Hamburg University of Applied Sciences Faculty of Life Sciences

A Constraint Optimization Approach to Model Based Supervisory Control for Renewable Energy Integration in Commercial Buildings

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ABSTRACT

With the recent focus on climate protection amidst rising energy prices, the need of the hour is the utilization of renewable energy sources. Many countries are already on the right track, however, new technical and economical challenges arise with the penetration of renewable energy sources. Once such challenge arises in commercial buildings having huge energy expenses and at the same time trying to be self sufficient. A huge portion of the energy expense in commercial buildings in countries with hot climate can be attributed to space cooling. An effective way to control and integrate the space cooling with a renewable source such as a PV plant has been demonstrated in this thesis. A hypothetical building with a water cooled chiller unit and a solar PV plant has been modeled. A supervisory controller has been designed to provide set points to the chiller unit considering the output of the PV plant and the price of electricity from the local grid. This supervisory control problem has been formulated as a standard linear optimization problem. Three model based control methods have been proposed to design the supervisory controller. The supervisory controller designed according to the three methods have been tested on the building model for different scenarios and the results have been compared. All the modeling, designing and simulations have been done in Matlab/Simulink.

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1. INTRODUCTION

1.1 Motivation

The building sector accounts for a substantial amount of a country's primary energy consumption. In the building energy consumption, a big percentage can be attributed to space cooling or heating purposes. This is the case alike in developed and developing countries. For this reason there is great amount of interest in the building energy efficiency sector, including optimization methods, energy efficient buildings, zero energy buildings, energy efficient equipments, etc.

Renewable energy has been a crucial topic for quite a few years now. European countries such as Germany, Denmark, Norway are covering a big percentage of their energy demands through renewable sources. Developing countries such as India and China have optimistic targets regarding the renewable sources. One main problem with the renewable sources is their availability and one solution to this problem is demand side management or matching the demand to the availability. Traditionally, the electrical systems worked with costumers as the basic focus and the generation was based on the demand. Now there is paradigm shift with the penetration of renewable energy in a way that the power generation is no more just dictated by the consumers and the present infrastructure is struggling to cope up with this. Several storage methods have also been tried out. But the best case is to use the energy when it is available.

With renewable installation such as PV plants, buildings now are not only energy consumers but also energy producers. A building energy system in a futuristic setup could look like the one in Fig. 1.1 adapted from [1]. The challenge now is to design intelligent controllers that can not only make use of the self generation but also coordinate the operation of the various loads in the buildings such as a chiller, boiler, storage systems etc., in order to make the building energy efficient and self sustaining at the same time. This is the main challenge taken up in this thesis.



Fig. 1.1: A futuristic building energy system

1.2 Thesis objective

The main objective of this thesis is to design a supervisory controller for a futuristic building energy system. At the beginning of this thesis, it was planned to design the supervisory controller based on a real building energy system. After some initial attempts to obtain data from a real building energy system which were not successful, it was decided to design a generalized supervisory controller and test this on a hypothetical model of a building energy system with one load (chiller) and two sources (public electric grid and solar PV). To this end, the main objectives are the following:

- Model a building energy system with a chiller unit, solar PV and public electric grid based on first physical equations assuming suitable values for the different parameters reflecting real cases.
- Design a supervisory controller for such a system which would provide setpoints for the chiller considering the output from solar PV and the price of electricity from public grid.
- Test the supervisory controller on different test scenarios.

1.3 Supervisory control system

Some basic background on control systems is given in this chapter upon which the subsequent chapters are based. The designing process of a control system can be summarized in the following steps:

- 1. Establishing the objectives
- 2. Obtaining the model of the process
- 3. Designing the controller
- 4. Simulation

The chapters of this thesis are organized in the same way as the steps above.

1.3.1 Establishing the objective

The first step in controller design is to know the goal or formulate the control objective. Every controller in a system is trying to achieve something (control the speed, temperature, etc.) by controlling some variables of the system. For example, in a room heating system, a thermostat is a controller which tries to control the room temperature by varying the mass flow rate of the hot water through the radiator. It does so by providing control signals to an actuator, which in this case could be an electric valve. Here, the objective of the thermostat is to maintain the room temperature as close as possible to the setpoint specified by the user.

The user who is specifying the setpoint to the thermostat can also be considered as a controller. The user's objective may be to maintain the thermal comfort of the room and he does so by varying the setpoint of the thermostat. The user may also have multiple objectives such as maintaining the thermal comfort spending the least amount of energy. Now, the user may give different setpoints to the thermostat than the first case with a single objective. It is already easy to imagine in a complex system such a building energy system with different loads, there could be a web of objectives which would be not manageable for a human user. This is where the supervisory control system assists the human user. From this explanation, a supervisory controller can be thought of as a higher level controller controlling lower level controllers towards achieving a common higher objective, which could be cost efficiency, energy efficiency or multiple objectives. This can be represented as a hierarchy of controllers as in Fig. 1.2 from [2]. This schematic is a representation of a controllers hierarchy in a processing plant. It can be seen from the figure that each level has a different objective and works on a different time scale.



Fig. 1.2: Hierarchy of controllers in a processing plant

For a building energy system a simplified version of this is shown in Fig. 1.3. In [2] the author explains how the Model Predictive Control (MPC) method is used in an industrial control system as a local controller, controlling the lower level distributed PID controllers. In this thesis, a model based method similar to the MPC has been used to design the supervisory controller.



Fig. 1.3: Supervisory control system

Now, the description of a supervisory controller defined above will be adapted to the building energy system shown in Fig 1.1 which would form the basis of further discussion. In this case the supervisory controller needs to provide the setpoints for the local controllers controlling the various units such as chiller, boiler, storage systems etc. The objective of the supervisory controller considered in this thesis is to maximize the utilization of the self generation and minimize the energy expense to maintain the indoor comfort of the building.

1.3.2 Obtaining the model of the process

The model of the process is needed to get a relationship between the objective and the parameter varied to obtain the objective. Going back to the heating system example, the model needed for the thermostat to control the room temperature needs to be a set of equations relating the mass flow of water with the room temperature and the mass flow rate with voltage on the valve control circuit. A supervisory controller, whose objective is to maintain the room temperature within comfort levels spending the least amount of energy needs a model relating the room temperature to the energy spent.

The model of a physical process or a system can be obtained from the physical equations (white box modeling) or it could be a data based model (black box modeling) or a mix of both (gray box modeling) [3]. For modeling a building energy, system various software tools are available such as EnergyPlus, TRNSYS, Dymola which have detailed libraries for common components of a building energy system. Modeling of a building energy system is described in [1]. A gray box model for an heating, ventilation and air conditioning (HVAC) system is given in [3] and [4] gives a model for an HVAC system in India. A model for a hybrid renewable energy system is given in [5]. There are different ways to express a physical process such as a set of ordinary or partial differential equations, difference equations, transfer functions, state space equations etc. In this thesis, a state space representation is used. More on modeling is given in chapter 2.

1.3.3 Designing the controller

The controller needs to provide input signals to an actuator depending on the setpoint of the controller and in case of closed loop systems, the feedback received from the plant. A good representation of a basic closed loop control system is given in Fig. 1.4 from [7]. There are various ways in which a controller can be designed. Traditional methods include the proportional, integral and differential controllers which act on the error between the reference and the feedback signals to provide the control signal.



