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The Accuracy of a Simplified Calculation of Aircraft Climb Time

Calculating the time to climb to altitude for jet aircraft – accurately and with little effort.

PURPOSE

The time it takes an aircraft to climb (Figure 1) to a specific altitude (the climb time) can be calculated using a formula that, for simplicity, assumes that the climb rate decreases linearly with increasing altitude throughout the entire climb. The objective of this study is to determine whether the assumption of a linearly decreasing climb rate is realistic and, if not, what errors result from this assumption.

METHODOLOGY

As altitude changes, parameters such as air density, drag (Figure 2), and thrust change, and with them the optimal flight speed for the climb. These parameters influence one another. Thrust is calculated using three different methods, provided by Bräunling, Scholz (Figure 3), and Howe. The study analyzes the variation of thrust with altitude and the variation of climb rate with altitude for each of the three thrust calculations. Finally, for each thrust calculation, the climb time is compared as it results a) from the simple one-step formula and b) from a stepwise integration.

FINDINGS

Starting from the same initial thrust, the three thrust calculations yield different thrust values with altitude. Bräunling's method incorporates more parameters than the other two methods. It can be assumed that Bräunling's method is more accurate, but this cannot be proven. The thrust according to Scholz and Howe decreases almost linearly with altitude. The thrust curve according to Bräunling shows significant nonlinearity. The climb time from 0 km to 11 km altitude is calculated according to a) and b), using each of the three thrust calculations. The difference in climb time is determined for each case. Due to the nonlinearity in the thrust curve, Bräunling's method shows the greatest difference between the calculation methods at 7.1%. A thrust calculation according to Scholz yields 1.7%, and according to Howe, 1.4% difference. Please find the comparison calculated at Mutschall (2018). If simplifications, e.g., regarding engine thrust, were already made at the outset, it is possible – and in some cases sensible – to perform the climb time calculations using a linear decrease in vertical velocity, given the effort involved and the results to be achieved. It is expressly noted that this concerns a comparison of two methods for calculating climb time and not an evaluation of methods for thrust calculation (for which no comparative values were available).

$$\frac{T}{T_0} = a\sigma^n \qquad a = -0{,}0253\mu + 0{,}7291$$

$$n = 0{,}0033\mu + 0{,}7324$$

$$\sigma = \left(1 - \frac{LH}{T_0}\right)^{\left(\frac{g}{RL}\right)-1} \qquad \sigma(h_{abs}) = \sqrt{\frac{n}{E_{max}\, a\,(T/W)_{CR}}} => h_{abs}$$

$$L = 6{,}5\,\frac{\mathrm{K}}{1000\mathrm{m}}$$

$$T_0 = 288{,}15\mathrm{K}$$

$$g = 9{,}80665\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$R = 287{,}053\,\frac{\mathrm{m}^2}{\mathrm{s}^2\mathrm{K}}$$

$$\boxed{V_T = v = \sqrt{\frac{1}{6A}\left(T \mp \sqrt{T^2 + 12AB}\right)} \qquad ROC = V_V = \frac{T-D}{W} V_T}$$

Figure 3: Calculating trust, T (from Scholz), maximum speed in climb, V_T and vertical speed, V_V = ROC (rate of climb).

PRACTICAL IMPLICATIONS

It was found that a simple formula for calculating climb time can be applied with a small margin of error – especially when methods for thrust calculation are available in which thrust decreases approximately linearly with altitude. However, with a high level of effort and a realistic analysis, e.g., according to Bräunling, the linear approach leads to an excessive error. In such cases, the climb time should be calculated using stepwise integration.

$$V_V = \frac{dh}{dt} \qquad dt = \frac{1}{V_V}\,dh \qquad \boxed{t = \int_{h_0}^{h} \frac{1}{V_V(h)}\,dh}$$

$$\boxed{V_V(h) = V_{V0}\,\frac{h_{\text{abs}} - h}{h_{\text{abs}} - h_0}}$$

$$\boxed{\frac{V_V}{V_{V0}} = \frac{h_{\text{abs}} - h}{h_{\text{abs}} - h_0}}$$

$$\boxed{t = -\frac{h_{\text{abs}} - h_0}{V_{V0}} \ln\left(\frac{h_{\text{abs}} - h}{h_{\text{abs}} - h_0}\right)}$$

$$V_V(h_0) = V_{V0} \text{ and } V_V(h_{\text{abs}}) = 0$$

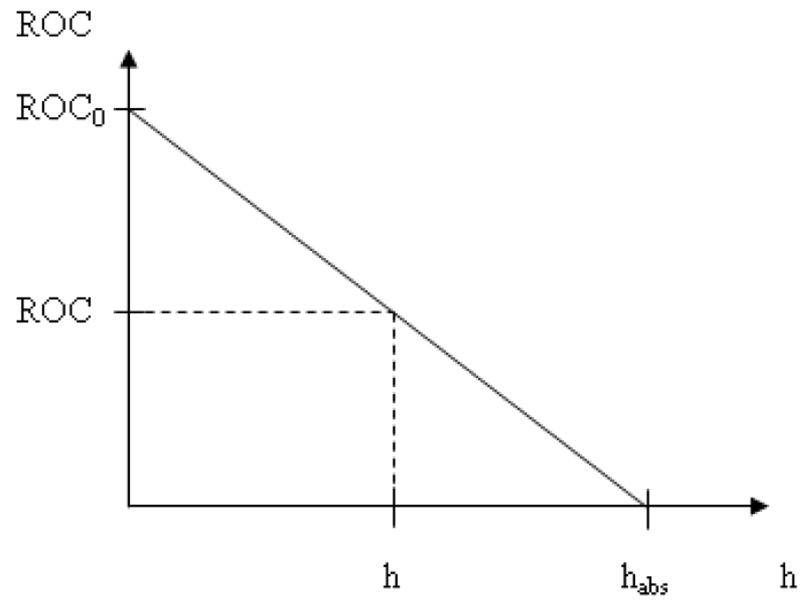

Figure 1: Calculating time to climb, t from altitude to climb, h starting from an altitude h_0. ROC = V_V.

