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## Aircraft Cabin Ventilation Theory

### Abstract

Ventilation on board of an aircraft is governed by the ventilation equation. In the steady state case, a concentration of any substance depends only on the source strength and the effective air flow rate for ventilation. Not all air for ventilation is effective and helps to lower concentration. Some air leaves the cabin without mixing and rinsing. This is expressed by the ventilation efficiency. The dynamics follows an exponential function and is expressed by a time constant that depends on the air change rate and the ventilation efficiency. The (theoretical) air change rate is the air flow rate divided by the volume of the room. With full mixing (i.e. ventilation efficiency of 1), the concentration is reduced to 36.8% after one air change.

### Ventilation Theory

The ventilation theory in an aircraft cabin is the same as the one in any room. Proper **ventilation is important for buildings and for cleanrooms**. For this reason, literature about ventilation can be found for these applications.

The fundamental **ventilation equation** is summing all the mass of a particular substance in a volume per time – the mass generated inside the volume, the inflow of that substance minus the outflow of that substance results in the change of that substance in the volume

$$S + Q_e C_{out} - Q_e C = V \frac{dC}{dt} .$$

$S$ : source strength in kg/s

$Q_e$ : effective air flow rate for ventilation in m<sup>3</sup>/s

$C$ : concentration of CO<sub>2</sub> or any other substance in kg/m<sup>3</sup> in the room

$C_{out}$ : concentration of CO<sub>2</sub> or any other substance in kg/m<sup>3</sup> outside of the room

$V$ : volume of the room

The air change rate  $n$  (in 1/h) is

$$n = \frac{Q}{V}$$

$Q$ : air flow rate for ventilation in m<sup>3</sup>/s

The time for one theoretical air exchange,  $t_{n1}$  is

$$t_{n1} = 1/n .$$

If  $C_{out}$  is zero, the respective term can be deleted from the equation. The same is true, if  $C$  is understood as the difference of the concentration to the outside (ambient) concentration.

$$S - Q_e C = V \frac{dC}{dt}$$

In case of a **steady state** situation (no change in concentration  $C$ ), the equation simplifies to

$$C = \frac{S}{Q_e}$$

**We learn:** The concentration is independent of the volume  $V$  and depends only on the source strength  $S$  and the effective air flow rate  $Q_e$ .

The **effective air flow rate** can be determined from the **measured** CO<sub>2</sub> concentration on the aircraft during a steady state situation. With the source strength,  $S$  known or artificially introduced

$$Q_e = \frac{S}{C} .$$

The source strength,  $S$  is calculated from the people on board the aircraft. Each person has an emission of  $0.02 \text{ m}^3/\text{h}$  of  $\text{CO}_2$  while resting or with low activity of work – i.e. at a respiration rate of  $0.5 \text{ m}^3/\text{h}$  (IDC 2012, 3.14.3). This at standard conditions (1013.25 hPa and  $0^\circ\text{C}$ ). The density of  $\text{CO}_2$  at these conditions is  $1.98 \text{ kg}/\text{m}^3$ .

The ventilation efficiency,  $\eta$  is subsequently calculated from

$$\eta = \frac{Q_e}{Q} = \frac{S}{C n V} \quad \text{or} \quad Q_e = \eta Q = \eta n V$$

See Appendix A for a sample calculation showing the ventilation efficiency in an aircraft based on measuring  $\text{CO}_2$  concentration. The ventilation efficiency has typically values as low as 25% ... 50%.

The simplified **equation for the unsteady case** can be written as

$$S(t) - \eta n V C(t) = V \frac{dC(t)}{dt}$$

This is a first order ordinary differential equation (ODE) with constant coefficients. Laplace transformed:

$$S(s) - \eta n V C(s) = V C(s) s$$

$$\frac{S(s)}{V} - \eta n C(s) = C(s) s$$

$$\frac{S(s)}{V} = C(s) (s + \eta n)$$

$$\frac{C(s)}{S(s)} = \frac{1/V}{s + \eta n}$$

The time constant,  $T$  of this PT1-System can be identified as

$$T = \frac{1}{\eta n}$$

**We learn:** The speed with which the system reacts to change is characterized by the effective air change rate  $\eta n$ .