Fig. 1.4: Closed loop control system

In this thesis, a model based control similar to the MPC is used. In a model based control, process inputs are computed so as to optimize future plant behavior over a time interval known as the *prediction horizon*. In the general case any desired objective function can be used. Plant dynamics are described by an explicit process model which can take, in principle, any required mathematical form. Process input and output constraints are included directly in the problem formulation so that future constraint violations are anticipated and prevented. The first input of the optimal input sequence is injected into the plant and the problem is solved again at the next time interval using updated process measurements [2]. A MPC solves a quadratic optimization problem to minimize the error between the predicted outputs and reference outputs to compute optimum process inputs. In this thesis, the model based controller solves a linear op-

timization problem to compute the inputs or in the supervisory sense the 'setpoints'. Chapter 3 discusses the controller design in detail. The supervisory control system as designed in this thesis, is represented in Fig. 1.5.



Fig. 1.5: Supervisory control system for a building energy system with renewable sources

1.3.4 Simulation

The supervisory controller designed needs to be tested before it can be implemented in the hardware. The testing can be done as a computer simulation. The mathematical programming and simulation tool Matlab/Simulink is chosen since it has a number of toolboxes with extensive features for modeling and designing controllers and developing mathematical models. It also provides tools for solving optimization problems which have been used in this thesis. The supervisory controller has to be simulated in various scenarios depicting the real cases in which it might be used. More on simulation is given in chapter 4.

2. MODELING OF THE COOLING SYSTEM AND COOLING SPACE

Tab. 2.1: List of Abbreviations									
Abbreviation	Expansion								
AHU	Air Handling Unit								
CCU	Cooling Coil Unit								
COP	Coefficient Of Performance								

	Tab. 2.2: List of Symbols	
Symbol	Description	Unit
\dot{m}_a	mass flow rate of air	kg/s
\dot{m}_w	mass flow rate of water	kg/s
c_w	specific heat capacity of water	J/kgK
c_a	specific heat capacity of air	J/kgK
C_a	heat capacity of air	J/K
C_{ccu}	overall heat capacity of the CCU	J/K
U_{ccu}	heat transfer coefficient of CCU	J/m^2K
A_{ccu}	effective surface area of CCU	m^2
T_{wi}	temperature of the chilled water from the chiller	°C
T_{wo}	temperature of the return water from the CCU	°C
T_{ai}	temperature of the air input to the AHU	°C
T_{ao}	temperature of the air output from the AHU	°C
T_a	ambient temperature	°C
T_r	room temperature	°C
C_b	overall heat capacity of the building	J/K
K_b	overall heat transfer coefficient of the building	J/m^2K
P_C	chiller power	kW
H_r	heat extracted by the chiller	kW
W_c	chilled water from the chiller	J/s
W_o	return water from the AHU	J/s
A_i	air input from AHU to the room	J/s
A_o	air recirculation to AHU from the room	J/s

Tab. 2.2: List of Symbols

The scope of the model developed during this thesis is to be a base model for the supervisory controller and not to be an accurate model of a commercial building cooling system with all the components and heat flows. As mentioned in the previous chapter, the supervisory controller is designed to work on different systems and the model developed in this chapter will act as one test case. The model is based on the common cooling systems in commercial buildings in countries with tropical climate. Due to lack of data, the model is not validated. Suitable parameters have been assumed to provide a reasonable test case for the supervisory controller. The system considered here has three main components as follows :

- 1. Chiller system,
- 2. Air Handling Unit (AHU) with Cooling Coil Unit (CCU),
- 3. Thermal zone which is a single room.

The simplified cooling system and the room considered in this thesis can be represented by the schematic diagram in Fig. 2.1.



Fig. 2.1: Schematic representation of the chiller and room model

The chiller is considered to be a chilled water type, which is common in commercial buildings. The chiller block is a complex system with various other components such as compressor, cooling tower, evaporator, pumps, distribution systems etc. These are not considered in this model because the controlling of the sub components is the task of the local controllers. It is assumed that the local controllers are able to operate the chiller at the setpoint given by the supervisory controller. There are different variables in a chiller system such as chilled water temperature, condenser water temperature etc., which can be adjusted to influence the chiller power [8]. One such variable is the chilled water temperature T_{wi} which will be considered in this thesis. The chiller efficiency can be defined in various ratios such as the energy efficiency ratio, part load ratio, coefficient of performance (COP) etc. Here the COP is considered which is defined as the ratio of heat extracted to the mechanical input [9]. For electrical motor driven chillers, the mechanical input is defined in terms of the electrical input to the motor. The higher the COP, the higher the efficiency. Chillers are more efficient at higher leaving water temperatures [8].

The following assumptions are made regarding the chiller system :

- 1. The chilled water temperature is adjustable from 5 to $10^{\circ}C$
- 2. The COP of the chiller varies from 3 to 5.5 corresponding to $5^{\circ}C$ and $10^{\circ}C$ respectively
- 3. Constant flow rate of water is considered from and to the chiller
- 4. Simple power consumption model for the chiller is assumed

In [10] and [11] detailed models for chiller power consumption are given. These models however require other parameters such as the condenser water temperature, part load ratio, rated chiller power etc. Since these variables are not available the power consumption of the chiller P_c is approximated corresponding to the COP and the heat extracted H_r as

$$COP = H_r/P_c ,$$

$$H_r = m_w c_w (T_{wi} - T_{wo}) ,$$
(2.1)

where m_w is the mass flow rate of water, c_w is the specific heat capacity of water, T_{wi} , T_{wo} are temperatures of the chilled water and the return water respectively.

An Air Handling Unit supplies cool air input to the room by circulating the air from the room over a cooling coil. During the circulation, some fresh air is added to maintain the air quality in the room. The input to the AHU is the chilled water from the chiller unit whose temperature is fixed by the supervisory controller. In the AHU only the cooling coil unit is considered as it is the main part where the actual heat transfer takes place and it is modeled based on the equations from [13] which uses a lumped parameter approach. The schematic diagram of the CCU is given in Fig. 2.2.



Fig. 2.2: Schematic Representation of the CCU

The CCU is modeled based on equations from [13]. Here the CCU is considered as a perfectly mixed vessel, therefore the outflow water temperature is same as the mean temperature of the water content of the coil. The energy balance on the water side of the coil, is given by

$$C_{ccu}\dot{T}_{wo} = \dot{m}_{w}c_{w}(T_{wi} - T_{wo}) - U_{ccu}A_{ccu}(T_{wo} - T_{ao})$$
(2.2)

and the energy balance on the air side of the coil, is given by

$$C_a \dot{T}_{ao} = U_{ccu} A_{ccu} (T_{wo} - T_{ao}) - \dot{m}_a c_a (T_{ao} - T_{ai}) , \qquad (2.3)$$

where C_{ccu} is the overall heat capacity of the CCU, C_a is the heat capacity of air, U_{ccu} is the heat transfer coefficient of CCU, A_{ccu} is the effective surface area of CCU, T_{ao} is the temperature of the air input to the room and the temperature of the input air to the AHU T_{ai} is considered to be the same as that of the room T_r . Constant mass flow of air and water is considered. To simplify the model, the quality of the air fed to the room by the AHU is not considered. As mentioned before, the purpose of this model is to act as a simple model reflecting the real cooling process in a building to a reasonable extent for the supervisory controller. The setpoint given by the controller may vary if there is fresh air added, but the controller algorithm remains the same. This is another test case for the controller, however this has not been tested in this thesis.

Instantaneous heat transfer between the air and the cooling coil is assumed, therefore

(2.3) can be written in terms of T_{ao} as

$$T_{ao} = \frac{U_{ccu}A_{ccu}T_{wo} + \dot{m}_a c_a T_{ai}}{U_{ccu}A_{ccu}\dot{m}_a c_a} .$$

$$(2.4)$$

For simplicity, the cooling space is modeled as a single thermal zone. The temperature change in the thermal zone can be expressed in terms of the heat transferred to the zone \dot{Q}_{room} . The thermal zone will be referred to as the room. The following equations

$$Q_{room} = Q_{gain} - Q_{lost} , \qquad (2.5)$$

where

$$\dot{Q}_{room} = C_b \dot{T}_r , \qquad (2.6)$$

$$Q_{gain} = K_b(T_a - T_r) , \qquad (2.7)$$

$$Q_{lost} = \dot{m}_a c_a (T_r - T_{ao}) , \qquad (2.8)$$

describe the dynamics of the room and are based on the equations from [12].

