turbine (*c*). (*c**) marks a commercial turbine that is in operation without a teeter system. A turbine that has not been built so far is a theoretical turbine (*t*).

Turbine	Year	Status	Rotor θ	Lock number γ	Data sources
MOD-0	1975	<i>r</i>	39 m	7.0	[9, 27]
Growian	1981	<i>r</i>	100 m	9.9	[15, 16, 28]
WTS-4	1982	<i>r</i>	78 m	11.0	[12, 29, 30]
CART2 (Westinghouse0600)	1985	<i>c</i>	42 m	10.0	[21, 31]
MOD-5B	1987	<i>c</i>	100 m	10.4	[14]
Windflow 500	2003	<i>c</i>	33 m	16.0	[32]
2-B Energy 2B6	2016	<i>c</i> *	141 m	19.0	2-B Energy
2B20HAW-T	2021	<i>t</i>	257 m	30.8	[33]

Equation (3) shows that natural teeter frequency and rotor frequency are the same if there is no PTC ($K_\theta = 0$). PTC connects the teeter angle with the pitch angle in order to reduce the teeter movement. This is done by a factor, x_{PTC} , which is called the PTC coefficient [2]. It is defined by the ratio of the pitch angle difference $\Delta\theta$ and the teeter angle ζ :

$$x_{PTC} = \Delta\theta / \zeta \quad (4)$$

PTC reduces the mean teeter angle [3]. It can be implemented either mechanically or via the controller. Former teetered turbines have used mechanical methods for PTC. These are either a mechanical linkage between pitch and teeter axis or the use of a δ_3 -angle, which is an inclination of the teeter axis leading to a linear change in pitch angle depending on teeter angle [1, 3, 8, 16]. x_{PTC} is a fixed value for both methods. Implementing PTC via the controller gives the opportunity to change x_{PTC} when necessary. For example, x_{PTC} could be increased for higher wind speeds or larger teeter angles.

An additional opportunity is to take the teeter velocity instead of the teeter angle as a control input. The signal is similar to the teeter angle but with a valuable positive phase shift that enables an earlier reaction to teeter movement and, with this, a damping of the teeter motion instead of a spring-like reduction. It has been utilized by Larsen et al. [18] with the result that teeter angles can be reduced even better compared with classical PTC. The method will, herein, be called PTVC with a PTVC coefficient x_{PTVC} described by

$$x_{PTVC} = \Delta\theta / \dot{\zeta} \quad (5)$$

Despite the advantages of PTC and PTVC, there have indeed been some teetered turbines without PTC [23]. This is rather disadvantageous but nonetheless possible as aerodynamic damping (Equation (7)) is also an important teeter restoring force [3].

When characterizing (two-bladed) wind turbines, there is a very important dimensionless number that describes the ratio of aerodynamic forces to inertia forces. It is called the Lock number [25] and has formerly been used for helicopter aerodynamics.

$$\gamma = \frac{2\rho R^4 C_{la} c}{J} \quad (6)$$

The Lock number depends on air density ρ , rotor radius R , blade chord c , mass moment of inertia around the teeter axis J , and the linear slope of the lift curve C_{la} . The calculation of the Lock number in this paper refers to Eggelston et al. [26], who used the blade chord at 75% blade length, and a value of 2π for the slope of the lift curve C_{la} . Table 1 shows Lock numbers for various turbines.

The aerodynamic damping C_a and stiffness K_θ from the equation of teeter motion (1) can be expressed in a simplified way when assuming a constant blade chord c and a constant slope of the lift curve C_{la} ($\Delta C_l / \Delta\alpha$) [3, 23]. Then

$$C_a = \frac{1}{4} \rho \Omega C_{la} c R^4 \quad (7)$$

$$K_\theta = \frac{1}{4} \rho \Omega^2 C_{la} c R^4 x_{PTC} = \frac{1}{4} \rho \Omega^2 C_{la} c R^4 \Delta\theta / \zeta \quad (8)$$

Equations (7) and (8) do both contain the slope of the aerodynamic lift curve, defined by the change of lift coefficient ΔC_l due to a change in angle of attack $\Delta\alpha$, in the term C_{la} . The change in the angle of attack $\Delta\alpha$ in Equation (8) is caused by the change in pitch angle $\Delta\theta$ from PTC in Equation (4).

Similarly, the change in pitch angle $\Delta\theta$ can also be connected to the teeter velocity coupling from PTVC in Equation (5):

$$C_\theta = \frac{1}{4} \rho \Omega^2 C_{la} c R^4 x_{PTVC} = \frac{1}{4} \rho \Omega^2 C_{la} c R^4 \Delta\theta / \dot{\zeta} \quad (9)$$

Consequently, the teeter equation of motion (1) can be rewritten to include PTVC, which can be described as an additional aerodynamic damping:

$$J\ddot{\zeta} + (C_a + C_\theta)\dot{\zeta} + (K_\theta + K_\theta)\zeta = M(t) \quad (10)$$

When comparing the teetering behavior of different turbines, one important parameter is the damping factor ξ . This damping can be derived from the equation of teeter motion (10) by applying the dynamic characteristics of a mass-spring-damper system [4] and substituting the natural frequency ω_n by Equation (3):

$$\xi = \frac{C_a + C_\theta}{2J\sqrt{\Omega^2 + K_\theta/J}} \quad (11)$$

Inserting the simplified aerodynamic damping coefficient C_a of Equation (7), as well as the pitch coupled aerodynamic stiffening and damping, K_θ and C_θ , of Equations (8) and (9), and substituting variables by the Lock number γ of Equation (6) leads to

$$\xi = \frac{\gamma(1 + \Omega x_{PTVC})}{16\sqrt{1 + \frac{1}{8}\gamma x_{PTC}}} \quad (12)$$

As described by Anderson et al. [2] and Equations (12) and (8), PTC actually reduces the damping of the teeter motion, even though it acts like an aerodynamic stiffness. The PTVC, in turn, acts as an additional damping but does not increase the stiffness of the teeter motion, as evident from Equations (12) and (10). Note that Equation (12) does not take the time lag of a controller or actuator into account. Nonetheless, a certain amount of teeter damping could be added by PTVC, whereas PTC enlarges stiffness but reduces damping.

For the case without pitch couplings when $x_{PTVC} = 0$ and $x_{PTC} = 0$, Equation (12) results in

$$\xi_0 = \frac{1}{16}\gamma \quad (13)$$

This underlines the impact of the Lock number on aerodynamic damping. If the Lock number of a turbine is larger than 16, the teeter movement will be overdamped. Looking at the Lock numbers of Table 1, it can be seen that most teetered turbines so far have been underdamped.