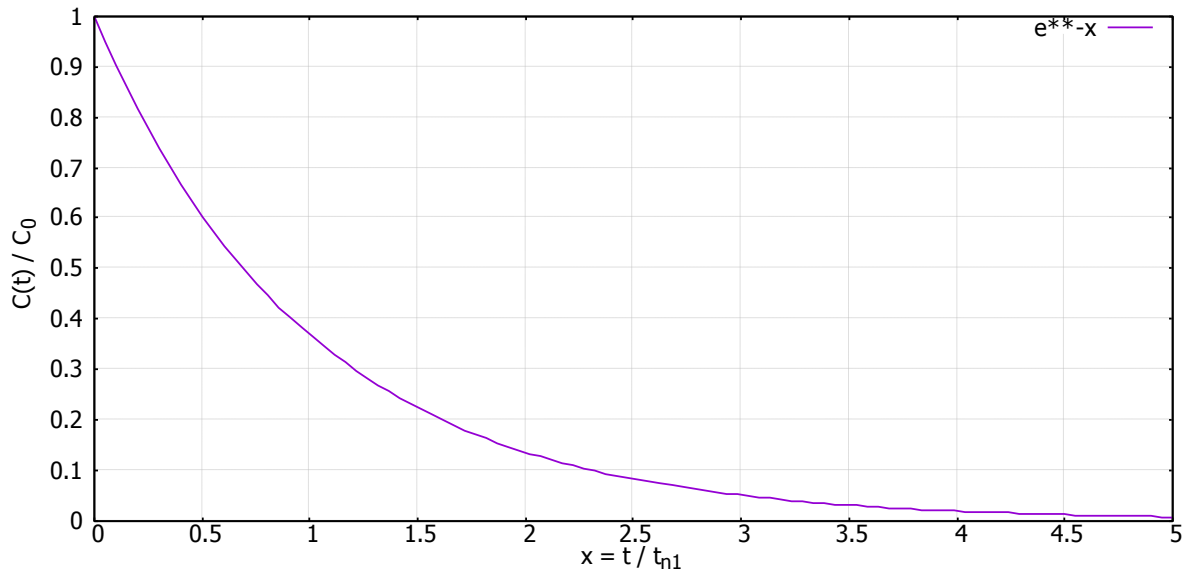
The transfer function is the pulse response to an initial concentration at  $t = 0$  with  $C_0 = S/V$ . Transforming back into the time domain

$$\frac{C(t)}{C_0} = e^{-1/T \cdot t} = e^{-\eta \cdot t} = e^{-\eta \frac{t}{t_{n1}}} \quad .$$

With this we can fill Table 1 and draw Figure 1.

**Table 1:** Relative remaining concentration for a ventilation efficiency of  $\eta = 1$  versus relative time

$t = x \cdot t_{n1}$	$x = 0.1$	$x = 1/3$	$x = 1/2$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$
$C(t)/C_0$	90.5%	71.7%	60.7%	36.8%	13.5%	5.0%	1.8%	0.67%



**Figure 1:** Relative remaining concentration for a ventilation efficiency of  $\eta = 1$  versus relative time

Hence, rinsing is an asymptotic process. A relative concentration will only reach the value 0% of the initial amount after an infinitely long time.

If a certain relative remaining concentration is given (e.g. 12%) and a time (e.g. 4 min.) a calculation of the time for one (theoretical) air change can be calculated

$$t_{n1} = -\frac{\eta t}{\ln(C(t)/C_0)} \quad .$$

Assuming a ventilation efficiency of  $\eta = 0.75$ , we would get the time for one (theoretical) air change as low as 1.4 min. from the above numbers.