2.1 Simulink Model

Simulink is a modeling and simulating tool with a graphical programming interface. Simulink/Matlab is one of the effective software that has advanced possibilities to design thermodynamic models and controllers for indoor climatic conditions [12]. The physical equations (2.2) to (2.5) describing the cooling system and the room has been implemented in Simulink. Fig. 2.3 shows the Simulink implementation of the cooling system and the room.



Fig. 2.3: Simulink model of the cooling system and the room

It can be seen from the above figure that there are two blocks in the Simulink model representing the AHU and room respectively. The block representing the room has two inputs, the ambient temperature T_a and the temperature of the input air from the AHU T_{ao} . Equation (2.5) is implemented in the block representing the room. This can be seen in Fig. 2.4.



Fig. 2.4: Equations representing the room in Simulink

Equations (2.2) and (2.4) are implemented in the block representing the AHU as seen in Fig. 2.5. The block has two inputs, the temperature of the chilled water from the chiller T_{wi} and the temperature of the air input to the AHU which is also the room temperature T_{ai} .



Fig. 2.5: Equations representing the AHU in Simulink

The parameters required by the Simulink model are given in a Matlab m-file. The model is simulated with temperature data from a sample file and a constant chiller set point of $8^{\circ}C$ for a period of 7 days. Fig. 2.6 shows the result of the simulation. It can be seen that the room temperature varies with the ambient temperature.



Fig. 2.6: Continuous time simulation of the cooling system

If heat gained by the room from the ambient is not removed by the cooling system (the chiller is turned off), the room temperature quickly approaches the ambient temperature. This can be seen in Fig. 2.7.



Fig. 2.7: Room temperature approaching ambient temperature

2.1.1 State-Space representation and discretization

A state-space model is just a structured form or representation of the differential equations of a system. It is especially useful in Multi Input, Multi Output 'MIMO' systems modeling and analysis. According to [14] a linear, time-invariant system in continuous time can be modeled by a set of differential equations of type

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{U}(t) ,$$

$$\mathbf{Y}(t) = \mathbf{C}\mathbf{X}(t) + \mathbf{D}\mathbf{U}(t) ,$$

$$\mathbf{X}(0) = \mathbf{X}_0 ,$$

(2.9)

where the state vector $\mathbf{X} \in \mathbb{R}^n$, the input vector $\mathbf{U} \in \mathbb{R}^m$, the output vector $\mathbf{Y} \in \mathbb{R}^r$, the system matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$, the input matrix $\mathbf{B} \in \mathbb{R}^{n \times m}$, the output matrix $\mathbf{C} \in \mathbb{R}^{r \times n}$ and the feed through matrix $\mathbf{D} \in \mathbb{R}^{r \times m}$.

The set of differential equations representing the cooling system and the room from the previous section can be represented in the same form as (2.9). The Matlab command 'linmod' is used to obtain a continuous time linear state-space model of a Simulink model around a given operating point and initial condition. The command 'linmod' is used

on the Simulink model show in Fig. 2.8 to obtain the state-space representation of the cooling system and room. The initial values for the two states of this system T_{wo} and T_r are $12^{\circ}C$ and $24^{\circ}C$ respectively and inputs T_{wi} and T_a are $5^{\circ}C$ and $35^{\circ}C$ respectively.



Fig. 2.8: Simulink model used for linmod command

The supervisory controller is designed to give setpoints at discrete time of 15 minutes. This is a reasonable control interval for a cooling system, considering that the temperature change of the room is not too big in a 15 minute interval. Since the supervisory controller is working at a 15 minute interval, the continuous time state-space model has to be discretized at a 15 minute interval as well. The MATLAB command 'c2d' is used to discretize a continuous time system. The discretization interval has to specified in seconds. Here the 'c2d' command is used with a discretization interval of 900 seconds. The discretized state space model thus obtained is simulated with the same inputs as with the continuous time model in the previous section. The comparison of the results can be seen in Fig. 2.9



Fig. 2.9: comparison of continuous and discretized system

The discretized form of the equation (2.9) is,

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) + \mathbf{B}\mathbf{U}(k) ,$$

$$\mathbf{Y}(k+1) = \mathbf{C}\mathbf{X}(k) + \mathbf{D}\mathbf{U}(k) ,$$

$$\mathbf{X}(0) = \mathbf{X}_0 ,$$

(2.10)

where **k** is the time step. The chiller system can be represented by the following discrete state-space model :

$$\begin{pmatrix} X_1(k+1) \\ X_2(k+1) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} X_1(k) \\ X_2(k) \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} U_1(k) \\ U_2(k) \end{pmatrix} , \quad (2.11)$$

$$\begin{pmatrix} Y_1(k) \\ Y_2(k) \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} X_1(k) \\ X_2(k) \end{pmatrix} + \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \begin{pmatrix} U_1(k) \\ U_2(k) \end{pmatrix} , \quad (2.12)$$

where T_{wo} is state X_1 , T_r is state X_2 , T_{wi} is input U_1 and T_a is input U_2 . The input $U_1 \in \mathbb{U}$, where $\mathbb{U}=(5,5.5,6,\ldots,10)$.

The supervisory controller has to give the optimal input U_1 to the chiller unit at each time step, considering the room temperature X_2 does not exceed the allowable limits for all time steps k.

3. OPTIMIZATION

The supervisory control problem of providing the best setpoints to the chiller unit considering constraints on the room temperature is formulated as a linear optimization problem.

In order to follow the arguments made in this chapter, some basic mathematical definitions on polytopes from [15] are given below.

Polytope: A subset $\mathbf{P} \subseteq \mathbb{R}^d$ that can be presented as a V-polytope or as an H-polytope.

V-polytope: Vertex representation of a polytope is the convex-hull of a finite set $\mathbf{F} = (f^1 \dots f^n)$ of points in \mathbb{R}^d . Convex-hull \mathbf{P} of the set \mathbf{F} is,

$$\mathbf{P} = conv(\mathbf{F}) := \left\{ \sum_{i=1}^{n} \lambda_i f^i \, \middle| \, \lambda_i \ge 0, \sum_{i=1}^{n} \lambda_i = 1 \right\} \, .$$

H-polytope: Half-plane representation is a bounded solution set of a finite system of linear inequalities:

$$\mathbf{P} = \mathbf{P}(L, b) := \left\{ f \in \mathbb{R}^d \mid l_i^T f \le b_i \text{ for } 1 \le i \le m \right\} ,$$

where $L \in \mathbb{R}^{m \times d}$ is a real matrix with rows l_i^T , and $b \in \mathbb{R}^m$ is a real vector with entries b_i .

d-Polytope: A d-dimensional polytope. A 2-d polytope is a polygon, a 3-d polytope is a polyhedron and so on.

3.1 Linear Optimization

Linear optimization also known as linear programming is an important class of optimization problems in which all the objectives and constraints are linear [16]. According to [17] linear optimization deals with minimizing or maximizing the value of a function called as *objective function* by choosing some optimum values for the *decision variables* of the function. The values which these variables are allowed to take are defined by set of inequalities. Linear programming in its standard form as described in [17] for a n-dimensional space is formulated as follows :

Minimize

$$J = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \tag{3.1}$$

subject to

$$a_{11}x_{1} + a_{12}x_{2} + \dots + a_{1n}x_{n} \leq b_{1}$$

$$a_{21}x_{1} + a_{22}x_{2} + \dots + a_{2n}x_{n} \leq b_{2}$$

$$\vdots$$

$$a_{m1}x_{1} + a_{m2}x_{2} + \dots + a_{mn}x_{n} \leq b_{m}$$

$$x_{1}, x_{2}, \dots x_{n} \geq 0.$$

The variables in (3.1) are known as the decision variables and the vector containing these variables is known as the *decision vector*

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$$

and the constants in (3.1) are the costs for choosing a certain value for the variables in the decision vector. This is known as the *cost vector*

$$\mathbf{c} = (c_1 \ c_2 \dots c_n) \in \mathbb{R}^n$$
.