The mean teeter amplitude ζ^* is dependent on aerodynamic damping, the maximum teeter disturbing moment M_0 , and rotor speed Ω . According to Garrad [1], it is independent of the rotor mass moment of inertia and can, without PTC, be expressed as

$$\zeta_0^* = \frac{M_0}{C_a\Omega} \quad (14)$$

The mean teeter amplitude with PTC, ζ_1^* , and the mean teeter amplitude without PTC, ζ_0^* , have the following relation [3]:

$$\frac{\zeta_1^*}{\zeta_0^*} = \frac{1}{\sqrt{1 + x_{PTC}^2}} \quad (15)$$

Taking the full relation of mean teeter angle from [4]

$$\zeta^* = \frac{M_0}{J\omega_n^2\sqrt{\left(1 - \left(\frac{\Omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\Omega/\omega_n\right)^2}} \quad (16)$$

and substituting the undamped natural frequency ω_n by Equation (3) and the damping ξ by Equation (12), which includes PTVC, results in an extended new relation for ζ^* , the mean teeter angle including PTC and PTVC:

$$\frac{\zeta^*}{\zeta_0^*} = \frac{1}{\sqrt{(1 + \Omega x_{PTVC})^2 + x_{PTC}^2}} \quad (17)$$

As evident from Equation (17), increasing the PTC or PTVC will reduce the mean teeter angle. Likewise to the mean teeter angle ζ_0^* without PTC or PTVC, the effect of pitch-teeter angle and PTVC on ζ^* is also independent of rotor inertia and the Lock number, and thus independent of rotor size.

The phase lag ϑ of the teeter motion describes the angular shift between the rotor position of the teeter excitation force and the rotor position where the corresponding reaction of the teeter excursion occurs. Taking the phase lag ϑ definition from [4], and introducing the new equation of teeter motion (10), results in

$$\cot \vartheta = \frac{\omega_n^2 - \Omega^2}{2\xi\omega_n\Omega} = \frac{x_{PTC}}{1 + \Omega x_{PTVC}} \quad (18)$$

Consequently, a PTC will change the phase shift of the teeter movement, while avoiding PTC will result in a fixed phase shift of $\vartheta = \arccot 0 = 90^\circ$. This underlines that the maximum teeter angle at constant wind shear occurs, without couplings, at a 90° shift with respect to the teeter excitation when the rotor is horizontal [18].

A PTVC reduces the deviation of the phase shift away from 90° if PTC is present. Furthermore, also the phase shift is independent of the rotor size if the PTVC coefficient x_{PTVC} is chosen antiproportional to the rotor speed:

$$x_{PTVC} \propto \frac{1}{\Omega} \quad (19)$$

2 | Materials and Methods

Based on the equations of the general teeter dynamics, the scaling behavior of an increasing rotor size and its impact on teeter characteristics will be analyzed. A focus is put on the effect of wind shear, aerodynamic teeter damping, and two disparate cases of extreme load conditions leading to large teeter excursions.

2.1 | The Two-Bladed Teetered Turbines

The turbines used for this study are the CART2 [31], the 2B6 of 2-B Energy, which has been modified for this research project to accommodate a teeter system, and the turbine of the research project X-Energy, the 2B20HAW-T [33–36], which is based on the INNWIND 20MW turbine. Those turbines ensure a wide range of different rotor diameters, starting with 42m of the 600kW CART2, to 141m of the 6MW 2B6, and going to 257m of the 20MW two-bladed 2B20HAW-T turbine.

All numerical models are implemented in the aero-servo-elastic blade element momentum wind turbine simulations tool, Bladed 4.14, from DNV [37], modeled with multiple degrees of freedom to account for complex motions of bending and torsion. The

blade modeling includes a multipart blade definition with separated modal reductions along the blade to account for nonlinearities of more flexible large wind turbine blades [38]. The model of the two-bladed 20 MW turbine is made open source [33]. The utilized C++ control algorithms are very basic and rely on the subsequently described functionalities and equations. The pitch-actuator delays are chosen relative to the decrease in rated rotation speed. The controller time lag is set to 0.01 s, equal to the simulation time step.

2.2 | Teeter Angle and Wind Shear

According to Henderson et al. [3], the mean teeter angle can be estimated with a linearized wind shear k with the wind speed at the upper blade tip u_{up} , the wind speed at the bottom blade tip u_{bottom} , and the wind speed at hub height u_{hub} :

$$k = \frac{u_{up} - u_{bottom}}{2u_{hub}} \quad (20)$$

The mean teeter angle depending on wind shear and mean wind speed $\bar{u}$ can then be calculated with [4]:

$$\zeta_0^* = \frac{\bar{u}_{hub} k}{\Omega R} \quad (21)$$

So, the combination of wind shear and turbine size is an additional effect having an impact on teeter angles. As shown in Equation (21), the slope of the linearized wind shear has an impact on the mean teeter angle. It is, thus, highly dependent on the wind shear calculation method. A method to account for an “equal turbine location” is the logarithmic calculation by ground roughness (z_0) and respective height (h) [4]:

$$\bar{u}(h) = \bar{u}_{ref} \frac{\ln(h/z_0)}{\ln(h_{ref}/z_0)} \quad (22)$$

Inserting Equations (20) and (22) in (21) yields

$$\zeta_0^* = \bar{u}_{hub} \frac{\ln((h_{hub} + R)/z_0) - \ln((h_{hub} - R)/z_0)}{2 \ln(h_{hub}/z_0) \Omega R} \quad (23)$$

Figure 2 shows the wind shear for a wind speed of 10 and 20 m/s at 100 m and a roughness length of 0.001 m together with the dimensions of this study's largest and smallest turbine. Therein, the CART2 possesses a −13% lower wind speed at its hub height (u_{hub}), and a −26% lower linearized wind shear (k) than the 2B20HAW-T at the same place with the same roughness height $z = 0.001$. Further, the wind shear exponent (α) of the CART2 is +12% higher to match the given wind speeds at the upward and downward pointing blade tip by the wind shear power law [4], where $\bar{u}(h) \propto h^\alpha$. Larger roughness heights would increase these differences further.

2.3 | Scaling Behavior of Teetering

As a first step, it will be shown how the variables of the complete teeter motion of Equation (10) scale with rotor radius.

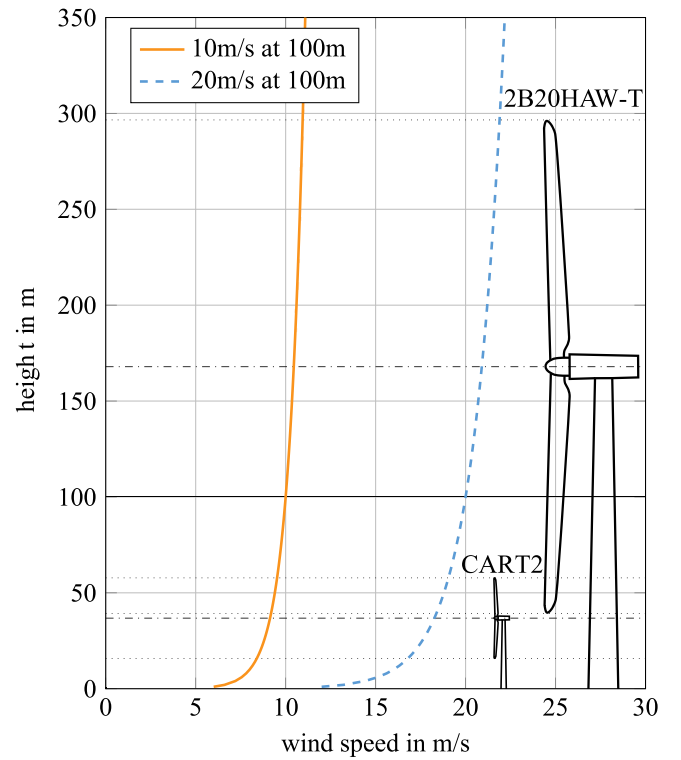


FIGURE 2 | CART2 and 2B20HAW-T dimensions in a logarithmic shear wind profile (roughness length $z_0 = 0.001$ and reference height $h_{ref} = 100$ m).