Also the ventilation efficiency could be calculated, if the parameters in the equation are given as follows

$$\eta = -\frac{t_{n1}}{t} \ln(C(t)/C_0) \quad .$$

ISO 14644-3 (Cleanrooms and Associated Controlled Environments - Part 3: Test Methods) defines a "recovery time" (German: Erholzeit). The recovery time is the time it takes a concentration to be reduced to 1%.

More details are given in the EudraLex, The Rules Governing Medicinal Products in the European Union, Volume 4, EU Guidelines to Good Manufacturing Practice (EU GGMP 2008). The recovery time is the time a concentration is reduced to 1% (Grade B cleanroom) or 10% (Grade C cleanroom). The EU GGMP include a recommendation that the concentration should decay in 15 min. to 20 min. For interpretation of EU GGMP see Whyte 2016. The decay time can be calculated from

$$t = -\frac{t_{n1}}{\eta} \ln(C(t) / C_0) = -\frac{t_{n1}}{\eta} \ln(0.01) = 4.605 \frac{t_{n1}}{\eta}$$

and can be compared with the EU GGMP-requirement. Reversed, the equation can be used to calculate the required time for one (theoretical) air change  $t_{n1}$ .

If  $\eta = 46.05\%$  is assumed (Wikipedia 2020)

$$t = 10.0 t_{n1}$$

We learn: The air in a room will never be "fully renewed", but a remaining concentration of 1% may be accepted to call this "fully renewed" (in accordance with ISO 14644-3). As a rule of thumb "fully renewed" is achieved during a time about ten times the time for one (theoretical) air change.

**If the time for one (theoretical) air change is 3 minutes, the air can be considered to be "fully renewed" in 30 minutes.**

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## Appendix A: Calculation of Ventilation Efficiency, Airbus A340-600

E_pax	0.02	m <sup>3</sup> /h	CO2 emission volume per time ( <b>IDC 2012</b> )
rho_CO2_0deg	1.980	kg/m <sup>3</sup>	at 0 °C
rho_CO2	1.877	kg/m <sup>3</sup>	at 15 °C
C_pax	0.038	kg/h	CO2 emission mass per time
n_pax	345		EASA 2017
n	20	1/h	EASA 2017
l_F	75.3	m	A340-600
l_F,press	60	m	length of pressurized equivalent cylinder, estimate
d_F	5.64	m	A340-600
mCO2-mAir	0.001		1000 ppm above ambient CO2 level ( <b>EASA 2017</b> )
rho_air	1.225	kg/m <sup>3</sup>	ISA
V	1499	m <sup>3</sup>	= $d_F^2/4 * \pi() * l_F_{press}$
eta	0.35		= $C_{pax} * n_{pax} / (mCO2_{mAir} * rho_{air} * n * V)$
C_pax_EASA	0.44	g/min	CO2 emission mass per time ( <b>EASA 2017</b> )
C_pax_EASA	0.026	kg/h	
V_EASA	470	m <sup>3</sup>	wrong, the whole pressurized fuselage need to be considered
eta_EASA	0.79		= $eta * C_{pax\_EASA} / C_{pax} * V / V_{EASA}$ (wrong due to wrong volume!)

### Calculation of C\_pax according to Lindner 2005

V_resp	9	l/min	Lindner 2005
V_resp	540	l/h	
m_resp	0.662	kg/h	
mCO2-mAir	0.04		4% CO2 in air exhaled ( <b>Lindner 2005</b> )
C_pax_Lindner	0.026	kg/h	

With this, the source strength, S = 0.026 kg/h per person is in agreement with the EASA value.

eta 0.25

As such the ventilation efficiency is not better than the one given in **Wikipedia 2020** for mixed (turbulent) ventilation (typical e.g. in offices):

eta                                      0.5

This means that the ventilation efficiency in an aircraft cabin (here A340-600) is as low as 25% ... 50%.

However, **Whyte 2016** claims it [the the ventilation efficiency] is "unlikely to be less than 0.7".