The linear inequalities form the constraints of the linear optimization problem. Each constraint is an half-plane and the set of feasible solution is just the intersection of the half-planes given by the constraints which is a polytope, a polygon in case of two decision variables and polyhedron in case of three decision variables and so on [17]. The solution to the linear optimization problem contains the optimum value for all the elements in the decision vector. The rest of this chapter shows how the model based supervisory control problem is formulated as a standard linear optimization problem.

3.2 Problem definition

Referring to the (2.11), the problem is to determine the input vector \mathbf{U}_1 at time step k, such that, the state variable \mathbf{X}_2 is within allowable limits defined by \mathbf{X}_{low} for the lower limit and \mathbf{X}_{up} for the upper limit for all the time steps k to $k + H_p$, where H_p is the prediction horizon. This problem of finding the optimum future input vector has to be formulated as standard linear optimization problem. This can be mathematically formulated as,

$$min \mathbf{W}.\mathbf{U}_{1}$$

$$subject \ to \ \mathbf{U}_{low} \leq \mathbf{U}_{1} \leq \mathbf{U}_{up}$$

$$\mathbf{X}_{low} \leq \mathbf{X}_{2} \leq \mathbf{X}_{up}$$
(3.2)

where the cost vector

$$\mathbf{W} = (W_k \ W_{k+1} \dots W_{k+H_p-1}) \in \mathbb{R}^{H_p}$$

contains the costs for choosing a particular U_1 . The costs can be real price in k/kWh or some virtual cost.

$$\mathbf{U_1} = \begin{pmatrix} U_1(k) \\ U_1(k+1) \\ \vdots \\ U_1(k+H_p-1) \end{pmatrix} \in \mathbb{U}^{H_p} ,$$

is the input vector time step k = 1 to $k = k + H_p - 1$,

$$\mathbf{U_{low}} = \begin{pmatrix} U_{low}(k) \\ U_{low}(k+1) \\ \vdots \\ U_{low}(k+H_p-1) \end{pmatrix} \in \mathbb{U}^{H_p} ,$$

is the lowest setpoint at which the chiller can operate and

$$\mathbf{U_{up}} = \begin{pmatrix} U_{up}(k) \\ U_{up}(k+1) \\ \vdots \\ U_{up}(k+H_p-1) \end{pmatrix} \in \mathbb{U}^{H_p} ,$$

is the highest setpoint at which the chiller can operate. From (3.1), it can be seen that all the constraints formulated as inequalities are in the decision variable, whereas here the decision variable for the objective function is \mathbf{U}_1 but the constraints are in variable \mathbf{X}_2 . In this case either the objective function has to be formulated in \mathbf{X}_2 or the constraints have to be formulated in \mathbf{U}_1 .

3.3 Formulation of the objective function

In this section three different methods of defining the objective function are discussed.

- 1. Method 1: Objective function with U_1 as decision variable and constraints transformed from variable X_2 to U_1
- 2. Method 2: Objective function with X_2 as decision variable
- 3. Method 3: Objective function with real price vector

The cost vector can be the real price or some virtual cost vector. The price vector has the real price in /kWh. The methods 1 and 2 work with a virtual cost vector, whereas method 3 works with a real price vector.

The objective function in terms of \mathbf{U}_1 is defined as

$$J_1 = \mathbf{W_1} \cdot \mathbf{U_1} , \qquad (3.3)$$

subject to

$$U_{low} \leq U_1 \leq U_{up} ,$$

where,

$$\mathbf{W}_{1} = (W_{1}(k) \ W_{1}(k+1) \dots W_{1}(k+H_{p}-1)) \in \mathbb{R}^{H_{p}}$$

is the cost vector that has the costs for achieving the setpoints U_1 , considering the price for electricity, renewable energy output. This cost vector does not reflect the real price of the electricity but rather a virtual price.

Here the constraint on X_2 are not specified. It has to be included in the constraints on U_1 . How can this be done ?

To answer this question, first a set **N** is defined which contains points from the space \mathbb{U}^{H_p} . The entries in **N** are the points from \mathbb{U}^{H_p} which when given to the chiller does not

violate the constraints on the room temperature over the prediction horizon H_p . For the condition when the constraint is not active, this set **N** would have all points in the space \mathbb{U}^{H_p} . For a $H_p=2$, the convex hull of **N** denoted by $conv(\mathbf{N})$ is shown in Fig. 3.1 which is a square.



Fig. 3.1: Convex hull of N without constraints on X_2

Now the constraints in \mathbf{X}_2 are considered, depending on the ambient temperature and the present state of the room temperature some points in \mathbb{U}^{H_p} could not be applied as it would violate the constraint on the room temperature. This would change the shape of the convex hull from a square as seen in Fig. 3.1 to a different one. To find out which points from the space \mathbb{U}^{H_p} does not violate the constraints, equation (2.11) is computed iteratively for each time step with all the values from U. With 'D' number of discrete levels which the input U_1 can take, a prediction horizon of H_p gives D^{H_p} possible input combinations. Here the set U has 12 discrete levels which means for a $H_p = 2$, there are 144 different combinations in \mathbf{U}_1 . Fig. 3.2 show the convex hull $conv(\mathbf{N})$ of \mathbf{N} at a time step k with active constraint on \mathbf{X}_2 . It can be seen that the shape of the polygon is no more a square. The vertices of this polygon is the feasible set of solution for the objective function (2.11)



Fig. 3.2: Convex hull of ${\bf N}$ with constraints on ${\bf X_2}$

In (3.3), $\mathbf{U}_1 \in conv(\mathbf{N})$, where $conv(\mathbf{N}) \in \mathbb{U}^{H_p}$. In this way now the constraints in \mathbf{X}_2 have been transformed to constraints in \mathbf{U}_1 .

3.3.2 Method 2

The method of obtaining the convex hull as explained in section 3.3.1 becomes complicated for higher dimensional spaces, that is for higher values of H_p . To overcome this problem, the cost function in this section is formulated as a function of the room temperature.

$$J_2 = \mathbf{W}_2 \cdot \mathbf{X}_2 , \qquad (3.4)$$

where,

$$\mathbf{W}_{2} = (W_{2}(k) \ W_{2}(k+1) \dots W_{2}(k+H_{p}-1)) \in \mathbb{R}^{H_{p}}$$

is the cost vector that has the costs for achieving the desired room temperature \mathbf{X}_2 , considering the price for electricity, renewable energy output. This cost vector does not reflect the real price of the electricity but rather a virtual price. The decision vector

containing the optimum room temperature for each time is,

$$\mathbf{X_2} = \begin{pmatrix} X_2(k) \\ X_2(k+1) \\ \vdots \\ X_2(k+H_p-1) \end{pmatrix} \in \mathbb{R}^{H_p}$$

The constraints are given by

$$\mathbf{X_{low}} \le \mathbf{X_{2}} \le \mathbf{X_{up}},$$

$$\begin{pmatrix} X_{2}(k) - X_{2}(k+1) \\ X_{2}(k+1) - X_{2}(k+2) \\ \vdots \\ X_{2}(k+H_{p}-1) - X_{2}(k+H_{p}) \end{pmatrix} \le \begin{pmatrix} P(k) \\ P(k+1) \\ \vdots \\ P(k+H_{p}-1) \end{pmatrix},$$

where, \mathbf{X}_{low} and \mathbf{X}_{up} are the vectors giving the lower and upper limits of the room temperature at each time step, the first values of which are $T_r(k)$. The second inequality constraint is required to consider the power constraint of the chiller. Therefore a limit on the difference between the room temperatures at the present time step and the next time step is specified as variable P(k). Here for simplicity all the elements in the vector $(P(k) \dots P(k + H_p - 1))^T$ are considered to have the same value, which can be obtained from the heat transfer equations mentioned in chapter 2 or by analysis of the energy data from the chiller and corresponding room temperature data if available.