Using general scaling relations [39], the rotational speed is antiproportional to the rotor radius, and it is simplified that the blade chord is proportional to the rotor radius, neglecting that the tip speed ratio may vary on different turbines:

$$\Omega \propto R^{-1} \quad (24)$$

$$c \propto R \quad (25)$$

The teeter angle can be expressed as a sine function with the mean teeter angle ζ^* :

$$\zeta = \zeta^* \sin(\Omega t) \quad (26)$$

Teeter velocity is the first derivative and proportional to R^{-1}

$$\dot{\zeta} = \zeta^* \Omega \cos(\Omega t) \propto \Omega \propto R^{-1} \quad (27)$$

Taking the second derivative from Equation (27) for teeter acceleration results in

$$\ddot{\zeta} = -\zeta^* \Omega^2 \sin(\Omega t) \propto \Omega^2 \propto R^{-2} \quad (28)$$

The following relations can be obtained by applying the proportions of Equations (24), (25), and (27) to the aerodynamic damping (7), PTC stiffness (8), and PTVC damping (9):

$$C_a \propto R^4 \quad (29)$$

$$K_\theta \propto R^3 \quad (30)$$

$$C_\theta \propto R^4 \quad (31)$$

The PTC stiffness K_θ is proportional to R^3 instead of R^4 due to the division by the teeter velocity $\dot{\zeta}$ in Equation (8) and the relation of Equation (27).

Finally, the scaling of the mass moment of inertia around the teeter axis must be put in relation to the rotor radius. Taking the common scaling rules, e.g., from [39], the blade weight scales with the cube of rotor radius R^3 . Taking a simplified equation of the rotor as a rotating beam

$$J = \frac{1}{12} mL^2 \quad (32)$$

leads to the relation:

$$J \propto R^5 \quad (33)$$

Combining all the relations from above into Equation (10) shows that all governing parameters of the teeter equation scale equally with rotor radius:

$$\underbrace{J\ddot{\zeta}}_{\propto R^5} + \underbrace{C_a\dot{\zeta}}_{\propto R^3} + \underbrace{C_\theta\dot{\zeta}}_{\propto R^3} + \underbrace{K_{qf}\zeta}_{\propto R^{2.3}} + \underbrace{K_\theta\zeta}_{\propto R^3} = M(t) \quad (34)$$

According to these scaling laws, the damping ratio of the teeter motion in Equation (12) would be independent of turbine size, and the Lock number would be constant. However, it has been shown that especially the masses of the blades scale well below R^3 when evaluating historic turbine data. Ashuri et al. [40] and Jamieson [22] mention a data-based blade mass upscaling trend of $R^{2.09}$ and $R^{2.06}$, respectively. Sieros et al. [41] and Jensen and Banner [42] show a trend of $R^{2.5}$ and Veers et al. [43] a trend of $R^{2.3}$. Here, $R^{2.3}$ will be used as a compromise.

The tendency to lighter blades also entails an effect on the Lock numbers of former and today's turbines. Table 1 shows some examples. Larger Lock numbers indicate that blades are more dominated by aerodynamic forces and less by inertia forces.

Analyzing the respective relations shown in Equations (32) and (25) under the assumption of the reduced blade mass scaling trend of $R^{2.3}$, the Lock number γ from Equation (6), and with this, the aerodynamic damping ratio ξ_0 of Equation (13) are no longer independent of rotor size. Instead, the following relations are obtained:

$$J \propto R^{4.3} \quad (35)$$

$$\gamma \propto R^{0.7} \quad (36)$$

$$\xi_0 \propto R^{0.7} \quad (37)$$

So when comparing wind turbines of different sizes, the dimensionless Lock number increases with rotor radius. Based on Equation (35), the former relations for the teeter motion in Equation (34) are altered to

$$\underbrace{J\ddot{\zeta}}_{\propto R^{2.3}} + \underbrace{C_a\dot{\zeta}}_{\propto R^3} + \underbrace{C_\theta\dot{\zeta}}_{\propto R^3} + \underbrace{K_{qf}\zeta}_{\propto R^{2.3}} + \underbrace{K_\theta\zeta}_{\propto R^3} = M(t) \quad (38)$$

Equations (36), (37), and (38) indicate that aerodynamic damping and pitch-teeter angle and velocity coupling become more dominating with a larger rotor size. The numerical evaluation of this effect is done in Section 3. The analytical Lock number and damping ratio of the simulated turbines are listed in Table 2.

2.4 | Extreme Event Clusters

Looking at teeter dynamics, it must be distinguished between normal operation and extreme loads. The teeter movement must be limited to a certain maximum teeter angle in order to prevent the rotor from hitting the tower. Former studies on rotor diameters up to 100m have shown that this should be done with a teeter stop consisting of a sufficiently large spring stiffness [23]. The design of the teeter stop is essential, as it has been shown that a bad design of the teeter stop can lead to much higher extreme loads.

Nonetheless, a reduction in teeter angle during extreme conditions may alleviate the necessity of a teeter stop. It could be achieved through the influence of the characteristics analyzed subsequently, namely the increased teeter damping from a continuously increasing Lock number, or an effective application of PTC and PTVC.

Situations that can lead to extreme teeter movement contain all kinds of unequal aerodynamic blade loading. Severe load cases are, e.g., an extreme coherent gust with direction change (ECD) or a stuck pitch. Turbine stops, starts, or idling without a teeter brake can also lead to massive teeter movement as the restoring teeter moments drop with decreasing rotor speed. This is not regarded here as many former studies have shown that a teeter brake is mandatory for every teetered turbine [1, 3, 16, 24].

All situations during rated rotor speed leading to extreme teeter movements can be distinguished into two cases [6] as shown in Figure 3:

- **Case 1.** The teeter angle (and the rotor) is still moving from one side to the other during the extreme event, which

TABLE 2 | Rotor diameter, Lock number, and damping ratio of different two-bladed wind turbines used for this study.

Turbine	Rotor ϕ	Hub height	Lock number γ	Damping ratio ξ_0
CART2	42.0 m	36.8 m	10.0	0.6
2B6	141 m	105.0 m	19.0	1.2
2B20HAW-T	257.0 m	167.9 m	30.8	1.9

accounts for most situations. Then, the large teeter movement is caused by external wind conditions, which impose an unequal blade loading to the rotor.

- **Case 2.** The rotor does not teeter anymore from one side to the other. Instead, the teeter angle remains at a large angle over several rotor rotations. This is caused by a stuck pitch if the remaining pitch is used to stop the rotation of the turbine.

In both cases, there is an unequal blade loading. In Case 1, it is caused by external wind conditions, and in Case 2, it is caused by a pitch fault. It must be noted that Case 2 could be avoided if the other well-functioning pitch actuator remains at the angle of the stuck pitch, whereas the yaw drive is used as a redundant secondary speed control by turning the entire turbine out of the wind to stop the rotation. Utilizing the yaw system during the extreme event of a stuck pitch may introduce higher loads on other components, which should ideally not exceed the ultimate loads from the other extreme load cases, to render this control strategy a reasonable solution. Furthermore, a yaw fault would lead to Case 1, as it has the same effect as a change in wind direction.