$$D = \frac{1}{2}\rho v^2 c_{D0} S + \frac{1}{2}\rho v^2 \cdot \frac{4m^2 g^2}{\rho^2 v^4 S^2 \pi A e} \cdot S$$

$$= \frac{1}{2}\rho c_{D0} S v^2 + \frac{2m^2 g^2}{\rho S \pi A e} \cdot v^{-2}$$

$$D = A \cdot v^2 + B \cdot v^{-2}$$

Figure 2: Calculating drag, D: The notation with A and B follows from the detailed equation above. Note: Variable "A" is used twice. "A" is aspect ratio in πAe. "A" is a drag constant together with "B".

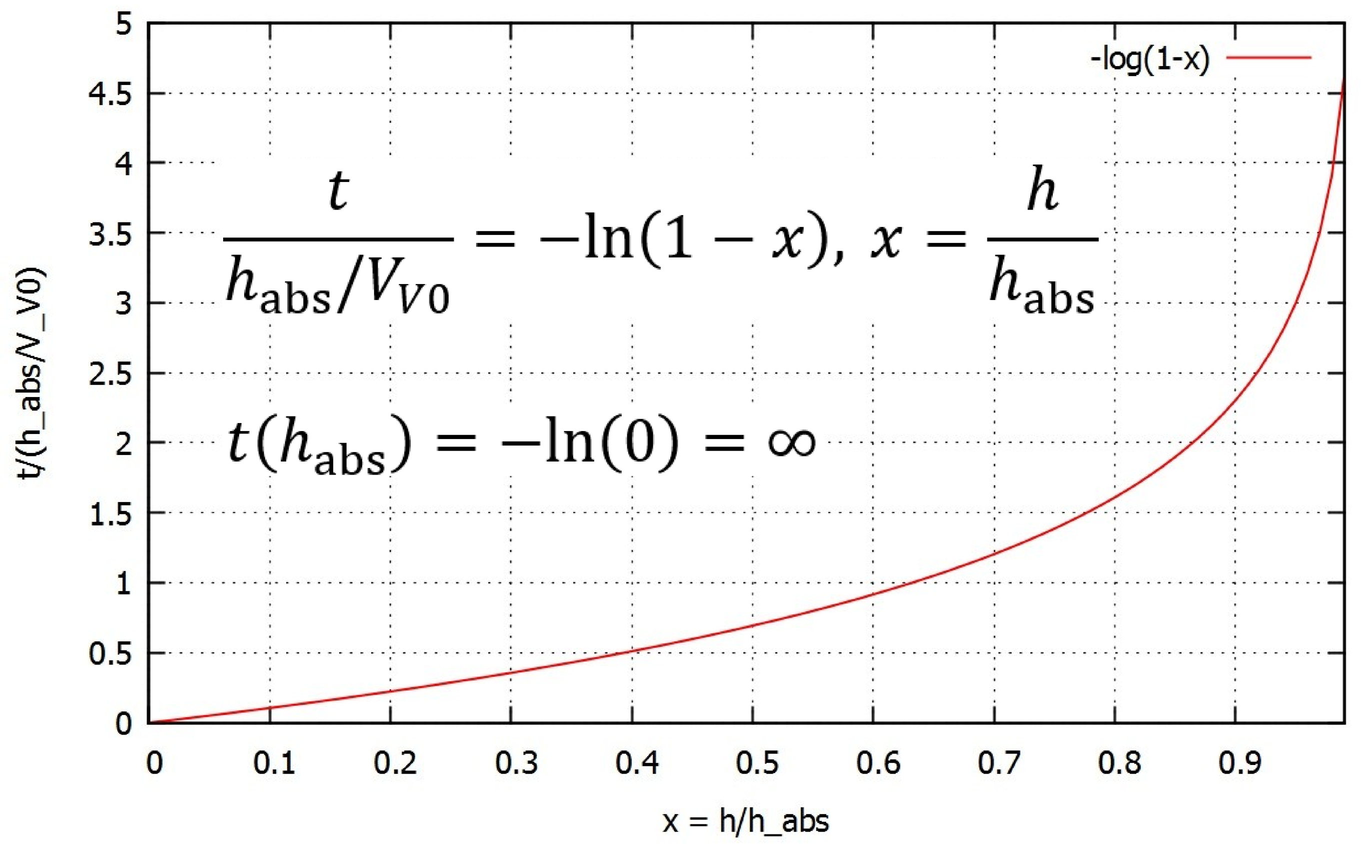

Figure 4: Plot of the nondimensional time to climb versus nondimensional altitude, x .

More details in the Project of Mutschall (2018):
https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2018-02-28.018

Associated research data (Harvard Dataverse):
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