The equation (2.11) is rearranged in terms of the state variables X_1 and X_2 to obtain the chiller set point U_1 as

$$U_1(k) = \frac{X_2(k+1) - a_{22} \cdot X_2(k) - a_{21} \cdot X_1(k) - b_{22} \cdot U_2(k)}{b_{21}}$$
(3.5)

$$X_1(k+1) = a_{11} \cdot X_1(k) + a_{12} \cdot X_2(k) + b_{11} \cdot U_1(k) + b_{12} \cdot U_2(k) .$$
 (3.6)

The value $X_2(k+1)$ from the decision vector \mathbf{X}_2 obtained by solving the optimization problem defined by (3.4) is substituted in (3.5) to compute the input U_1 to be applied to the chiller at the present time step.

Constant reference temperature

In the case where the electricity price is always constant and there is no self generation, the lowest cost of operation of the chiller unit would be achieved with the highest setpoint, such that the room temperature is always maintained at the upper end of the comfort requirements.

The linear inequalities to the cost function from (3.4) can be defined in a way that the decision vector has the same value for all the elements. The upper and lower bounds should be set to the same value which is the upper bound of the room temperature. In this case the solution to the optimization problem (3.4) is always the upper limit of the room temperature. The chiller set point is obtained by substituting this value in (3.5).

Assuming that the maximum allowable room temperature is $25^{\circ}C$, the decision vector will be

$$\mathbf{X_2} = \begin{pmatrix} 25\\25\\\vdots\\25 \end{pmatrix}$$

This approach is certainly not the best approach due to the following reasons. $\mathbf{W_2}$ depends on different variables

In a practical case where the building has a renewable energy source such as a solar plant, the cost vector \mathbf{W}_2 will depend on the following variables:

- Output E_{PV} of the solar plant
- Cost of electricity C_{ele} purchased from the utility
- Ambient temperature T_a
- Occupancy rate O_r of the building

The utilization of the self generation from solar plant has the highest priority and hence the room is cooled to its minimum allowed temperature when E_{PV} is at its maximum. With a fixed reference vector R this would not be possible. Also it is a good economic strategy to pre-cool the room when C_{ele} is low and stop cooling when it is high.

3.3.3 Method 3

In the previous sections the cost function J was defined such that the weights vector \mathbf{W} did not reflect the real cost of the energy purchased from the grid or from the solar plant. Here, the cost function J is defined such that both the public electric grid and the solar plant are considered as external power suppliers supplying electricity at a certain

kWh.

$$\mathbf{J}_3 = \mathbf{W}_{\mathbf{g}} \cdot \mathbf{E}_{\mathbf{g}} + \mathbf{W}_{\mathbf{s}} \cdot \mathbf{E}_{\mathbf{s}} , \qquad (3.7)$$

where,

$$\mathbf{W}_{\mathbf{g}} = (W_g(k) \ W_g(k+1) \dots W_g(k+H_p-1)) \in \mathbb{R}^{H_p},$$

is the cost vector of length H_p containing the costs of one unit of electricity purchased from the electric grid,

$$\mathbf{W}_{\mathbf{s}} = (W_s(k) \ W_s(k+1) \dots W_s(k+H_p-1)) \in \mathbb{R}^{H_p}$$

is the cost vector of length H_p containing the costs of one unit of electricity purchased from the solar plant,

$$\mathbf{E}_{\mathbf{g}} = \begin{pmatrix} E_g(k) \\ E_g(K+1) \\ \vdots \\ E_g(k+H_p-1) \end{pmatrix} \in \mathbb{R}^{H_p} ,$$

is the decision vector of length H_p containing the amount of electricity to be purchased from the electric grid at each time step and

$$\mathbf{E_s} = \begin{pmatrix} E_s(k) \\ E_s(K+1) \\ \vdots \\ E_s(k+H_p-1) \end{pmatrix} \in \mathbb{R}^{H_p} ,$$

is the decision vector of length H_p containing the amount of electricity to be purchased from the solar plant at each time step.

Equation (3.7) is subjected to the following inequalities :

$$\mathbf{E_{gl}} \le \mathbf{E_g} \le \mathbf{E_{gu}} \tag{3.8}$$

$$\mathbf{E_{sl}} \le \mathbf{E_s} \le \mathbf{E_{su}} \tag{3.9}$$

$$E_{min} \le \sum \mathbf{E_g} + \mathbf{E_s} \le E_{max} \tag{3.10}$$

$$\mathbf{E}_{\mathbf{kmin}} \le \mathbf{E}_{\mathbf{k}} \le \mathbf{E}_{\mathbf{kmax}} \tag{3.11}$$

 $\mathbf{E_{gl}}$ and $\mathbf{E_{gu}}$ in (3.8) are the vectors of length H_p containing the lower and upper limits

for the electricity to be purchased from the electric grid.

 $\mathbf{E_{sl}}$ and $\mathbf{E_{su}}$ in (3.9) are the vectors of length H_p containing the lower and upper limits for the electricity to be purchased from the Solar plant.

Equation (3.10) gives the upper and lower bounds for the electricity that has to be purchased from the electric grid and the solar plant over the entire prediction horizon from time step k to $(k + H_p - 1)$. E_{min} and E_{max} are the values of electricity needed if the chiller is operated at its highest and lowest setpoints respectively from time step k to $(k + H_p - 1)$. The total amount of electricity that has to be purchased from the electric grid and the solar plant from time step k to $(k + H_p - 1)$ is given by $\sum \mathbf{E_g} + \mathbf{E_s}$.

Equation (3.11) gives the upper and lower bounds for the electricity to be purchased at each time step. These upper are lower bounds are corresponding to the electrical energy required by the chiller unit to maintain the room temperature within the limits, where

$$\mathbf{E_k} = \mathbf{E_g} + \mathbf{E_s}$$

is a vector of length H_p containing the sum of electricity to be purchased from the electric grid and solar plant at each time step,

$$\mathbf{E_{kmin}} = \begin{pmatrix} E_{kmin}(k) \\ E_{kmin}(k+1) \\ \vdots \\ E_{kmin}(k+H_p-1) \end{pmatrix} \in \mathbb{R}^{H_p} ,$$

is a vector of length H_p containing the lower bounds of electric energy required by the chiller to maintain the room temperature at its maximum allowable limit at each time step and

$$\mathbf{E_{kmax}} = \begin{pmatrix} E_{kmax}(k) \\ E_{kmax}(k+1) \\ \vdots \\ E_{kmax}(k+H_p-1) \end{pmatrix} \in \mathbb{R}^{H_p} ,$$

is a vector of length H_p containing the upper bounds of electric energy required by the

chiller to maintain the room temperature at its minimum allowable limit at each time step.

The electricity consumption of the chiller is dependent on the setpoint of the chiller. Therefore the vectors $\mathbf{E}_{\mathbf{kmax}}$ and $\mathbf{E}_{\mathbf{kmin}}$ are specified in terms of the chiller setpoint by (2.1). The maximum and minimum allowable chiller setpoints are obtained as mentioned in section (3.3.1).