The main difference between both cases is that aerodynamic damping still plays an important role in Case 1, as the rotor is still

making the back-and-forth teeter movement. In Case 2, the rotor remains at an almost constant teeter angle so that the teeter velocity is nearly zero. Aerodynamic damping of the teeter movement vanishes as there is hardly any teeter velocity. If it is additionally the case that the rotor has a large Lock number, it will react even more sensitively to unequal blade loading, leading to larger teeter movements [6]. As mentioned, Case 2 could be avoided but is used here to show the effects of different rotor sizes.

3 | Results

3.1 | Influence of Wind Shear and Turbine Size on Teeter Angles

The linearized wind shear gradient of Equation (20) rises with larger turbine sizes (see Figure 2), which consequently increases the mean teeter angle according to Equation (21). Figure 4 shows the effect of rising teeter angles with rotor size for different rotor radii and wind speeds. The hub height is assumed to be rotor radius +39 m, which is the case for the 2B20HAW-T. The roughness length is 0.001 m.

Table 3 shows the values for the analytical and the numerical teeter angle ζ^* , according to Equation (23) and respective numerical

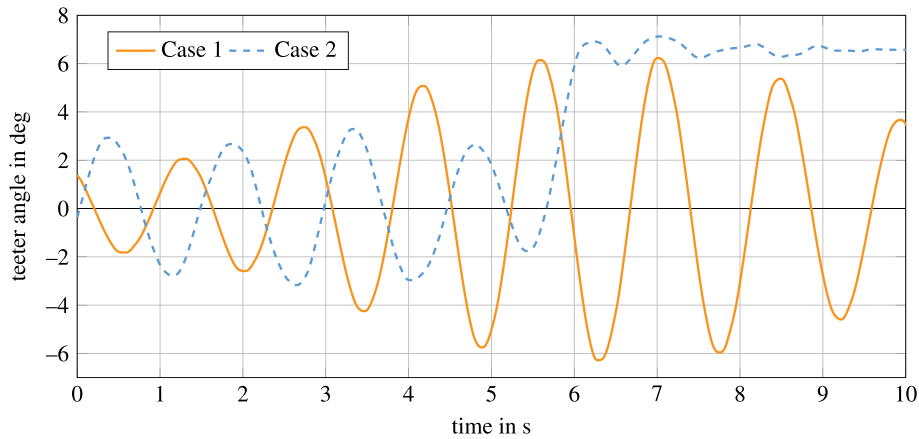


FIGURE 3 | Examples for different cases of teeter behavior on the CART2 based on [23]: Case 1 is caused by extreme wind shear, Case 2 by a stuck pitch.

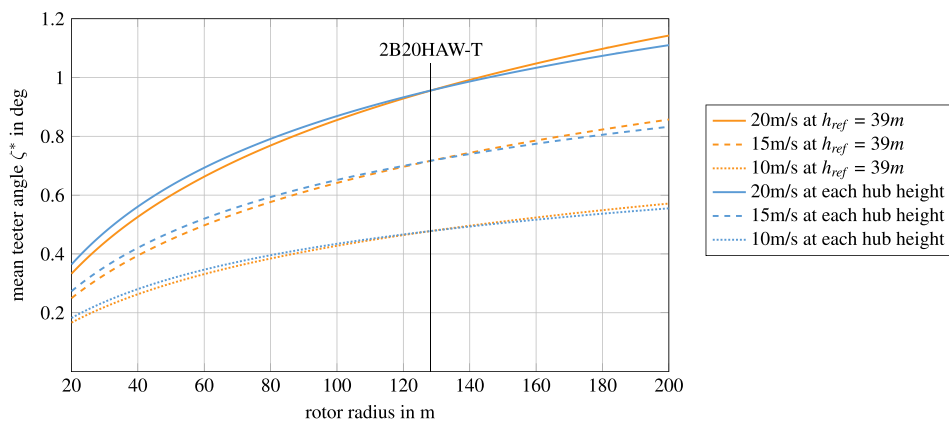


FIGURE 4 | Analytically obtained mean teeter angle according to Equation (23) depending on rotor radius for three different wind speeds at a constant reference height of 168 m and, for comparison, at a hub height that consists of the rotor radius +39 m. The roughness length is $z_0 = 0.001$ m.

simulations, in order to verify the analytical results numerically. The CART2 and the 2B20HAW-T, described in Section 2.1, are simulated during a steady wind of 15 m/s at hub height and roughness length of 0.001 m. As expected by the analytical results above, the mean teeter angle is larger for the 2B20HAW-T. The difference between the analytical and the mean numerical teeter amplitude in Table 3 may mostly be attributed to the significant increase in modeling complexity, e.g., the reduction of the actual blade shape to a simple box shape with continuous characteristics, or the wind shear linearization, or the absence of aerodynamic losses and blade flexibilities, and associated aerodynamic damping. Gravitational effects are neglected in these simulations because Equation (21) does not include a rotor mass center of gravity in front of or behind the teeter hinge. If gravity had been present, the blade prebend and rotor cone of the upwind-designed 2B20HAW-T would have resulted in a forward tilting moment when the rotor is vertical. At the same time, the wind shear causes a backward tilting moment. These opposed moments would have reduced the constant shear-based teeter angle for the 2B20HAW-T. For the downwind-designed 2B6, gravity would have caused the opposite effect.

To present the influence of the turbine size on teeter motion in more realistic conditions, including gravity, Table 4 shows the results from normal operation simulations with a wind speed of 15 m/s, an IEC 61400-1 class IC longitudinal turbulence intensity of 13.5% and a wind shear exponent α of 0.14. All three turbines have been simulated with gravity and with the same Kaimal turbulence wind field file centered on the respective hub height. No active teeter angle reduction methods are applied. The mean teeter angle of the 2B20HAW-T is again the largest, whereas the teeter angle of the 2B6 fits right in between. Having a larger mean teeter angle, it is obvious that the standard deviation (std) is larger as well. However, if the standard deviation is compared with the mean teeter angle, it can be observed that its share gets smaller for the larger turbines. The last column in

TABLE 3 | Wind gradient k , corresponding wind shear exponent α and analytical, according to Equation (23), and numerical teeter angles for a constant wind speed of 15 m/s at hub height with a roughness length of $z_0 = 0.001$ m.

Turbine	k	α	ζ^* analytical in deg	ζ^* simulation in deg
CART2	0.062	0.097	0.59	0.54
2B20HAW-T	0.084	0.086	0.72	0.64

TABLE 4 | Teeter statistics of simulations with 15 m/s at hub height, wind shear exponent of 0.14, and longitudinal turbulence intensity of 13.5%.

Turbine	ζ^* turb. in deg	Std	Rel. std
CART2	1.42	0.74	0.52
B6	1.56	0.78	0.50
B20HAW-T	1.64	0.81	0.49

Table 4 shows the relative standard deviation (rel. std): mean teeter angle divided by standard deviation. The results of the table indicate that the teeter activity of the 2B20HAW-T and the 2B6 are lower compared with the CART2. This is a sign of a larger increase in aerodynamic teeter damping compared with the increase of teeter inertia as indicated by Equations (37) and (38).