The vector $\mathbf{E}_{\mathbf{k}}$ obtained as a solution to linear optimization problem is used to compute the chiller setpoints by (2.1).

4. TESTING

The different methods for the supervisory controller described in the previous chapters have to be tested on the model system developed in chapter 2. Since the model has been implemented in the Simulink environment, the supervisory controller is also designed in Simulink.

4.1 Controller design in Simulink

An user defined function can be implemented as a S-function in Simulink. The S-function can be written in MATLAB, C, C++ and Fortran. In this thesis the S-function for the controller is written in MATLAB. 1-level S-function is used. Three different S-function for the three methods described has been developed. The algorithms of the S-functions for the different methods are given below:

4.1.1 Method 1

As described in section 3.3.1, the objective function here is defined in terms of the chiller setpoint.

- 1. Get the value of the state vector \mathbf{X} at the current time step k
- 2. Obtain the set **N** containing points from \mathbb{U}^{H_p} by solving (2.11)
- 3. Get the convex hull of N using 'convhull' command in MATLAB.
- 4. Solve (3.3) with the set $conv(\mathbf{N})$.
- 5. Find the minimum value of the resulting vector and the point in $conv(\mathbf{N})$ corresponding to the index of the minimum value is the optimum solution point.
- 6. Give the first coordinate of the optimum solution point as the setpoint.
- 7. Repeat for the next time step.

4.1.2 Method 2

The objective function is defined in terms of the room temperature and the constraints are in room temperature.

- 1. Get the value of the state vector \mathbf{X} at the current time step k.
- 2. Formulate the objective function, the equality and inequality vectors of length H_p from (3.4)
- 3. Solve the objective function (3.4) subject to the constraints using 'linprog' command in MATLAB.
- 4. Compute (3.5) with the X_2 obtained as a solution from the previous step.
- 5. Apply the first value of the resulting vector as the setpoint.
- 6. Repeat for the next time step.

The command 'linprog' requires all the inequalities and the equalities as vectors. The first entries of \mathbf{X}_{low} and \mathbf{X}_{up} are set to the value $X_2(k)$ which is the present value of the room temperature. The other values are set according to the ambient temperature.

4.1.3 Method 3

The cost function here is defined with the electricity used by the chiller as the decision variable and real electricity price as the cost vector.

- 1. Get the value of the state vector \mathbf{X} at the current time step k.
- 2. Formulate the objective function, the equality and inequality vectors of length H_p from (3.7)
- 3. Solve the objective function (3.7) subject to the constraints using 'linprog' command in MATLAB.
- 4. Compute the chiller setpoint from (2.1) by substituting the result from the previous step.
- 5. From the resulting vector of the previous step apply the first value to the chiller.
- 6. Repeat for the next time step

4.2 Simulation results

The simulation has been performed for different scenarios. Each scenario has been simulated for a period of seven days. For the simulation, the forecasts of the weather and solar power are needed. The performance of the three methods under the different scenarios will be compared with respect to a baseline based on the following criteria:

- Criterion 1: Maintaining the room temperature within constraints
- Criterion 2: Electricity used by the chiller
- Criterion 3: Total cost of operating the chiller

4.2.1 Scenario 1

In this scenario the chiller is considered to be oversized. The weather and solar energy data used for the simulation are shown in Fig. 4.1 and Fig. 4.2. The temperature data here depicts the ambient temperature profile in summer season.



Fig. 4.1: Weather forecast used for scenario 1



Fig. 4.2: Solar power generation forecast used for scenario 1

The forecast for the solar power is based on a typical bell shaped curve resembling the typical solar irradiation over a day. The power level of the solar plant is chosen to be higher than the full load of the chiller, such that the solar plant can supply the chiller on a stand alone mode when the irradiation is high.



Fig. 4.3: Electricity price scheme used for scenario 1

A variable cost for electricity is considered as shown in Fig. 4.3. The lowest tariff is $0.22 \ kWh$ during off peak hours, normal tariff is $0.24 \ kWh$ and during the peak hours it is $0.26 \ kWh$.



Fig. 4.4: Virtual cost vector

The cost vectors $\mathbf{W_1}$ and $\mathbf{W_2}$ are subsets of a virtual cost vector defined for the during of the simulation time. The virtual cost vector is shown in Fig. 4.4. The virtual cost vector defined here depends on the real price of electricity and the solar power generation forecast. It can be seen in the figure that the values that the elements of the virtual cost vector can take are -1, 0 or 1. -1 is during the case when there is sufficient solar power generation or when the electricity price is at its off peak. 0 is during the case when the electricity price is normal and there is no solar power. 1 is during the case when the electricity price is at its peak value and when there is no solar power generation.

Criterion 1

A performance indicator β , is defined to asses the performance of the different methods according to criterion 1.

$$\beta = 1 - \frac{\text{Number of time steps when } T_r(k) \ge 1.02 \cdot X_{up}(k)}{\text{Total number of time steps}} , \qquad (4.1)$$

where $T_r(k)$ is the room temperature and $X_{up}(k)$ is the upper limit of the room temperature at the present time step. A threshold of +2 % for the upper limit is assumed.

Baseline

As a baseline for scenario 1 the chiller set point is maintained at a constant value of $10^{\circ}C$.



Fig. 4.5: Baseline simulation with a constant chiller setpoint at $10^{\circ}C$

Method 1

The simulation result in Fig. 4.6 shows the performance of the supervisory controller based on method 1 for a $H_p = 2$. It can be seen from the figure that the supervisory controller sets the setpoint to the lowest possible value when the corresponding value of the virtual cost vector is -1 or 0 and the highest possible value when the value is 1. The room temperature is also within the constraints.



Fig. 4.6: Scenario 1: Method 1 supervisory controller performance

The simulation result in Fig. 4.6 shows the performance of the supervisory controller based on method 2 for a $H_p = 8$. Similar to the performance of method 1, it can be seen from the figure that the supervisory controller sets the setpoint to the lowest possible value when the corresponding value of the virtual cost vector is -1 or 0 and the highest possible value when the virtual cost vector is 1. The room temperature is also within the constraints.



Fig. 4.7: Scenario 1: Method 2 supervisory controller performance

Comparing Fig. 4.7 with Fig. 4.6, the difference in the setpoint given by both the methods can be seen. The setpoint given by method 1 was either 5 or 10, where as the setpoint given by method 2 takes different values from the set \mathbb{U} . This is because the objective function is in terms of the room temperature and the chiller setpoint is calculated from the room temperature set over the prediction horizon. The room temperature at the solution of (3.4) can be seen in Fig. 4.7 mentioned as 'Tr-setpoint'.

Method 3

Method 3 is different from the first two methods since it uses real price for electricity instead of the virtual cost as used in the methods 1 and 2. The simulation result in Fig. 4.8 shows the performance of the supervisory controller based on method 3 for a $H_p = 8$.



Fig. 4.8: Scenario 1: Method 3 supervisory controller performance

Comparing with methods 1 and 3 the setpoints given method 3 is most of the times at the upper end. This is because of the real electricity price. Since there is always a price associated with purchasing electricity from the gird, the controller decides to purchase the least amount of energy required to maintain the room temperature within limits. The controller chooses the lowest chiller set point when there is sufficient solar energy or during off peak times. It can be seen from Fig. 4.8, the room temperature is maintained closer to the upper limit compared to the first two methods. On the 6th day highlighted by a black square, when there is not sufficient solar energy the setpoint is at the upper end and it is set to the lowest setpoint when the electricity price is at the lowest value.

Criterion 2

The comparison of total electricity consumed by the chiller when controlled by the three methods with respect to the baseline is shown in Fig. 4.9. It can be seen that the electricity consumption of the chiller at the baseline scenario is the lowest, this is because the chiller is always operating at the highest possible setpoint. Among the three methods, method 3 has the best energy performance which is around 1.8 times that of



the baseline followed by method 2 which is around 2.14 times that of baseline. Method 1 is around 2.24 times that of baseline value.

Fig. 4.9: Scenario 1: Comparison of electricity consumption of the chiller among the three methods

Criterion 3

The performance of the different methods with respect to the price of electricity from the gird and solar plant is shown in Fig. 4.10. Here the price of one unit of electricity from the solar plant is assumed to be 0.10 \$, which is considered to be the feed-in tariff provided by the public grid. In case where there is no feed in tariff, the price for the electricity from the solar plant can be considered as 0. The performance of the controllers are similar to that in criterion 2. The baseline has the best value followed by method 3. The comparison of the different methods according to the criteria is summarized in Tab. 4.1



Fig. 4.10: Scenario 1: Performance of the methods with respect to the electricity price

		J F		
	Baseline	Method 1	Method 2	Method 3
Criterion 1: β value	1	1	1	1
Criterion 2: Total kWh	488	1095	1048	876
Criterion 3: Total \$	98	221	213	167

Tab. 4.1: Scenario 1: Summary of performance

4.2.2 Scenario 2

For scenario 2 the size of the room is increased, such that the cooling load on the chiller is increased. The chiller is considered to be sized rightly for the room, which is closer to the practical cases than scenario 1. The ambient temperature profile, solar power forecast and the virtual cost vector remains the same as from scenario 1. The upper limit for T_r is considered to be a constant value of $26^{\circ}C$ at all time steps.

Criterion 1

Baseline

As a baseline for scenario 2 the chiller set point is maintained at a constant value of 5°C. Fig .4.11 show the result of the simulation at the baseline case. It can be seen that the room temperature exceeds the limits at certain time steps because of the increased cooling load on the chiller. However, the β is 1 for the baseline case, since T_r does not go beyond the threshold at any time step. Since the baseline case is working at the lowest possible setpoint at all time steps and still not able to maintain the room temperature within limits, one can say that the other methods would also fail to keep the room temperature within limits during all the time steps. However, since this is more likely the scenario in real applications, it is interesting to see the performance of the different methods.



Fig. 4.11: Baseline simulation with a constant chiller setpoint at $5^{\circ}C$

The performance of method 1 supervisory controller for a $H_p = 2$ is shown in Fig. 4.12. The room temperature is maintained on the upper limit and it exceeds the upper limit at certain time steps but the the β is 1 similar to the baseline. Compared to Fig. 4.12, here the setpoints given by the chiller on the lower limit most of the times as expected.



Fig. 4.12: Scenario 2: Method 1 supervisory controller performance

Method 2

The performance of the method 2 supervisory controller for a $H_p = 8$ is shown in Fig. 4.13. As in method 1, the room temperature is maintained on the upper limit and it exceeds the upper limit at certain time steps but the β value is 1.



Fig. 4.13: Scenario 2: Method 2 supervisory controller performance

The performance of the method 3 supervisory controller for a $H_p = 8$ is shown in Fig. 4.14. From the figure it can be seen that the setpoints look lot different than the first two methods. Compared to Fig.4.8, here the chiller operates at its lowest values not only when sufficient solar power is available, but also during other time steps when the room temperature is near the upper limit. As a result the room temperature is almost always maintained on the upper limit and it exceeds the upper limit at certain time steps. The β value is 0.9911 for this method. In this the first two methods marginally perform better than method 3.



Fig. 4.14: Scenario 2: Method 3 supervisory controller performance

Criterion 2

The comparison of total electricity consumed by the chiller when controlled by the three methods with respect to the baseline is shown in Fig. 4.15. Contrary to scenario 1, the electricity consumption of the chiller at the baseline scenario is the highest since the chiller is always operating at the lowest possible setpoint. All the three methods perform better than the baseline. Among the three methods, method 3 has the best energy performance which is around 0.76 times that of the baseline followed by method 2 which is around 0.91 times that of baseline. Method 1 is around 0.92 times that of baseline value.



Fig. 4.15: Scenario 2: Comparison of electricity consumption of the chiller among the three methods

$Criterion \ 3$

The performance of the different methods with respect to the price of electricity from the grid and solar plant is shown in Fig. 4.16. Method 3 has the best price performance followed by method 1 and method 2. One may expect the method 2 to have a better price performance than method 1 since the electricity consumed by method 2 is lower than method 1. This is due to the fact that the method 2 gives setpoints at the lowest level for more time steps than method 1. The comparison of the different methods according to the criteria is summarized in Tab. 4.2



Fig. 4.16: Scenario 2: Performance of the methods with respect to the electricity price

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	Baseline	Method 1	Method 2	Method 3
Criterion 1: β value	1	1	1	0.9911
Criterion 2: Total kWh	1491	1368	1352	1130
Criterion 3: Total \$	308	276	282	221

Tab. 4.2: Scenario 2: Summary of performance

From the summary, one can see the method 3 supervisory controller performs better than the other methods even though its β value is not 1.

4.2.3 Scenario 3

In scenario 3, the performance of the different methods under conditions where the system is subjected to disturbance are simulated and compared. The disturbance in the system may be due to heat gains such as leaving a window open, adding an extra load or increased occupancy. There may also be error in the prediction of the future states due to modeling errors or due inaccurate forecasts. The disturbance is added to the

feedback to the supervisory controller. The system remains the same as in scenario 2. The disturbance profile is shown in Fig. 4.17.



Fig. 4.17: Disturbance profile added to the feedback

Criterion 1

Baseline

The baseline is the same as in scenario 2, the set point is maintained at $5^{\circ}C$. As seen in Fig. 4.18 the room temperature exceed the upper limit at certain time steps due to the disturbance and the β value is no more 1. Due to the disturbance, the β value is 0.9152.



Fig. 4.18: Scenario 3: Baseline simulation with a constant chiller setpoint at $5^{\circ}C$

From the Fig. 4.19, it can be seen that setpoints given remain as in scenario 2 for most of the time steps. In Fig. 4.12 on the 5th day which can be identified by the 5th peak on the ambient temperature, the setpoint is increased to $8^{\circ}C$ when the virtual cost is 1, whereas at the same time step in Fig. 4.19 the setpoint remains at $5^{\circ}C$ which indicates that, the supervisory controller adjusts the setpoints according to account for the disturbance. The β value is 0.8869.



Fig. 4.19: Scenario 3: Method 1 supervisory controller performance

The observation made for method 1 also applies here. On the 5th day, the setpoint remains at $5^{o}C$ in Fig. 4.20, whereas in Fig. 4.13 it is about $8^{o}C$. The β value is 0.8929.



Fig. 4.20: Scenario 3: Method 2 supervisory controller performance

Comparing Fig. 4.14 and Fig. 4.21, it can be seen the major difference in the setpoints occur during the last three days when the ambient temperature is higher than the other days. The β value is 0.8601 which is the lowest among all the case.



Fig. 4.21: Scenario 3: Method 3 supervisory controller performance

$Criterion\ 2$

As one would expect, with the introduction of disturbance, the chiller consumes more electricity than scenario 2 for all the methods. The interesting indicator here, is how much the consumption increases with respect to the corresponding values at scenario 2. For the baseline there is 2.15 % increase. For methods 1, 2 and 3 the consumption increases by 3.51 %, 4.07 % and 10.44 % respectively. Fig. 4.22 shows the energy performance of the different methods at scenario 3.