3.2 | Simulation Settings for All the Following Analyses

Subsequently, all turbines have been simulated with the following settings for better comparability:

- no tilt
- no gravity
- no constant wind shear
- no tower shadow
- no turbulence
- no teeter resistance or brake
- no constant pitch error or mass unbalance

These settings lead to a very artificial situation that no turbine would ever experience. If simulated more realistically, the application of tilt, gravity, and wind shear, described in the previous Section 3.1, would superimpose a sinusoidal teeter motion with a relatively constant amplitude to the plots in the following analyses. The tower shadow would influence the teetering motion similarly, but with a different shape of repetitive teeter motion. Wind turbulence would add stochastic changes in teeter motion. Its influence is indicated by the standard deviations in Table 4. A mass imbalance or a general pitch error, causing an aerodynamic imbalance, would overlay a constant teeter angle, which may be trivial to detect and avoid by retuning the constant pitch angles. The teeter resistance usually changes the teeter motion only for larger teeter excursions, and the brake only if it is engaged.

Omitting these settings improves the understanding of how significant the aerodynamic damping is and how it changes with the turbine size. It also enhances the comparability of the results for the very different turbine designs, including their upwind and downwind configurations. Consequently, these settings are disabled in all subsequent simulations to improve the analyses of cause and effect.

3.3 | Damping of the Teeter Movement

Figure 5 shows the influence of aerodynamic damping of the three teetered turbines. All turbines have been brought into constant teetering with a similar maximum teeter angle by a constant wind shear. Then, the wind shear has been switched off instantly in order to observe how long it takes until teetering is stopped by aerodynamic damping.

Looking at Figure 5, it can first be noted that aerodynamic damping has a massive impact on teeter movement. Even for the CART2, which has the lowest Lock number of the simulated turbines, teeter

movement is stopped after the first 1.5 rotations. The 2B6 and the 2B20HAW-T are so heavily damped that the teeter angle does not overshoot and stays on one side instead. Those two turbines show supercritical damping, which is quite significant, keeping in mind that it is of the highest priority to watch out that teeter angles do not get too large because they can lead to additional loads.

In most conditions, the higher aerodynamic damping due to a higher Lock number is very beneficial to reduce the teeter angles and dynamics. Nonetheless, there might be possible conditions where an instant push from the wind, e.g., an extreme gust or wind shear, causes larger teeter angles. So the next section takes a deeper look at extreme load cases.

3.4 | Extreme Load Cases

Figure 6 shows the effect of an extreme wind shear (EWS). This situation belongs to teeter Case 1 of Figure 3. So there is an unequal blade loading, leading to large sinusoidal fluctuating teeter angles. The simulations of Figure 6 are based on the extreme load case DLC 1.5, according to IEC 61400-1 for a duration of 12s, but with the artificial turbine conditions of the previous subsection 3.2

for better comparability. All teeter angles are normalized by the *global* maximum of the 2B20HAW-T. The simulations are performed for a variety of initial rotor azimuth angles to choose the time series with each turbine's worst-case teeter angle for the diagram. It can be observed that the 2B20HAW-T has the fastest raising teeter angle. In contrast to normal operation, it is now the case that the impact of a larger Lock number results in a rapid re-action of the rotor, leading to larger teeter angles for the 2B6 and the 2B20HAW-T than for the CART2. Nonetheless, there is a huge difference in the time periods relative to the rotation speed and, thus, in the wind conditions experienced by the turbines.

For a better comparison, the duration of the EWS in Figure 7 is respectively enlarged from 10 to 19s for the 2B6 and from 10 to 66s for the 2B20HAW-T by a factor

$$f_{\Omega} = \Omega_{ref} / \Omega \quad (39)$$

with the rotation speed of CART2 as a reference Ω_{ref} so that all turbines have the same amount of rotor rotations during the EWS. The maximum wind shear amplitude is taken from the 2B6 simulation and not scaled for the other turbine sizes. For this EWS with equal shear amplitudes at the blade tips, the maximum teeter

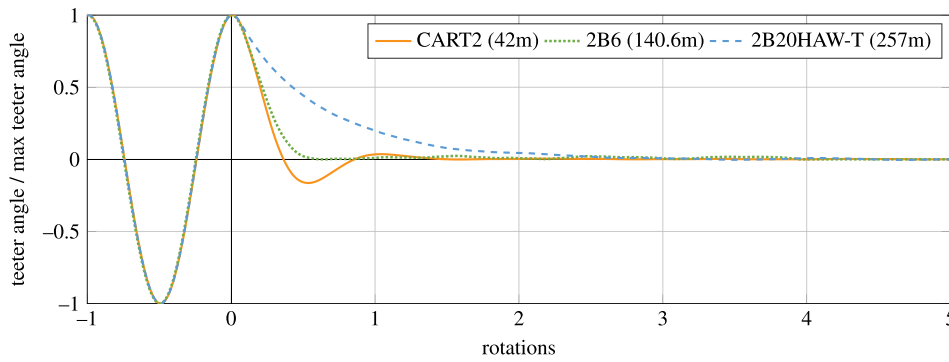


FIGURE 5 | Impact of aerodynamic damping on the teeter movement with a wind shear that stops at $t=0$; the x-axis shows rotor rotations for an easier comparison of the three turbines.

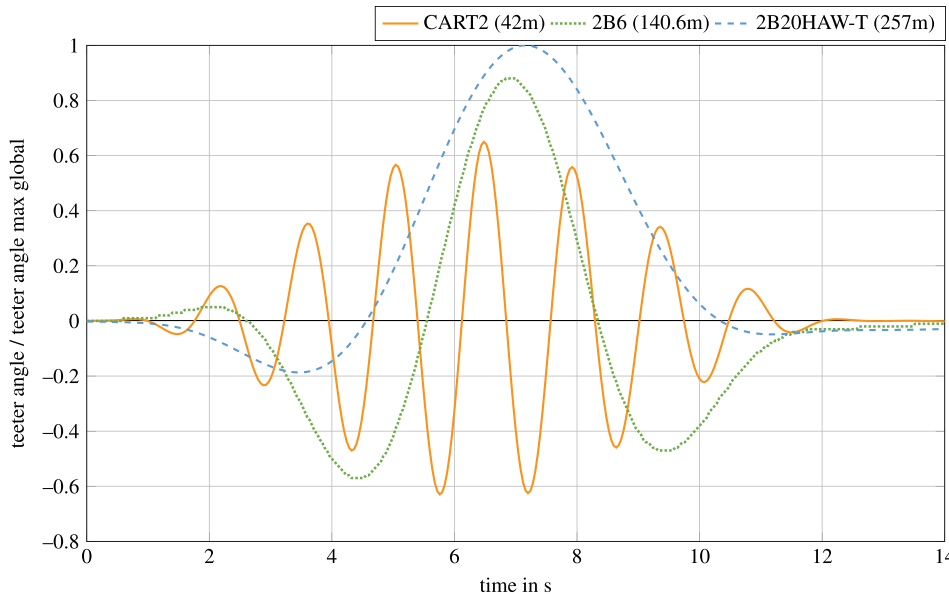


FIGURE 6 | Extreme wind shear DLC 1.5 for a duration of 12s with teeter angles normed by the global maximum of the 2B20HAW-T.

angles are almost equal for all turbines because the mean teeter angle is, according to Equation (21), only dependent on wind shear and independent of rotor mass or damping. Most importantly, it can now be seen that the teeter amplitude raises and decreases faster for the larger turbines, which have a higher Lock number. A higher Lock number not only causes a more instantaneous reaction of the rotor but also dampens the teeter movement once the excitation force (e.g., of the wind shear) is reduced.

Another severe load case for a teetered turbine is a stuck pitch [21]. There are two ways to handle this situation. One results in the teeter behavior of Case 2 from Section 2.4: The pitch angle of the broken pitch drive does not move anymore, and the healthy blade pitches out to feather, leading to an extreme pitch imbalance and severe unequal blade loading. The rotor will have a

more or less constant teeter angle after being stopped by a teeter end impact, which ensures that a certain maximum teeter angle is not exceeded. Teeter velocity $\dot{\zeta}$ is then almost zero, preventing teeter damping from having its positive effect. The simulated teeter angles for such a stuck pitch and one-sided pitch to feather but without teeter restraints or end impacts are shown in Figure 8. The pitch velocity is scaled for an equal pitch readjustment per rotor revolution by Equation (39). This is again a situation where it can be seen that a rotor with a large Lock number will react much faster on the unequal blade loading. Here, the consequence is an earlier and potentially more severe hit of the teeter end impact. It must be noted that pitching out in this situation is exactly what should not be done. The simulations of Figure 8 are only shown in order to have a comparison to the other option: yawing-out during this load case.

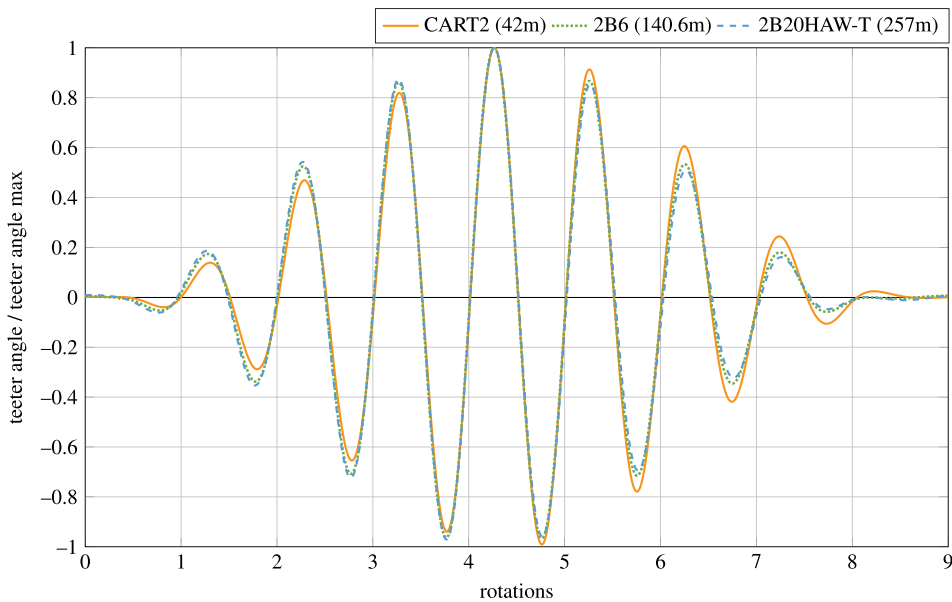


FIGURE 7 | Extreme wind shear DLC 1.5, with an extended duration for the 2B6 and 2B20HAW-T to compare rotor rotations and teeter angle.

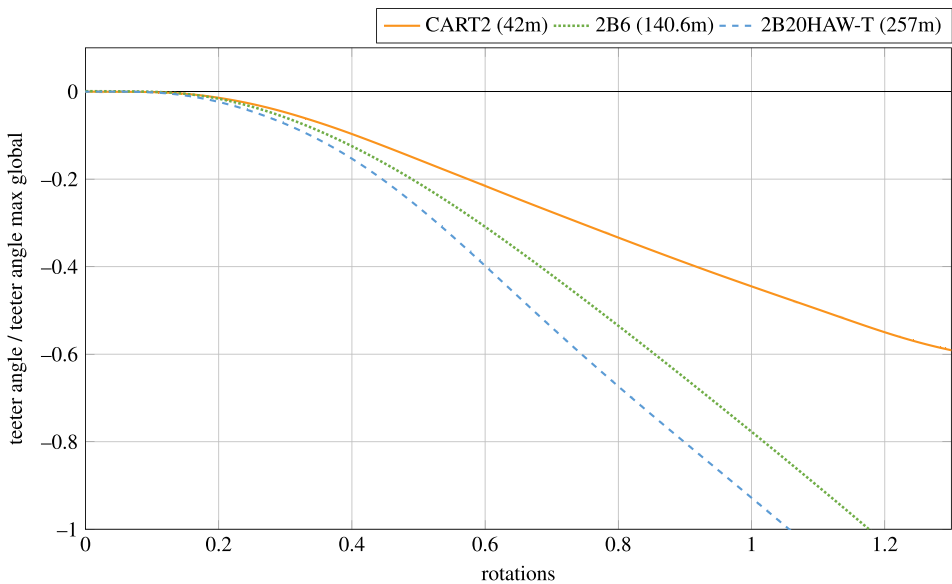


FIGURE 8 | Stuck pitch and turbine shut down at 15 m/s with a pitch rate of 2 deg/s for the 2B20HAW-T as a reference and scaled pitch rates proportionally to the rotor speeds.

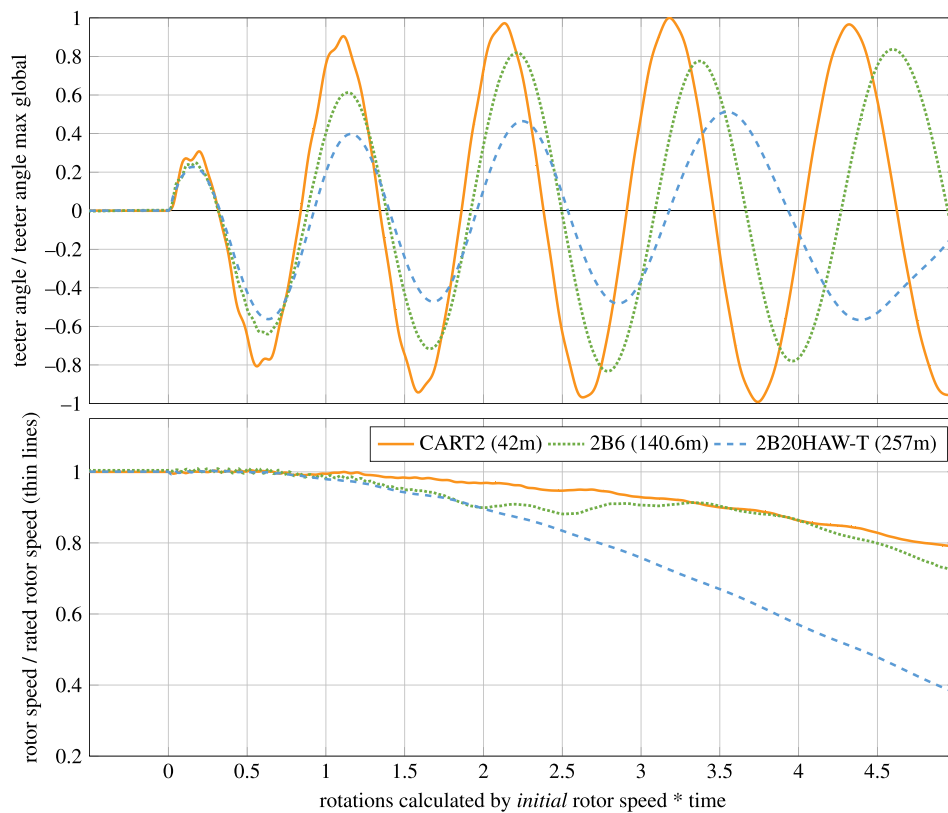


FIGURE 9 | Stuck pitch and turbine shut down at 15 m/s with a yaw rate of 2 deg/s for the 2B20HAW-T and scaled proportionally to the other rotation speeds by Equation (39); the x-axis is normed by the initial rotation speeds at zero.