Fig. 4.22: Scenario 3: Comparison of electricity consumption of the chiller among the three methods

$Criterion \ 3$

As with criterion 2, the price paid for the electricity increases under all the methods with method 3 having the greatest increase as in criterion 2. The price performance of the different methods is given in Fig. 4.23. The comparison of the different methods according to the criteria is summarized in Tab. 4.3



Fig. 4.23: Scenario 3: Performance of the methods with respect to the electricity price

Tab. 4.5. Scenario 5. Summary of performance								
	Baseline	Method 1	Method 2	Method 3				
Criterion 1: β value	0.9152	0.8869	0.8929	0.8601				
Criterion 2: Total kWh	1523	1416	1407	1248				
Criterion 3: Total \$	315	289	293	249				

Tab. 4.3: Scenario 3: Summary of performance

4.2.4 Effect of H_p

The prediction horizon H_p plays an important role in the setpoints given by the supervisory controller as it decides how far the controller is looking into the future. A large H_p would lead to high computational time and a small H_p may not be the most economical.

As mentioned before the supervisory controller based on method 1 has been designed for a $H_p = 2$. If the prediction horizon is increased the convex hull of the set **N** will vary. This is because, with an increase in H_p , the number of points in \mathbb{U}^{H_p} will decrease. The supervisory controllers based on method 2 and method 3 have been designed to work for different prediction horizons. The simulations for different scenarios were done for a $H_p = 8$. Now, the H_p is changed to 4 and scenario 2 is simulated again.



Fig. 4.24: Method 3: Effect of H_p

The effect of varying the H_p on the setpoints given by supervisory controller based on method 3 is shown in Fig. 4.24. The highlighted region shows the time steps when the supervisory controller decides to cool down in case of $H_p = 4$ and wait until there is sufficient PV output in case of $H_p = 8$.

		<i>p</i>
	$H_p = 4$	$H_p = 8$
Criterion 1: β value	0.9955	0.9911
Criterion 2: Total kWh	1178	1130
Criterion 3: Total \$	233	221

Tab. 4.4: Scenario 2: Effect of H_n

From Tab. 4.4 it can be observed that with an higher H_p the energy consumption is low and also the economical performance is better. This can be explained with an example. Considering the $H_p = 4$, with a time step of 15 minutes, this would mean the controller is looking 1 hour into the future. Now, if the solar PV forecast is such that from 10 a.m. the solar output is high enough to operate the chiller at the lowest setpoint. With $H_p = 4$, at 8 a.m., the controller would not see the solar output at 10 a.m. and may decide to cool down the room. If the $H_p = 8$, at 8 a.m., the controller would see that the solar output is high at 10 a.m. and it would delay the cooling to this point. This would explain why a small H_p may not have the best economical performance. However, the β value for $H_p = 4$ is marginally better.



Fig. 4.25: Method 2: Effect of H_p

From Fig. 4.25, it can be seen that the H_p does not have a big influence on the setpoints given by supervisory controller based on method 2. This is because the objective function of method 2 is formed with the room temperature as the decision variable and a virtual cost vector is used. The value of the room temperature obtained as a solution to the optimization problem depends only on the value of the virtual cost vector at the current time step.

5. CONCLUSION

A supervisory controller has been designed which provides optimum setpoints for a chiller unit considering the price of electricity and output from a PV plant. This problem has been formulated as a standard linear optimization problem. Three different methods have been proposed and three different supervisory controllers based on these methods have been designed and tested.

5.1 Results

- The results show that the supervisory controller designed based on method 3 has the best economic performance.
- The supervisory controller performs well on a rightly sized chiller unit. If it is oversized it can be set to operate at the highest possible set point for the lowest economic performance and if it is undersized the supervisory controller cannot keep the room temperature within constraints.
- All the methods fail to keep the room temperature within constraints when the system is subjected to disturbance because of the increased cooling load, however, all the methods changed the input trajectory given to the chiller when the disturbance is introduced.
- The performance of the supervisory controller can be tuned by varying the prediction horizon H_p .

5.2 Limitations

- The model built in this thesis is not validated due to lack of data.
- The state space model of the building energy system is valid only for the conditions when the chiller is in operation since the physical model has been linearized about this operating point. Therefore the supervisory controller is not turning off the chiller because the test model is not valid for such a condition. This is the reason why in scenario 1, even though the chiller is oversized, the supervisory controller does not turn off the chiller.

- For simplicity, single zone thermal model is used in this thesis. The working of the supervisory controller remains the same for a detailed building model.
- The electricity consumption of the chiller is based on a simple approximation of the heat rejected by the chiller.
- The building energy system is considered to have only one load and two sources.

5.3 Recommendations for further work

A basic framework for a supervisory controller has been provided in this thesis. The adaptability of the methods when other loads and sources of the building energy system are included should be further studied. Following chapter 3, one should be able to formulate objective function and constraints to include other loads and sources. In this thesis the setpoint of the chiller is considered to be the chilled water temperature, however in practice there might be other variables which could be setpoints to the chiller and the methods described in this thesis could be adapted to provide setpoints other than chilled water temperature as a setpoint. Also an accurate model for the chiller power consumption based on the chiller setpoint is needed.

6. OUTLOOK

This thesis will be used as a basis for further research in supervisory control problems at Envidatec GmbH. It has been planned to adapt the methods developed in this thesis to include other loads and sources in subsequent bachelor and master works resulting in a modular supervisory controller in which prediction models for different loads and sources can be added or removed to suit a particular application.

Simultaneously the incorporation of the supervisory control system as a feature or a tool box of the JEV is energy monitoring software will be studied. The JEV is system is the monitoring software of the Envidatec GmbH, which has been specifically developed for the evaluation and processing of energetically and operationally relevant process variables. The JEV is system can communicate with a variety of data sources, read-out data and control the analysis and visualization of the processed data. The captured data are stored in a database within the JEV is system. Initially it has been proposed to develop an interface between JEV is and Matlab and use the Matlab code developed in this thesis with certain adaptations as needed. This is an offline optimization setup where the JEV is system transfers the setpoints computed by Matlab and the Matlab models use data from the JEV is system. The performance requirements for such an intermediate setup will be studied. Later the possibilities of incorporating the Matlab based supervisory controller to a setup native to the JEV is system and its performance requirements will be identified.

One immediate application at Envidatec GmbH is the on going research project 'Green Power Efficiency (GPE)' funded by the German Ministry of Commerce. GPE aims at providing cost effective energy and water supply through operational optimization of solar thermal facilities. The limited access to electricity and drinking water is a key issue in the development of many regions worldwide. The use of solar energy by local solar thermal power plants and desalination plants as a solution to this problem has been observed. Despite the elimination of fossil fuels, the full potential of the solar plants have not yet been tapped, because of the absence of solutions for the safe and simultaneously energy efficient operation in practice on the spot. This prevents demand based dimensioning and leads to high capital and operating costs. The aim of this project is a new management system that facilitates a cost efficient use solar energy for electricity and water supplies. The approach is based on optimizing the efficiency by forecasting the load volume and energy input. It uses an estimation method, which allows for both the assessment of the energy status and the predictive detection of defects with minimum additional equipments. The system is largely neutral to the plant and is thus particularly suitable for retrofitting. The suitability of this thesis to the GPE project will be studied.

Apart from the R&D applications mentioned above, the results from this thesis can also be utilized as a value added service to the customers of Envidatec GmbH, especially in the south east Asia region. South east Asia is a dynamic and a fast growing region facing huge challenges towards sustainability. To address this, Envidatec GmbH has been active in the south east Asia region, especially in Thailand and Vietnam. A branch of Envidatec GmbH has been recently established in Bangkok. Several commercial buildings with high energy costs have expressed interests in the services offered by Envidatec. A common trait identified in these buildings has been the huge cooling costs partly due to inefficient operation. The suitability of this thesis for such commercial buildings in this region will be studied in a series of pilot projects.

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