As mentioned, another way to handle such a stuck pitch is to yaw the turbine out of the wind. Then, this load case behaves rather like Case 1 of Figure 3 because the teeter angle will not directly go into the end impact and instead keeps its teeter velocity. The result can be seen in Figure 9. For comparability, the yaw speed of the turbines is scaled proportionally to the respective initial undisturbed rotor speed by Equation (39). In this case, there is still aerodynamic damping, which rises with the Lock number. So here, teeter angles decrease and get less harmful for larger rotors. Additionally, the large aerodynamic damping of the 2B6 and the 2B20HAW-T leads to a faster drop in rotation speed per revolution, making this extreme event shutdown strategy more suitable for newer turbines with higher Lock numbers.

3.5 | Effects of PTC and PTVC

The above-presented teeter behaviors can be reduced by an active or passive adaption of the pitch angle, which is especially important for reducing extreme teeter angles during extreme load scenarios. Figure 10 shows the impact of PTC and PTVC for the 0.6 MW CART2 and the 20 MW 2B20HAW-T during the same EWS conditions as in Figure 7. For PTC, a coupling factor from Equation (4) of $x_{PTC} = 0.5$ is applied. For PTVC, the pitch coupling is proportional to the teeter velocity, which decreases with the rotor speed and is antiproportional to the turbine size. For better comparability of the respective effects on the resulting teeter angle, the velocity coupling coefficient x_{PTVC} of Equation (5) is tuned to result in an equal pitch activity as $x_{PTC} = 0.5$ in the shown load case, assessed by the accumulated

pitch adjustments. The resulting x_{PTVC} values differ very precisely by the rotation speed factor of Equation (39). Similarly, the reaction time lag of the pitch actuator is scaled by Equation (39) to enable a proportionally fast pitch activity. The unscaled electronic controller time lag of 0.01 s is generally very small compared with the teeter movement, but a phase shift from, e.g., a low-pass filter (not applied herein) would presumably also increase proportionally to the rotation speed.

Most evident in Figure 10 is the much higher effect of PTVC compared with PTC. Although using the same amount of blade pitching as PTVC, PTC reduces the maximum teeter angle in this load case only marginally. From a mechanical point of view, showcased earlier in Equations (9) and (10), PTVC acts as a damper as its input is the teeter velocity. It applies the required pitch difference at an earlier, thus better time than PTC. For classical PTC, the maximum pitch angle and, thus, aerodynamic force difference will occur at the time when the teeter angle is at maximum. There, it will push the teetered rotor back at a higher speed, compared with no coupling, until a teeter angle of zero is reached and consequently causes a higher teeter overshoot. Instead, a force against the overshoot of the teeter movement would be considerably more beneficial. This is how PTVC works. It already starts to slow down the teeter movement when the teeter angle is maximal and will have the maximum pitch difference and aerodynamic counter teeter force at a teeter angle of zero when the teeter velocity is at its maximum. The mechanism of the PTVC is similar to the higher damping effect of larger turbines with an increased Lock number, e.g., presented in Figure 5.

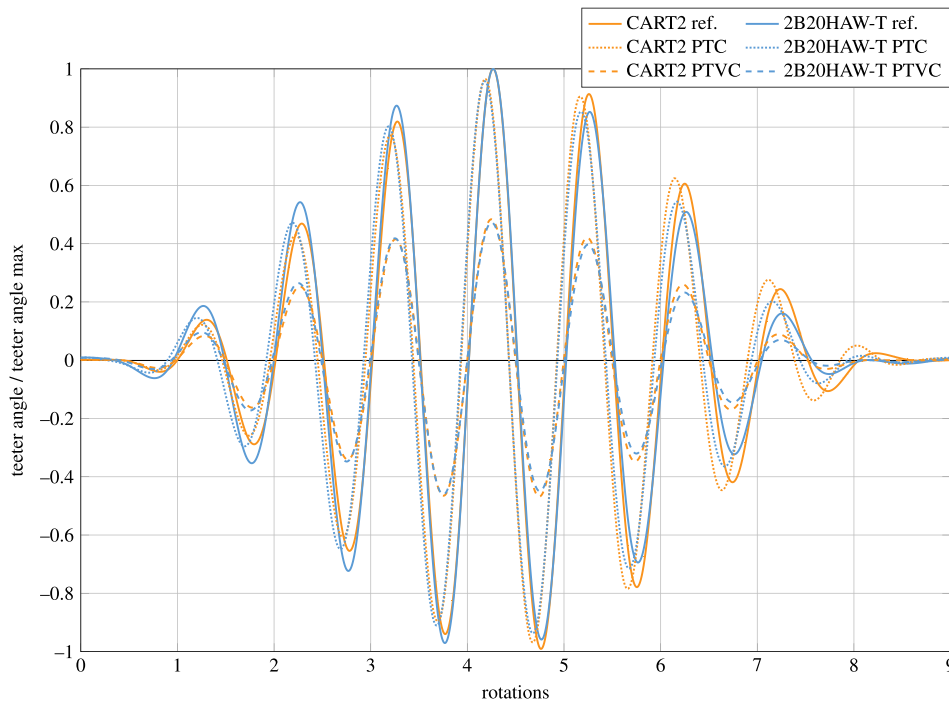


FIGURE 10 | Pitch-teeter coupling (PTC) and pitch-teeter-velocity coupling (PTVC) during an extreme wind shear for the CART2 and the 2B20HAW-T.

Furthermore, a high teeter velocity typically occurs before an extreme teeter angle, enabling the implementation of a trigger to detect extreme cases and act accordingly before the ultimate load or teeter angle takes place.

Another observation from Figure 10 is that the effect of PTC and PTVC is independent of rotor size. This also aligns well with Equation (17), which contains no size-dependent parameter for the effect of PTC or PTVC on the teeter angle reduction (if x_{PTVC} is antiproportional to the rotor speed Ω , as applied by utilizing Equation (39) in this example). The scaling-dependent effect of the larger Lock number is already present in the reference teeter movement without the couplings in Figure 7. Even though not included in the analytical approach of Equation (12), the size independence of the results may also indicate that the filter-less controller time lag is too small to affect the performance of the active couplings, whereas the scaled actuator dynamics appear to yield an equal outcome.

Furthermore, a closer look at Figure 10 unveils that the PTC first reduces and later increases the teeter angle, compared with the respective baseline curves without any couplings. It is a result of an increased aerodynamic stiffening from PTC, described as the stiffness K_θ in Equation (10). The effect is similar to a lower Lock number. The poorer reduction of the existing teeter angle can also be seen as a result of the lower damping from PTC, derived by the damping ratio of Equation (12), which accompanies a higher aerodynamic stiffness. Interestingly, PTC decreases the aerodynamic damping even more for turbines with larger Lock numbers.

Equation (12) shows that having a PTC coefficient x_{PTC} of 1, the damping ratio of the CART2 will be reduced from 0.6 to 0.4, whereas the damping ratio of the 2B20HAW-T will be reduced

from 1.9 to 0.9. This is slightly visible in Figure 10, where the differences of teeter angles with PTC compared with the baseline curves without couplings are a bit larger for the 2B20HAW-T. PTVC, on the other side, increases the damping directly, similar to the effect of a higher Lock number. Through their parallels with the more advantageous PTVC, the benefits of larger turbines with higher Lock numbers become more evident, especially in contrast to the PTC in Figure 10.

Finally, a phase shift of the teeter motion due to PTC can be recognized in Figure 10, where the rotor azimuth angles are equal for all simulations with CART2 or 2B20HAW-T, respectively. PTVC shows no teeter angle phase shift. These phase shifts are in good agreement with Equation (18).

4 | Discussion

The paper's introduction shows that the Lock number, which is the ratio between aerodynamic forces and inertia forces, increases for larger (and newer) rotors. It is a consequence of a mass scaling that is, in reality, less than cubic with the rotor size. Subsequently, the influence of rotor size and Lock number on the teeter movement is demonstrated analytically and numerically, leading to the following observations:

The wind shear-driven wind speed difference between the upper and lower blade tip positions increases for larger turbines. Since the teeter angle in constant shear and constant wind speed is independent of general rotor size characteristics, it consequently rises for larger turbines.

Today's modern turbines with larger rotor diameters typically have larger Lock numbers. This leads to a higher contribution

of aerodynamic damping for larger rotors, which is, under normal operation, besides the centrifugal force, the most important teeter restoring mechanism that removes energy from the teetering rotor. Hence, the variation in teeter angle, the relative standard deviation, becomes smaller for larger turbines. Larger (or newer) teetered turbines do thus run more smoothly in normal conditions.

The effects of PTC and PTVC have been shown to be independent of rotor size, analytically and numerically. However, PTVC reduces teeter angles more significantly and more efficiently than PTC. Teetering is already a highly effective method to eliminate most fatigue loads from imbalanced aerodynamic rotor forces. The teeter control methods are not expected to reduce fatigue loads further or improve the energy yield [18] compared with a passive teetering system. Instead, the great value of PTC and PTVC is a reduction of large teeter movements, a primary issue of teetering systems in extreme scenarios, causing high ultimate loads by an adverse run into teeter restraints, brakes, or end impacts [23]. When a pitch-teeter-(velocity) coupling is implemented actively by a controller, it enables activating its usage only for extreme scenarios to avoid excessive pitch activity in normal operation. A high teeter velocity occurs mostly before an extreme teeter angle, enabling the implementation of an effective trigger to detect extreme cases and act accordingly, e.g., with a PTVC, before the ultimate load or teeter angle takes place.

A large Lock number is not in every aspect advantageous. It makes the rotor very prone to any kind of unequal aerodynamic blade loading, leading to much faster reactions than heavier turbines. This can be seen during the EWS. Nevertheless, fast-reacting rotors with a low rotational speed may open up opportunities for advanced control methods beyond PTC and PTVC since the teeter movement remains relatively slow and the teeter velocity decreases with size.

The very critical stuck pitch fault can be handled by yawing the turbine out of the wind. The unusual maneuver may cause increased loads on some components during this load case, which should still be lower than those in other ultimate cases for its practical application.

The flexible hub joint effectively avoids severe loads during yawing, which would otherwise be caused by the fluctuating moment of inertia of a two-bladed rotor about the yaw axis.

At least in normal wind conditions, speed control by yawing should work sufficiently. Other wind conditions or a yaw-system fault are, according to the DNVGL-ST-0437 [44], not expected to occur simultaneously with the rare event of a single-blade stuck pitch fault during power production. It is still an issue of safety and certification, which must be considered when designing such a turbine. If certifiable, this solution works better for larger turbines with larger Lock numbers. As shown by the yaw-out simulations in Figure 9, a larger Lock number forces the rotor to stop quicker while reducing the teeter amplitude.

Caution is advised when correlating Lock numbers and turbine size. An important reason for higher Lock numbers of today's turbines is certainly progress in composite materials and much better possibilities for aeroelastic and structural simulations. If,

nowadays, someone would develop a turbine of the same rotor diameter as the CART2, it would certainly have a Lock number in the order of the 2B6 or the 2B20HAW-T with a respective teeter behavior. Thus, it could at least be stated that two-bladed teetering turbines benefit from technological improvements from the last decades.

5 | Conclusion

This paper expands the existing teeter equations by PTVC, which is an important contribution in understanding the effects of modern control methods analytically. The comparison of different turbine sizes in this study leads to a more detailed understanding of teeter behavior.

The conclusion that teetering can be handled more easily on larger turbines may certainly be drawn from this study. Higher Lock numbers of larger (or newer) rotors raise the contribution of aerodynamic damping, which is, under normal operation, a remarkably important mechanism to remove energy from the teetering rotor. However, it must be considered that teetering gives the rotor an additional degree of freedom. In theory, this enables the rotor to gain a large amount of kinetic energy when rotating around the teeter axis. It must be ensured in every situation during the turbine's lifetime that this energy will be removed within a certain teeter range. A reliable teeter brake and teeter end impact design, combined with cleverly designed control strategies, will be essential to handle a teetered turbine well. The newer PTVC proved herein and in other studies to achieve a better handling of extreme cases. It is up to turbine developers to decide whether the effort of implementing a teetered system is worth the benefits of extreme and fatigue load reductions.

Nevertheless, considering today's large turbines and advanced possibilities for control, the authors come to the conclusion that teetering should actually be much more interesting today than it was almost 50 years ago.

Author Contributions

Fabian Bölle: conceptualization, formal analysis, methodology, software, validation, investigation, data curation, visualization. Vera Schorbach: conceptualization, formal analysis, investigation, writing – original draft, validation, supervision, project administration.

Acknowledgments

We would like to thank 2-B Energy for valuable discussions about two-bladed turbines and for sharing the simulation results of the 2B6 model, which has been modified for this research project to accommodate a teetering system. We want to send special thanks to the Federal Ministry of Education and Research, Siemens Gamesa Renewable Energies, and Energie Baden Württemberg (EnBW) for funding and support of “X-Rotor – two-bladed wind turbines” (13FH1I04IA) and “X-Rotor – two-bladed floating wind turbines” (ref. 13FH1I11IA). These projects have been the basis for the current study. Open Access funding enabled and organized by Projekt DEAL.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Peer Review

The peer review history for this article is available at <https://www.webofscience.com/api/gateway/wos/peer-review/10.1002/we.70069>.

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